Appendix B

Table 5. Tests for Lag Length

a) Successive three-year averages

Model	*l*	*AIC*	*L-statistic*	*df*	*p-value*
Investment equation	3	6.99	0.25	1	0.617
	2	5.84	0.14	1	0.708
	1	4.65	–	–	–
Export share equation	3	1.08	0.11	1	0.741
	2	1.07	0.01	1	0.920
	1	0.83	–	–	–
Labor productivity growth equation	3	1.13	0.22	1	0.639
	2	1.37	0.09	1	0.764
	1	1.20	–	–	–

b) Annual Data

Dep. Var.	*Indep. Var.*	*l*	*AIC*	*L-statistic*	*df*	*p-value*
Exports	Investment	4	1.08	0.01	2	0.995
Exports	Investment	3	1.09	–	–	–
Investment	Exports	4	2.82	1.69	2	0.429
Investment	Exports	3	1.13	–	–	–
Labor prod.	Exports	4	7.59	0.17	2	0.918
Labor prod.	Exports	3	7.76	–	–	–
Exports	Labor prod.	4	1.01	0.08	2	0.961
Exports	Labor prod.	3	1.08	–	–	–

Notes: AIC is the Akaike information criterion. *L*-statistic is defined as restricted sum of squared residuals minus the unrestricted sum of squared residuals, and it has a X^2 distribution under the null hypothesis of linear restrictions on lag lengths. The table shows the results for the sequence lag length **1** = 3 (versus 2), and l = 2 (versus 1). *df* is the degrees of freedom.

Sectoral Public Power and Endogenous Growth

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Abstract: We study growth in an economy composed of sectors producing specific goods with advantage under fragmented competition. The government allocates public inputs. Sectorial political power defines government objectives and restrictions, and consists of passive resistance (bounding taxation), organized resistance (an effective minimum welfare demand), and socially organized power (pushing Sectorial objectives). Income distribution and growth, mechanisms and incentives for public investment allocation, and political organization incentives, are strikingly different functions of input dependence and political power in open and closed economies. Long-term political economy equilibria and tendencies in political transition due to technical or trade policy changes can be modeled.

Resumen: Estudiamos el crecimiento en una economía compuesta de sectores que producen bienes específicos con ventajas, bajo competencia fragmentada. El gobierno asigna insumos públicos según objetivos y restricciones, funciones del poder sectorial, originado en resistencia pasiva (que acota impuestos), resistencia organizada (demanda efectiva de bienestar mínimo) y poder socialmente organizado (que propugna objetivos sectoriales). Las dependencias económicas intersectoriales y el poder político implican distribución del ingreso, crecimiento, mecanismos e incentivos de inversión pública, e incentivos de organización política muy diferentes en economías abiertas y cerradas. Pueden modelarse equilibrios político-económicos de largo plazo y transiciones políticas debidas al cambio técnico o de política comercial.

1. Introduction

This article studies growth and distribution when the differentiated access of productive sectors to the economic benefits of power is of strategic importance. Our interest is centered in market economies in which the political system is characterized by a stable balance of

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power, established between various productive sectors, which determines the economic actions of the government.

The role of public spending in an endogenous growth model was first studied by Barro [5]. Futagami, Morita and Shibata [12] consider public investment instead of spending in an endogenous growth model. Jison Lee [20] considers both in a model which finds two types of equilibria, each emphasizing one of the modes of government participation in the economy. The theoretical and empirical importance of public investment has been substantiated by several studies (see for the former Arrow and Kurz [3], for the latter Aschauer [4], Iwamoto [16]), as well as the impact of institutional structures on the provision of specific public goods (*e.g.* Gorter and Zilberman [14]). In this article we are concerned with the strategic nature of the government's participation in the economy, in the context of a balance of power between sectors. Such strategic activity is mainly reflected in investment decisions which affect distribution and growth.

The study of the political economy of growth in endogenous growth models is well established (see for example Verdier [30]). In a survey on the recent literature on the political economy of growth, Alesina and Perotti [2] find the main theoretical and empirical studies to be on the linkages between income distribution and growth; political rights, democracy and growth; savings, investment, and political instability. Instead we consider governments which embody a specific, stable balance of power between sectors based on branches of the economy. We find that the balance of power and whether the economy is open or closed characterize government economic action and determine sectoral income distribution and some aspects of growth as well as the mechanisms and criteria of public investment and the incentives for political organization.

In the context of our assumptions rent-seeking is structured in a given, non-competitive and non-democratic manner. Thus our study is somewhat more associated with corporativist than with decentralized, pluralist frameworks of interest intermediation (see Schmitter and Lehmbruch [28]). The study of rent-seeking by specific interest groups is extensive (*e.g.* Riaz, Shogren and Johnson [25]; Zhou [32]; Congleton and Bennet [10]; Sturzenegger and Tommasi [29]; Hinich and Munger [15]). In the endogenous theory of trade tariffs specific economic sectors (owners of industry-specific factors) act with political coherence to obtain the economic benefits of certain policies (*e.g.* Brainard and Verdier [9]). In the sectoral structure we define, income distribution

will result from the political power of the sectors as well as from their economic potential and independence.

The analysis of economic history often involves sectors of the economy. For example, Pipitone [24] uses the concept of economic sectors extensively in a comparative historical analysis of the success or the failure of the transition to development in England, Holand, Belgium, France, and especially in the case of the "late-comers" Sweden, Denmark, Japan, Italy, India, Nigeria, Brazil and Mexico. Likewise, Evans [11] analyses a series of case histories of development by economic sectors. Our model will clarify the nature of the long-term political and economic relationships between sectors.

Section 2 defines the concept of a sectoral political economy and its interaction with the economy. Section 3 states and solves an endogenous growth model with a sectoral economy. Section 4 discusses the government as sector; transitions due to changes in trade policy or to technical change; fragmented perfect competition; and offers some insights on EC protectionism and the US Civil War. Section 5 contains some final remarks. Sections 8, the Appendix, contains the proofs of theorems and propositions, and occasionally some additional formulae.

2. The politico-economic sectoral structure

We characterize a sectoral structure by the following assumptions, which must hold within the horizon of economic planning, which we refer to as the long term. *Politically, each sector is coherent, and the balance of power between sectors determines government economic decisions. Economically, each sector specializes in some branch of production in which it has an advantage.* Together these assumptions mean that the economic and political system is jointly structured in sectors, and that the structure provides a stable horizon of economic planning. The definition implies that *inter-sectoral* competition is weak. However, we study the case in which there is perfect *intra-sectoral* competition. Thus our market structure is *fragmented competition*. Examples of such sectors could be the agrarian and urban sectors, industrial sectors such as large and small-scale industry, diverse special interest stable lobbying groups, or capital sectors such as financial versus productive capital. We shall say "sectoral economy" rather than "sectoral political economy" for short.

The advantages each sector has in production could take many different forms, such as advantages in knowledge resulting from learning by doing, transaction costs, entry costs, imperfect credit, etc. For example, in the case of the agrarian and urban sectors, it is often the case that the access of capitalists or workers from each sector to the other sector may involve a many types of economic barriers. Similarly in the cases of large and small-scale industry, or financial versus productive capital, intersectoral competition may be limited by problems with credit, knowledge, mobility of labor, scale, entry costs and so on.

In the situations in which we are interested the government has the power to strategically influence capital accumulation. Public spending and investment are important components of production and consumption, and may consist of public goods or of private goods which are produced by the government. The government acts to further certain objectives, in our model by maximizing some function of the sector welfares, according to certain constraints. These objectives and constraints will derive from the political power of each sector. We consider for each sector three aspects of power which we shall call *the power of passive resistance, the power of organized resistance and socially organized power*. These generally correspond to increasingly complex levels of collective action. More specifically, with respect to their effects on the economic sphere the first is a sector's capacity to oppose taxation, especially in the presence of government transfers to other sectors. The second is a sector's capacity to impose upon the government constraints guaranteeing it a minimum degree of welfare. The third is a sector's capacity to include its own welfare amongst the government's objectives.

The concept of sectoral structure is quite distinct from the concept of class structure. If we proceeded to analyze the determinants of collective action which give rise to political organization and power, undoubtedly the identification between members of the same classes (by occupation, communality of economic and political interests, etc.) plays a central role. However the analysis of the space of income negotiation between labor and capital is impossible within the framework of competitive production, which allocates income shares according to the production function shares, except perhaps in the case of human capital, which can involve a public input (and is of strategic importance because as an investment it is a determinant of growth). Thus the analysis of the dynamics of power including such bargaining possibilities is much more complex. Even to model class in simpler ways simultaneously with a sectoral structure would complicate the issues.

Hence we have chosen to deal with the sectoral structure on its own. The approach taken in this study can be understood as an analysis of the dynamics of power amongst the dominant classes, as owners of different kinds of capital. Alternatively, our approach can model social sectors forming coherent units even if composed of different classes, when the capital owners play the leading roles.

Olson [23] applies the theory of collective action to analyze a wide variety of historical events, in terms of the presence of collusions and organizations. He analyses how these collusions and organizations come to have some of the properties we ascribe to sectors. "Stable societies with unchanged boundaries tend to accumulate more collusions and organizations for collective action over time." "Encompassing organizations have some incentives to make the society in which they operate more prosperous, and an incentive to redistribute income to their members with as little excess burden as possible..." "Distributional coalitions, once big enough to succeed, are exclusive and seek to limit the diversity of incomes and values of their members." (*Ibid.*, chapter 3).

2.1. Sectoral structure-definition

We consider market economies in which the following scenario exists. *For the planning horizon, the political system is characterized by a stable balance of power established between various productive sectors, which determines the economic actions of the government.* To analyse such a scenario, we define the concept of *sectoral structure* as follows.

A country has a (politico-economic) sectoral structure if it can be divided into sectors which satisfy two hypotheses:

Political coherence. Each sector forms a politically coherent unit vis a vis other sectors in the government's spending decisions, which are a function of the resulting balance of power.

Productive specialization with economic advantage. Each sector specializes in some branch of production within which it has certain economic advantages with respect to the other sectors.

We contend that this is a minimal set of assumptions compatible with our scenario. The reasons are the following. If within some branch of production some subset of its firms can access government favor while the remainder cannot, the favored subset will out-compete the remainder, so that for any long-term steady state (compatible with the economic planning horizon) all of the firms in the productive branch must have equal access to the public investment goods *(political coherence)*. Hence, if there are to be several sectors, each must be identified with a specific good (or bundle of goods), each of which is necessary for the other sectors, either for consumption or production or both *(productive specialization)*. Moreover, since sectors are thought to be at least somewhat stable with respect to the process of economic competition, there must be barriers to the encroachment of their economic activities by other sectors. In the absence of such barriers the economic basis of a weaker sector can be eroded and we would analyze instead more fluid economic and political states of affairs *(economic advantages)*.

Thus only particular subsystems of a given country may be considered sectors in the sense just defined, although some kinds of sectors (as in the case of agricultural or urban sectors) may divide into subsectors geographically or otherwise. Our analysis examines politico-economic equilibria in which various market imperfections (which may themselves derive from the political system) define productive sectors within which economic competition remains and which, for social, political or practical reasons form coherent political units on the national scene.

We abandon the concept of a benevolent government acting for the general good, instead depicting it as pursuing the interests of certain sectors according to the balance of power. However, we eschew the sources of political coherence within sectors, on the one hand, and of government coherence with sectoral objectives resulting from some sectoral balance of power on the other. Note, however, that some of this coherence (meaning equal access to the benefits of public investment within sectors) simply follows from the accessibility of public goods (which may be local not only geographically but in the sectoral sense) or of private goods supplied by local or sector-specific government investment (on human capital, health, infrastructure, energy, and so on).

Our assumptions are probably more relevant in developing countries than in democratic, developed countries in which political and economic rigidities may be less dramatic. Nevertheless, we apply some of our results to the EC and the US.

2.2. Economic effects of power

We shall describe the economy by means of an endogenous growth model. The government will be a Stackelberg leader in a game in which the remaining agents are representative families each owning capital for production in its respective sector. We shall suppose that the government's objective is to maximize a function having as arguments the utilities of representative families of each sector. The objective function is a characteristic of the current political system. It is some combination of the sectors' welfare (and could be identical to just one of these). Maximization will also be subject to some restrictions, themselves part of the characterization of the political system.

The government has two sources of economic power in our model. The first is that it can tax families. The second is that it is the only source of some kinds of public investment which some of the sectors require for production and which therefore determine their income and growth. The goods that the government invests in need not be public goods. What is necessary is that no other agent has the incentives and the capability to invest in them. The specificity of the necessary public investment goods implies that the government might favor some sectors' production over others. The government funds its investment by raising taxes from families in each sector.

When it exercises the public budget and decides on investments and tax rates, the government determines and distributes wealth amongst the sectors. If it favors one, the remainder loose. The limit to the amount of taxes it can raise from each sector, a minimum bound to the welfare of each sector, and the government's objectives are economic effects of power. These attributes close the description of the economy, and thus we focus on them to describe the political power of each sector. We define:

The power of passive resistance. The capacity of a sector to impose bounds on the tax rate paid to the government.

The power of organized resistance. The capacity of a sector to impose as a restriction on the decisions of the government a minimum level of welfare for its sector.

Socially organized power. The capacity of a sector to include its welfare in the government's objectives.

These three sets of political "data" on the political system can be seen as resulting from qualitatively increasingly complex levels of collective action and organization. We shall refer to the first two as *power of resistance* and to the last two as *organized power*.

The power of passive resistance requires simple action which need not take the form of collective action proper, nevertheless setting an effective limit on taxation. It can be based, for example, on individual tax evasion, complicity and corruption. The power of organized resistance involves collective action whose effect can be based, for instance, on the imposition of costs (an example could be trade unionism) or on political maneuvering, the resort to public opinion, protests, legal action, and so on. Such resistance need not concert the actions of the sector in a fully unified way, but may effectively set a minimum level to the sector's welfare. Finally, socially organized power reflects a degree of organization capable of channeling effectively large amounts of public resources to specific economic projects in a socially articulated and unified way.

For the purposes of this article the political structure will be considered exogenous. To endogenize it would involve a description of costs of collective action and possibly of political dynamics which are beyond the scope of this paper.

3. The model

In this section we define an endogenous growth model with a sectoral structure. To avoid the mathematical complexity of more than just a few variables, we restrict our theoretical framework to the simplest interesting case of a sectoral structure, in which it can be optimal for a less dependent sector to transfer resources to a more dependent one.

3.1. Aggregate flow

In a general sectoral economy, each sector produces a specific good which all families consume and which can also be used as private or public capital or cause externalities in the production of any good. However, we study the following simplified situation. We consider an economy in which there are only two sectors, producing goods a and b. Production in the first sector does not require capital inputs other than private

capital of good a, while in the second sector capital requirements are private capital of type b and public capital of type a. Thus the first sector does not depend on the second for capital inputs and only needs it for consumption, while the second sector also depends on the investment of public capital of type a. This scheme could for example model a process in which sector 1 pursues policies modernizing sector 2. We shall suppose that the first sector can grow independently and that the second sector can grow if the public investment it needs occurs.

The aggregate physical equations of production are

$$K_1' = \varphi^1 (K_1) - C_a - I_a - a_X \qquad (a)$$

$$K_2' = \varphi^2 (K_P, K_2) - C_b - b_X \qquad (b) \qquad (1)$$

$$K_P' = \varepsilon_P I_a \qquad (c)$$

where the variables are the following:

K_1 is the aggregate private capital of sector 1, consisting of good a.
K_2 is the aggregate private capital of sector 2, consisting of good b.
K_P is the aggregate public capital of sector 2, consisting of good a.
C_a, C_b are the aggregate private consumption of goods a and b.
I_a is the aggregate public investment of good b.
$\varepsilon_P \in [0, 1]$ is the efficiency of public investment.
a_X, b_X are the net exports of goods a and b respectively.

φ^1 and φ^2 are assumed to be homogenous functions of degree 1. Hence $\varphi^1 (K_1) = A_1 K_1$. We shall use the Cobb-Douglass form $\varphi^2 (K_P, K_2) = A_2 K_P^{1-\theta} K_2^{\theta}$ for some formulae. These are available to families of each sector for producing their corresponding goods. We do not specify the production functions available for families to produce goods corresponding to other sectors, since as they embody a disadvantage they will not be used. Thus all capital K_1, K_2 is owned by families in the corresponding sectors. We are implicitly assuming that sector 1 is a growth sector independently, while sector 2 requires public investment in good a for growth. Sector 1 is economically independent of sector 2, while sector 2 is economically dependent on sector 1, needing capital inputs form this sector. Since it is the government that provides them, it is also politically dependent.

We follow Barro [8] and Futagami, Morita, and Shibata [12] in the manner of including public capital in the production function φ^2. It exhibits constant returns to scale with diminishing returns to each factor, and the benefits of public capital are provided free of charge. However, we differ in that we consider that the government is investing in congestible goods. We let the representative per-capita production function be $\psi^2 (K_P/K_2)\ k_2$, where $\psi^2 (K_P/K_2) = \varphi^2 (K_P/K_2, 1)$ and k_2 is per-capita capital in sector 2. Thus the private production function is homogeneous of degree 1, while the effect of public capital on production depends on its ratio to the global amount of private capital, with decreasing returns. Families in sector 2 perceive the marginal returns of k_2 to be higher than they actually are since they do not take into account the decrease which occurs in $\psi^2 (K_P/K_2)$ with additional investment in k_2. Thus investment decisions in K_2 are distorted.

The coefficient ε_P includes such considerations as government inefficiency, consumption and corruption. We assume these are proportional to its investment activities, which in this case comprise its full budget.

We have excluded public services because from a sectoral point of view in which preferences are uniform their main effect is to redistribute resources (something which is already present in the model through differentiated sectoral taxation) and because they involve multiple equilibria (high public expense with low growth, or viceversa, Jison Lee [23]) which would complicate the exposition.

We have supposed that public capital can be provided by the government completely selectively. That is, the development of sector 1 has no externalities which could take the place of the public capital input for good b, and the public capital provided to sector 2 does not benefit production in sector 1. This is realistic when the productive sectors are sufficiently different for it to be possible for public policy to be discriminatory, as when their needs or location are different.

In models of endogenous growth with a single equation of state there is no a priori limit to the amount of investment which can be made in a limited time. For symmetry and simplicity, we make the same supposition in the case of public investment. This has the consequence that optimization may be enhanced by instantaneous transfers of capital which would all occur at time $t = 0$ (see Kamien and Schwartz [18], section 18 for a treatment of jumps in state variables). We have included the possibility of these transfers, because they simplify the transitional properties of the model.

Let us take good a as numeraire and let p be the price of good b.

The aggregate budgetary restrictions for the families in the first and second sectors, and for the government, are respectively

$$K_1' = \varphi^1 (K_1) - C_1 - T_1$$

$$K_2' = \varphi^2 (K_P, K_2) - \frac{1}{p} C_2 - \frac{1}{p} T_2 \tag{2}$$

$$K_P' = \varepsilon_P (T_1 + T_2)$$

where the variable we have introduced are the following:

$C_i = c_i N_i$ is the aggregate consumption of sector i.
$c_i = (a_i + pb_i)$ is the per-capita consumption in sector i.
a_i, b_i is per-capita consumption of goods a and b by families in sector i.
T_i is the aggregate lump-sum tax on sector i, in units of good a.

The relation between the physical and monetary quantities when the markets clear and the government's budget and trade are in balance is the following.

$$C_1 + T_1 = C_a - a_X + I_a$$

$$C_2 + T_2 = p (C_b - b_X) \tag{3}$$

$$T_1 + T_2 = I_a$$

From the first and last equations, $C_1 = C_a - a_X + T_2$. Substracting $a_1 N_1$ from each side,

$$pb_1 N_1 = a_2 N_2 - a_X + T_2 \tag{4}$$

We will consider two cases for the determination of the relative price p between goods a and b. In the first, the economy is closed and p is formed in the internal market. Then $a_X = 0$ and equation (4) is the market clearing condition, since families in sector 1 buy an amount b_1 of good b, while the families in sector 2 in effect buy an amount $a_2 + T_2$ of good a (since the government will spend all of the taxes of sector 2, which originate in the sale of good b, purchasing goods of type

a for public investment). In the second, the economy is open and p is the international price, which we shall assume is constant. Then equation (4) implies the condition $a_X + pb_X = 0$ of balanced trade (obtained by using the second and third equations in (3) to replace T_2 and simplifying).

3.2. *The agents' decisions*

The government is a Stackelberg leader in a game in which the remaining agents are representative families each belonging to one of the sectors. The families are identical and maximize the utility functional

$$U[a_i, b_i] = \int_0^\infty u(a_i, b_i)\, e^{-\rho t}\, dt \tag{5}$$

where

$$u(a, b) = \frac{a^\alpha b^\beta - 1}{1-\sigma}, \text{ with } \alpha + \beta = 1 - \sigma \tag{6}$$

and a_i, b_i are the respective consumption streams of families in sector i.

The problem for families in sector 1

$$\max_{a_1, b_1} U_1 = U[a_1, b_1] \tag{7}$$

s.t. $$k_1' = \varphi^1(k_1) - nk_1 - a_1 - pb_1 - \tau_1$$

where $k_1 = K_1/N_1$ is the per-capita capital stock while $\tau_1(t) = T_1/N_1$ (lump-sum tax per-capita) and $p(t)$ are given. $k_1(0)$ is given and we use the transversality condition $\lim_{t\to\infty} k_1 e^{-(\bar{r}_1 - n)t} = 0$, where $\bar{r}_1 = (1/t)\int_0^t r_1(s)\, ds$, $r_1 = \varphi^{1\prime}(k_1)$.

The problem for families in sector 2

$$\max_{a_2, b_2} U_2 = U[a_2, b_2] \tag{8}$$

s.t. $$k_2' = \psi^2\left(\frac{K_P}{K_2}\right) k_2 - nk_2 - \frac{1}{p} a_2 - b_2 - \frac{1}{p}\tau_2,$$

where $k_2 = K_2/N_2$, is the per-capita capital stock while $\tau_2(t) = T_2/N_2$ and $p(t)$ are given. $k_2(0)$ is given and we use the transversality condition $\lim_{t\to\infty} pk_2 e^{-(\bar{r}_2 - n)t} = 0$, where $\bar{r}_2 = (1/t)\int_0^t r_2(s)\, ds$, $r_2 = \psi^2$.

The government's problem (closed economy)

We suppose that the government announces a lump-sum tax stream independent of the families' decisions. This last assumption is realistic in that governments adapt tax rates to budgetary requirements and greatly simplifies the mathematical problem since it ceases to be one of control with feed-back. We write $U_i = U_i^*(\tau_1, \tau_2, p, k_P)$, $i = 1, 2$, for the utilities of the families in each sector after optimization. The government's problem is

$$\max_{k_1(0),\, k_P(0),\, \tau_1,\, \tau_2} U_G = \chi U_1 + (1-\chi) U_2$$

s.t. $$k_P' = \varepsilon_P\left(\frac{N_1}{N_2}\tau_1 + \tau_2\right) - nk_P,$$

$$U_i \geq z_i, \qquad i = 1, 2$$

$$(k_{10} - k_1(0))\, \underline{\lambda}_{10} + \int_0^\infty \tau_1 \underline{\lambda}_1\, dt \leq \omega_1 \int_0^\infty \underline{c}_1 \underline{\lambda}_1\, dt \tag{9}$$

$$\int_0^\infty (1/p)\, \tau_2 \underline{\lambda}_2\, dt \leq \omega_2 \int_0^\infty (1/p)\, \underline{c}_2 \underline{\lambda}_2\, dt$$

p given by market conditions,

$$k_1(0) + \frac{N_2}{N_1\,\varepsilon_P}\,k_P(0) \le k_{10} + \frac{N_2}{N_1\,\varepsilon_P}\,k_{P0},$$

$$k_2(0) = k_{20}$$

It turns out for technical reasons that the problem must be posed somewhat differently in the closed and open economy cases. Here we have stated the problem for the closed economy. Below, we state the problem for the open economy (see equation 42).

The *power of passive resistance* is expressed as a limit on transfers from each sector to the government, expressed in terms of the total wealth as a proportionality constant ω_i times the discounted value of the consumption stream of each sector $i = 1, 2$. We shall suppose for simplicity that this limit is considered by the families a lump-sum, which the government cannot play with strategically. Thus $\underline{\lambda}_i$, $\underline{c}_i$ are fixed functions of t which equal the results of optimization: $\underline{\lambda}_i = \lambda_i^*$, $\underline{c}_i = c_i^*$. The transitional dynamics are simplified if instantaneous reallocation of capital of type a is possible between private and public capital (which are of the same type of good), so we shall allow this possibility. Thus in the case of sector 1 the limit is on the total value of discounted transfers. We write $k_{10} = (K_{10}/N_{10})$, $k_{P0} = (K_{P0}/N_{20})$ for the initial per-capita levels of private capital in sector 1 and public capital in sector 2, and $k_1(0) = K_1(0)/N_{10}$, $k_P(0) = K_P(0)/N_{20}$ for the per-capital levels of private capital in sector 1 and public capital in sector 2 *after* a possible positive or negative instantaneous transfer of private to public capital dictated by the government. For simplicity we suppose that the transformation rate ε_P which holds when private capital is transformed into public capital is reversible. Alternatively we could consider separately the cases in which there is initially too little or too much public capital.

The restrictions $U_i \ge z_i$, $i = 1, 2$, represent the *power of organized resistance* of sector i, setting a minimum level of welfare.

The parameter χ in the government's objective function U_G reflects the relative level of *organized social power* of the sectors. If $\chi = 1$ or 0 we say sector 1 or sector 2 governs respectively. We shall say that there is *shared government* if χ takes an intermediate value *and* if the solution of the problem occurs when the inequalities representing passive and organized resistance are lax.

Proposition 1. Given any smooth trajectories τ_1, τ_2, p, k_P, solutions to the problems of families in sector 1 and 2 exist and are unique. Let λ_1, λ_2, be the shadow prices of k_1, k_2 respectively. The first order conditions are

$$\begin{aligned} &u_a(a_1, b_1)\,e^{-\rho t} = \lambda_1, && u_a(a_2, b_2)\,e^{-\rho t} = \frac{1}{p}\lambda_2, \\ &u_b(a_1, b_1)\,e^{-\rho t} = p\lambda_1, && u_b(a_2, b_2)\,e^{-\rho t} = \lambda_2 \\ &\lambda_1' = \left(n - \varphi^1_{k_1}\right)\lambda_1 && \lambda_2' = (n - \psi^2)\,\lambda_2 \end{aligned} \tag{10}$$

The first two lines imply

$$\begin{aligned} a_1 &= c_a\left(p^{\beta}\,\lambda_1\,e^{\rho t}\right)^{-1/\sigma}, \quad & b_1 &= c_b\left(p^{1-\alpha}\,\lambda_1\,e^{\rho t}\right)^{-1/\sigma} \\ a_2 &= c_a\left(p^{\beta-1}\,\lambda_2\,e^{\rho t}\right)^{-1/\sigma}, \quad & b_2 &= c_b\left(p^{-\alpha}\,\lambda_2\,e^{\rho t}\right)^{-1/\sigma} \end{aligned} \tag{11}$$

where

$$c_a = \left(\frac{\alpha^{1-\beta}\,\beta^{\beta}}{1-\sigma}\right)^{1/\sigma}, \qquad c_b = \left(\frac{\alpha^{\alpha}\,\beta^{1-\alpha}}{1-\sigma}\right)^{1/\sigma} \tag{12}$$

From these equations we obtain

$$\frac{b_i}{a_i} = \frac{1}{p}\frac{\beta}{\alpha}, \qquad c_i = \left(1 + \frac{\beta}{\alpha}\right)a_i, \qquad i = 1, 2. \tag{13}$$

Consumption follows the differential equations

$$\begin{aligned} \frac{c_1'}{c_1} &= \frac{\varphi^1_{k_1} - n - \rho - \beta\,(p'/p)}{\sigma} \\ \frac{c_2'}{c_2} &= \frac{\psi^2 - n - \rho + (1-\beta)\,(p'/p)}{\sigma}. \blacksquare \end{aligned} \tag{14}$$

3.3. The closed sectoral economy

Before solving the government's problem when the economy is closed, we can say something about the distribution of wealth and about the price p in the closed economy.

Theorem 1. In the case of the closed sectoral economy, the distribution of aggregate consumption between sectors is proportional to the (absolute value of the) elasticity of the marginal utility of its product, modified by the intensity of taxation in sector 2.

$$\frac{C_1}{C_2} = \left(1 + \frac{\tau_2}{a_2}\right)\frac{\alpha}{\beta} \tag{15}$$

or alternatively

$$\frac{C_1 - T_2}{C_2 + T_2} = \frac{\alpha}{\beta} \tag{16}$$

from which it follows by (3) that the price p of good b in units of good a is given by

$$p = \frac{\beta}{\alpha}\frac{C_a}{C_b} \tag{17}$$

In this expression the price equals the ratio of quantities consumed corrected by the ratio of elasticities of consumer preferences. Let us define $\vartheta = (1 + \tau_2/a_2)\,\alpha/\beta$ as a parameter measuring the tax rate in sector 2. In per-capita terms,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \vartheta\frac{N_2}{N_1} \tag{18}$$

This implies the relation between shadow prices

$$\frac{\lambda_2}{p\lambda_1} = \left[\vartheta\frac{N_2}{N_1}\right]^{\sigma}. \tag{19}$$

Family's utilities can also be written in terms of C_a, C_b and ϑ:

$$u(a_1, b_1) = \frac{\left[\frac{\vartheta}{N_1[1+\vartheta]}\right]^{1-\sigma} C_a^{\alpha} C_b^{\beta} - 1}{1-\sigma}$$

$$u(a_2, b_2) = \frac{\left[\frac{1}{N_2[1+\vartheta]}\right]^{1-\sigma} C_a^{\alpha} C_b^{\beta} - 1}{1-\sigma} \tag{20}$$

The differential equations for p, C_a, C_b are:

$$\frac{p'}{p} = \varphi^1_{k_1} - \psi^2 - \sigma\frac{\vartheta'}{\vartheta}$$

$$\frac{C_a'}{C_a} = -\left(\frac{1}{1+\vartheta} - \beta\right)\frac{\vartheta'}{\vartheta} + \frac{(1-\beta)\,\varphi^1_{k_1} + \beta\psi^2 - \rho}{\sigma} \tag{21}$$

$$\frac{C_b'}{C_b} = \left(\frac{\vartheta}{1+\vartheta} - \alpha\right)\frac{\vartheta'}{\vartheta} + \frac{(1-\alpha)\,\psi^2 + \alpha\varphi^1_{k_1} - \rho}{\sigma}. \blacksquare$$

The asymmetry between the effects of τ_1 and τ_2 stems from the fact that, since all taxes go to public investment and all public investment is in good a, the flow of taxes from sector 2 involves a market exchange of good b for good a, creating an additional demand for good a.

The distribution of aggregate consumption between sectors is determined by the economic potential of each sector, rather than by initial wealth. In this case this "economic potential" depends on the relative elasticity (of the marginal utility) of the good it produces (which measures the relative demand for its product), modified by demands brought about by taxation, which themselves may result from relative political power. The relative price between goods is given by a relation between aggregate physical consumptions C_a, C_b also weighted by preferences. Utilities of families in both sectors are very similar functions of the aggregate consumption of both goods, differing in the distributional ratio ϑ. This will imply that governments of any sector will have the same investment policies, but will differ in their distribution of the tax burden.

The dynamic equation for p states that the price of good b rises if sector 2 is less productive than sector 1, so that $\varphi^1_{k_t} - \psi^2 > 0$, or if the tax on sector 2 decreases.

We now solve the government's problem, by showing that it is equivalent to a simple optimization problem. To do this we pose three associated problems.

Associated Problem A

$$\max_{C_a, C_b, I_a, v, K_1(0), K_P(0)} \quad U_G^A = \int_0^\infty \left[\chi u_1 + (1-\chi)\, u_2\right] e^{-\rho t}\, dt \tag{22}$$

subject to the state equations (1a), (1c) with $a_X = b_X = 0$ and (1b) replaced by

$$K_2' = \psi^2\left(\frac{K_P}{\underline{K_2}}\right) K_2 - C_b \tag{23}$$

and subject to

$$K_1(0) + \frac{1}{\varepsilon_P} K_P(0) \le K_{10} + \frac{1}{\varepsilon_P} K_{P0}$$

$$K_2(0) = K_{20}$$

$$\lim_{t\to\infty} \underline{\Lambda_1}\, K_1 = \lim_{t\to\infty} \underline{\Lambda_2}\, K_2 = \lim_{t\to\infty} \underline{\Lambda_P}\, K_P = 0 \tag{24}$$

$$\vartheta' = v,$$

$$U_i \ge z_i, \qquad i = 1, 2,$$

and to

$$\left[K_{10} - K_1(0)\right] \underline{\Lambda_1}(0) + \int_0^\infty \left(I_a - \frac{\frac{\beta}{\alpha}\vartheta - 1}{1+\vartheta}\, \underline{C_a} \right) \underline{\Lambda_1}\, dt \le \overline{\omega}_a \int_0^\infty \frac{\vartheta}{1+\vartheta}\, \underline{C_a}\, \underline{\Lambda_1}\, dt$$

$$\int_0^\infty \frac{\frac{\beta}{\alpha}\vartheta - 1}{1+\vartheta}\, \underline{C_a}\, \underline{\Lambda_1}\, dt \le \overline{\omega}_b \int_0^\infty \underline{C_a}\, \underline{\Lambda_1}\, dt, \tag{25}$$

where $\overline{\omega}_a = \omega_1\left(1 + \frac{\beta}{\alpha}\right)$, $\overline{\omega}_b = \frac{\beta}{\alpha}\,\frac{\omega_2}{1+\omega_2}$, and where

$$U_i = \int_0^\infty u_i\, e^{-\rho t}\, dt \qquad i = 1, 2,$$

$$u_1 = \frac{\left[\frac{\vartheta}{N_1\,[1+\vartheta]}\right]^{1-\sigma} C_a^\alpha C_b^\beta - 1}{1-\sigma} \tag{26}$$

$$u_2 = \frac{\left[\frac{1}{N_2\,[1+\vartheta]})\right]^{1-\sigma} C_a^\alpha C_b^\beta - 1}{1-\sigma}$$

Each underlined quantity is considered *given* by the optimizing agent (that is, a function of t, but must equal the corresponding non-underlined quantity *after* optimization. In other words, we solve the optimization problem together with the conditions $\underline{K_2} = K_2^*$, $\underline{\Lambda_i} = \Lambda_i^*$. This "underlining technique" allows us to reproduce the distortions in investment in K_2, and to eliminate (for simplicity) feedback effects in the underlined terms Λ_i.

We shall show that Associated Problem A generates the aggregate flow in the closed economy, and the solutions to the families' and government's problems. This will also show in what sense the market allocation is optimal. The last two restrictions in (25) will be shown to be equivalent to the passive resistance restrictions on taxes.

Associated Problem B

$$\max_{C_a, C_b, I_a K_1(0), K_p(0)} \quad V = \int_0^\infty u\,(C_a, C_b)\, e^{-\rho t}\, dt \tag{27}$$

subject to restrictions (1a), (1c), (23), (24), and

$$\left[K_{10} - K_1(0)\right]\underline{\Delta_1}(0) + \int_0^\infty \left(I_a - \frac{\frac{\beta}{\alpha}\vartheta - 1}{1+\vartheta}\underline{C_a} \right)\underline{\Delta_1}\, dt \tag{28}$$

$$= \overline{\omega}_a \int_0^\infty \frac{\vartheta}{1+\vartheta}\underline{C_a \Delta_1}\, dt$$

Here $\vartheta \in$ R is a given parameter of the restriction on I_a, whose corresponding Lagrange multiplier will be written Ξ_a. Underlined quantities are treated as in the previous problem. This associated problem defines $V(\vartheta)$ and the functions $U_i(\vartheta)$

$$U_1(\vartheta) = \left[\frac{\vartheta}{N_1[1+\vartheta]}\right]^{1-\sigma} V + \frac{V-1}{\rho(1-\sigma)}$$
$$U_2(\vartheta) = \left[\frac{1}{N_2[1+\vartheta]}\right]^{1-\sigma} V + \frac{V-1}{\rho(1-\sigma)} \tag{29}$$

which will form the Pareto frontier in the government's decision problem.

Associated Problem C

$$\max_{\vartheta \in R} \quad U_G^C = \chi U_1(\vartheta) + (1-\chi)\, U_2(\vartheta)$$

s.t.

$$U_i \geq z_i \qquad i = 1, 2 \tag{30}$$

$$\frac{(\beta/\alpha)\,\vartheta - 1}{1+\vartheta} \leq \frac{\beta}{\alpha}\frac{\omega_2}{1+\omega_2}.$$

The optimal values of this problem will be shown to equal those of Associated Problem A. We write the corresponding Lagrangian in the form

$$\mathcal{L}^C = \chi U_1 + (1-\chi)\, U_2 + \sum_{i=1,2} \eta^i\,(U_i - z_i) + \Xi_b\left[\frac{\beta}{\alpha}\frac{\omega 2}{1+\omega 2} - \frac{(\beta/\alpha)\,\vartheta - 1}{1+\vartheta}\right]. \tag{31}$$

Theorem 2. The solutions to Associated Problem A imply ϑ is constant and generate the solutions to the government's and the families' problems. In turn, Associated Problem A is equivalent to Associated Problem C. which generates the same optimal values and multipliers $U_1^*, U_2^*, \vartheta^*, \Xi_b^*, \eta^{i*}$, together with the values Ξ_a^* of the multipliers and trajectories and initial values $C_a^*, C_b^*, I_a^*, K_1^*(0), K_P^*(0)$ appearing in the definition of $V(\vartheta)$ through Associated Problem B. The implicit relation between U_1 and U_2 defined by (29) is decreasing, concave and once differentiable in the government's feasible region of values ϑ. Since ϑ is constant, equations (21) are replaced by

$$\frac{p'}{p} = \varphi_{k_1}^1 - \psi^2,$$

$$\frac{C_a'}{C_a} = \frac{(1-\beta)\,\varphi_{k_1}^1 + \beta\psi^2 - \rho}{\sigma} \tag{32}$$

$$\frac{C_b'}{C_b} = \frac{(1-\alpha)\,\psi^2 + \alpha\varphi_{k_1}^1 - \rho}{\sigma}.$$

The government's first order condition is the efficiency condition

$$\varphi_{K_1}^1 = \frac{\varepsilon_P}{1+\Xi_a}\, p\varphi_{K_P}^2. \tag{33}$$

p satisfies the autonomous differential equation

$$\frac{p'}{p} = A_1 - (\psi^2 \circ [\psi^{2\prime}]^{-1})\left(\frac{A_1(1+\Xi_a)}{p\varepsilon_P}\right) \tag{34}$$

and converges to some value $\tilde{p}$ at which the efficiency condition

$$\varphi^1_{k_1} = \psi^2 \tag{35}$$

is satisfied. When φ^2 is Cobb-Douglass, $\tilde{p}$ is given by

$$\tilde{p} = \left[\frac{A_1}{A_2}\right]^{\frac{1}{1-\theta}} \frac{1+\Xi_a}{(1-\theta)\,\varepsilon_P}\,. \tag{36}$$

When the tax restriction of sector 1 is slack (so $\Xi_a = 0$) the aggregate physical flows C_a, C_b, K_1, K_2, K_P (and therefore p) are independent of ϑ, which only affects distribution and not the levels of production.■

Theorem 2 means that *in the closed economy the conditions of power, defined by the government objectives and by the political restrictions, effectively conform the social utility function (and restrictions) which is maximized by the politico-economic system*. The price p tends to an equilibrium value at which the (distorted) efficiency condition (35) holds, meaning that private investment in both types of capital K_1 and K_2 is equally profitable. Thus the sectoral advantages in production need only be small in equilibrium for there to be no incentive for families in each sector to invest in the other sector's production. The government's optimality condition (33) is equivalent to a centralized efficiency condition if $\varepsilon_P = 1$ and $\Xi_a = 0$, that is, if the government is efficient and if the taxation restriction for sector 1 is slack. Under the later of these conditions, the level of investment is given by efficiency conditions and is independent of the relative power of the sectors, and the production flows are independent of the level of taxation of sector 2, which determines the intersectoral tax burden and therefore the distribution of income. If the total resistance to taxation imposes a restriction on the government, $\Xi_a > 0$ and the equilibrium price of good b rises; by (17), less is being produced of good b.

Let us examine the incentives for political organization. If sector 1 is in power, sector 2 can guarantee a certain level of income on the basis of its resistance to taxation (power of passive resistance), even if its organized power is small. A similar statement holds if the power of the sectors is reversed, because sector 2 may try to obtain a subsidy by transferring resources from sector 1 above the level of taxation which

is optimal for investment. Note that if sector 1 is in power it will be counterproductive for it to resist taxation. Thus *when each sector is in power the other can guarantee its respective income based only on the power of passive resistance*. The fixed cost of organization necessary to raise income further may or may not be worth expending.

Finally let us outline the mechanisms leading to public investment and its efficiency. *In a closed sectoral economy economic incentives tend to lead sectors to a consensus as to the efficient levels of public investment, which are judged by aggregate economic performance* (rather than price signals). There is, however, conflict as to the distribution of the investment costs.

We have not introduced the possibility of the government taking measures (such as taxation of investment in K_2) to remove the distortion originated in the need for public investment, for several reasons. The first is that it is often unrealistic for such measures to be taken. The second is to keep the analysis as simple as possible, without mixing the several motives for taxation. The third is to introduce the "underlining technique" for the solution *with* distortion, which is interesting in its own right.

3.4. The open sectoral economy

The economy may be open to trade or to capital flows. When trade is open, the economic (but not the political) interdependence between sectors is removed since inputs and consumption may now be freely imported. We consider two cases, the first with and the second without restrictions to capital flows. It is clear that even if financial markets are open there may be many other kinds of restrictions to capital flow, such as credit restrictions which may be different for different sectors. (See for example the human capital model in section 3.2 in Barro and Xavier [8], which applies to non-tradeable capital in general, for which open trade with only partially open capital markets applies.) We show that the political problem reduces to how much public in- vestment there will be and who will pay for it. Here if sector 2 resists taxation to restrict transfers towards sector 1, it may do so at the cost of restricting investment on the public good that it needs for production. However, we shall consider that to restrict such transfers without restricting taxation involves some type of organized power. Thus such a situation is described by the appropriate parameters z_2, χ.

Open trade with restricted capital flows

We first suppose that the economy is open to trade only (capital flows are restricted in some way). Sector 2 may now become an engine of growth, since its inputs may be imported. Suppose the international price p of good b is constant. When φ^2 is Cobb-Douglass, the natural growth rates for each sector (when the taxation limits do not interfere) are

$$\gamma_1 = \frac{A_1 - n - \rho}{\sigma} \tag{37}$$

$$\gamma_2 = \frac{A_2((1-\varphi)\, p\varepsilon_p)^{1-\varphi} - n - \rho}{\sigma}$$

(see equations 14 and 47). Using (36) (with $\Xi_a = 0$)

$$\gamma_1 \leq \gamma_2 \iff \tilde{p} \leq p\,. \tag{38}$$

If $p > \tilde{p}$, the engine of growth will be sector 1 when the economy is closed and sector 2 when it is open to trade and capital flows are restricted. Each sector may have its own growth rate. However, if for example a dominated sector grows faster than a dominant sector, we must think this means that the sectoral structure will tend to change in some fundamental way.

In the case under discussion the economic relation between the two sector's essentially reduces to the transfer of resources. The optimization problem in which the government chooses functions τ_1, τ_2 is ill-posed in the case of τ_1, since it would be a Pareto improvement for the sector with lesser growth to transfer all of its capital to the other sector, obtaining repayment in the form of taxes. Therefore we set

$$\tau_1 = \tau_{10}\, \Upsilon, \tag{39}$$

where $\Upsilon > 0$ is some predetermined function of t (so there is no feedback on this term), and optimize in τ_{10}, τ_2 (without sign restrictions). Correspondingly, we cannot allow initial transfers of capital between sectors. On the other hand we will let the government choose between functions τ_2, and since sector 2 can now buy good a with good b abroad, we allow for the instantaneous transfer between K_2 and K_P at time

$t = 0$. This induces an efficiency condition between capitals of these types which is internal to sector 2. As a technical device, we also reformulate the problem of families in sector 2 by incorporating in it the differential equation for k_p. This implies an additional first order condition

$$\lambda_P' = n\lambda_P - \varphi^2_{k_p}\, \lambda_2. \tag{40}$$

We can now write

$$U_i = U_i\,(k_1\,(0),\, k_2\,(0),\, k_P\,(0),\, \tau_{10},\, \tau_2,\, p\,(0)), \quad i = 1, 2. \tag{41}$$

Thus we rewrite

The government's problem (open economy)

$$\max_{k_2(0),\, k_p(0),\, \tau_{10},\, \tau_2} \quad U_G = \chi U_1 + (1-\chi)\, U_2 \tag{42}$$

s.t.

$$U_i \geq z_i,\; i = 1, 2,$$

$$\tau_{10} \int_0^\infty e^{\gamma_1 t}\, \underline{\lambda}_1\, dt \leq \omega_1 \int_0^\infty \underline{c}_1\, \underline{\lambda}_1\, dt,$$

$$(\,k_{20} - k_2\,(0))\, \underline{\lambda}_{20} + \tau_{20} \int_0^\infty (1/p)\, e^{\gamma_2 t}\, \underline{\lambda}_2\, dt \leq \omega_2 \int_0^\infty (1/p)\, \underline{c}_2\, \underline{\lambda}_2\, dt,$$

$$(1/\varepsilon_P)\, k_P\,(0) + p\,(0)\, k_2\,(0) \leq (1/\varepsilon_P)\, k_{P0} + p\,(0)\, k_{20},$$

We solve this problem by reducing it to a simple optimization problem. To do this we pose two associated problems.

Associated Problem D

$$\max_{a_2,\, b_2,\, \tau_2,\, k_2(0),\, k_p(0)} \quad U_2 = U\,[a_2, b_2] \tag{43}$$

s.t.
$$k_2' = \psi^2\left(\frac{k_P}{\underline{k_2}}\right) k_2 - nk_2 - (1/p)\, a_2 - b_2 - (1/p)\, \tau_2$$

$$k_P' = \varepsilon_P\left(\frac{N_1}{N_2}\tau_1 + \tau_2\right) - nk_P$$

$$\tau_1 = \tau_{10}\, \Upsilon$$

$$(k_{20} - k_2(0))\, \underline{\lambda}_{20} + \tau_{20}\int_0^\infty (1/p)\, e^{\gamma_2 t}\, \underline{\lambda}_2\, dt \le \omega_2 \int_0^\infty (1/p)\, \underline{c}_2\, \underline{\lambda}_2\, dt,$$

$$(1/\varepsilon_P)\, k_P(0) + p(0)\, k_2(0) \le (1/\varepsilon_P)\, k_{P0} + p(0)\, k_{20}.$$

This problem defines the function $U_2(\tau_{10})$, while $U_1(\tau_{10})$ is defined directly from the problem for families in sector 1 with the given function τ_1.

Associated Problem E

$$\max_{\tau_{10}\in R} \quad U_G = \chi U_1(\tau_{10}) + (1-\chi)\, U_2(\tau_{10}) \tag{44}$$

s.t.
$$U_i \ge z_i, \quad i = 1, 2$$

$$\tau_{10} \le \omega_1\, c_1(\tau_{10})\Big|_{t=0}$$

Here $c_1(\tau_{10})$ is the consumption stream of families in sector 1 paying taxes $\tau_{10}\,\Upsilon$, and we are using $\int_0^\infty \underline{c}_1\, \underline{\lambda}_1\, dt = c_1(\tau_{10})\,|_{t=0} \int_0^\infty e^{\gamma_1 t}\, \underline{\lambda}_1\, dt$.

Theorem 3. The government's problem (42) is equivalent to solving Associated Problem E, choosing the optimal function τ_2 and initial values $k_2(0)$, $k_P(0)$ that arise from the definition of $U_2(\tau_{10})$ in Associated Problem D. The multipliers of these problems also coincide with the multipliers of the original problem. $U_1(\tau_{10})$, $U_2(\tau_{10})$, are respectively increasing and decreasing concave functions. The government's first order conditions are

$$x_1\int_0^\infty e^{\gamma_1 t}\, \lambda_1\, dt + \Xi_a \int_0^\infty \Upsilon\lambda_1\, dt = x_2 \int_0^\infty \varepsilon_P\, (N_1/N_2)\, e^{\gamma_1 t}\, \lambda_P\, dt$$

$$\frac{1}{p}(x_2 + \Xi_b)\, \lambda_2 = x_2\, \varepsilon_P\, \lambda_P. \tag{45}$$

These imply

$$p\varepsilon_P \frac{x_2}{x_2 + \Xi_b}\varphi^2_{k_P} = \psi^2$$
$$\lambda_P' = (n - \psi^2)\, \lambda_P, \tag{46}$$

and in the case when φ^2 is Cobb-Douglass,

$$\frac{k_P}{k_2} = (1-\theta)\, p\varepsilon_P \frac{x_2}{x_2 + \Xi_b},$$
$$\psi^2 = A_2\left[(1-\theta)\, p\varepsilon_P \frac{x_2}{x_2+\Xi_b}\right]^{1-\theta} \tag{47}$$

$$\tau 2 = \frac{n\dfrac{1}{\varepsilon P} kP + \left[(1-\theta)\, p\dfrac{x2}{x2+\Xi b}\right]\left[\psi^2\, k2 - nk2 - \dfrac{1}{p}a2 - b2\right] - \dfrac{N1}{N2}\tau 10\, e^{\gamma 1 t}}{\left[1 + (1-\theta)\, \varepsilon P \dfrac{x2}{x2+\Xi b}\right]} \tag{48}$$

When sector 1 exercises power alone it takes sector 2 to its limit utility $U_2 = z_2$, while when sector 2 exercises power alone it takes sector 1 either to its limit utility $U_1 = z_1$ or to its taxation limit, whichever restriction is stronger. ■

Again the conditions of power conform the social utility function (and restrictions) maximized by the politico-economic system. Note that the dependence of sector 2 on public inputs now means that *its income depends on its organized power, independently of its passive power of resistance*. Whatever the balance of power between sectors, the growth rate of each sector will be determined by the level of international prices (equation 38). We qualify this by observing that if sector 2 counts initially with a large amount of public capital which it cannot transform into private capital, but has very little organized power, its utility demand may warrant no further investment in public capital, so that its growth rate may be negative. Such a situation may occur after a closed sectoral economy opens to trade, independently of sector 2's natural growth rate. After some time has elapsed and public capital has deteriorated enough, the utility demand may again warrant public investment, especially if the sector has decided to organize, and positive growth may be recovered. The simultaneous occurrence of different growth rates between sectors means that the sectoral structure has a tendency to break down. In the first place, because different growth rates coincide with different rates of return to capital between the sectors, which mean the sectoral advantages of production can break down. In the second, because the exponential growth of the income of one sector relative to the other means that eventually the structures maintaining the sector with diminishing relative income in existence must break down. Indeed, we can solve the model exactly when φ^2 is Cobb-Douglass, if we set $\Upsilon = e^{\gamma t}$. The solutions are given by

$$a_1 = a_{10}\, e^{\gamma_1 t}, \quad b_1 = b_{10}\, e^{\gamma_1 t}, \quad k_1 = \frac{a_{10} + p b_{10}}{A_1 - n - \gamma_1} e^{\gamma_1 t} + \frac{\tau_{10}}{A_1 - n - \gamma} e^{\gamma t} \tag{49}$$

$$\frac{a_{10} + p b_{10}}{A_1 - n - \gamma_1} + \frac{\tau_{10}}{A_1 - n - \gamma} = k_{10}, \qquad \frac{a_{10}}{b_{10}} = \frac{p\alpha}{\beta}$$

$$k_2 = \frac{\left[1 + \frac{\zeta}{1 + \zeta\varepsilon_P}\right]\left[\frac{1}{p} a_{20} + b_{20}\right]}{\tilde{A}_2 - n - \gamma_2} e^{\gamma_2 t} - \frac{\frac{N_1}{N_2}\left[\frac{1}{1 + \zeta\varepsilon_P}\right]\frac{1}{p}\tau_{10}}{\tilde{A} - n - \gamma} e^{\gamma_1^{t}}$$

$$\left[1 + \frac{\zeta}{1 + \zeta\varepsilon_P}\right]\left[\frac{1}{p} a_{20} + b_{20}\right] - \frac{N_1}{N_2}\frac{1}{1 + \zeta\varepsilon_P}\frac{1}{p}\tau_{10} = k_2(0), \qquad \frac{a_{20}}{b_{20}} = \frac{p\alpha}{\beta} \tag{50}$$

$$\left[1 + \frac{\zeta}{\varepsilon_P}\right] k_2(0) = \frac{1}{p(0)\varepsilon_P} k_{P0} + k_{20}$$

where $\zeta = (1 - \theta)\, x_2/(x_2 + \Xi_b)$ and $\tilde{A}_2 = \psi^2\, [1 - \zeta/(1 + \zeta\varepsilon_P)]$. The multipliers have still to be worked out according to different cases. Here k_1 becomes negative if $(\tau_{10}/(A_1 - n - \gamma)) < 0$ and $\gamma_1 < \gamma$, while k_2 becomes negative if $\tau_{10}/(A_1 - n - \gamma) > 0$ and $\gamma_2 < \gamma$, showing that the structure must break down, since the slower-growing sector has a tendency to disaccumulate its capital.

Open trade with unrestricted capital flows

Suppose now that the economy is open to trade and that capital flows are unrestricted. To extend our model to this case we must include the possibility that families borrow or invest in international assets returning a fixed interest rate r which must be set at A_1 for consistency. Also, the linear returns perceived by families in sector 2 must be replaced by a functional form reflecting decreasing returns. However, the results are quite clearly the following. Profit rates between sectors will be equalized, and sector 2 will borrow from or invest in the international asset until this happens. Thus there will be balanced growth between the sectors, and also a transfer of resources between them, resulting from the balance of power and determining who pays for the public investment. In this case sector 2 will still be reduced to the utility level warranted by its organized power since even if it does not pay taxes, a low level of public investment will reduce its well-being.

Conclusions on open trade

We can conclude that in a sectoral economy open to trade, *a dependent sector not in government will be reduced to the welfare level it can obtain by its organized power, its economic size being determined accordingly, while an independent sector will obtain the maximum of the levels implied by its three kinds of power.*

If there are restrictions to capital flows, *one of the sectors will grow faster than the other, according to whether the international price favors it when compared to the closed economy equilibrium price*. In this case, *the sectoral structure will tend to break down, independently of the initial distribution of power*. In the case when a politically weak sector 2 is initially well-endowed with public capital (as in transitions caused by opening trade), *its economic growth potential may be initially reversed by its lack of power.*

When capital flows are unrestricted, there will be balanced growth and the sectoral structure will be stable.

In either case, *only in shared government will the welfare of all sectors be lifted above the levels implied by their powers of resistance. In an open sectoral economy distribution is a result of the relative economic independence and of the political power of each sector.*

Finally let us outline the mechanisms leading to public investment and its efficiency. *In an open sectoral economy public investment results from organized political power. In trying to minimize the costs of meeting welfare demands, the government will tends towards the usual intra-sectoral efficiency conditions, but not necessarily towards the inter-sectoral efficiency conditions.*

4. Some consequences of the model

4.1. The government as sector

The government itself can be conceived of as a separate sector on which the others depend for the provision of such public goods as social organization, collective action, normative cohesion, power brokerage, law and order, conflict resolution, contractibility, national identity, etc. This models the basic dependence of any productive sector on the government. However, this kind of good is usually not importable, so should tend to be allocated according to the efficiency criteria corresponding to the closed economy. These goods can be "imported" if, for example, some sector manages to bring in international providers of these goods (*e.g.* other governments, the media, etc.) increasing the allocation it receives and therefore its income. In the presence of additional sectoral structure a game in which alliances are possible between different sectors could be considered.

4.2. Trade liberalization of sectoral economies

We have found that income distribution, the incentives for political organization, and the mechanisms and efficiency of public investment differ substantially in closed and open sectoral economies. Thus we

ask what happens when a closed sectoral economy in a steady state decides to open. Besides the direct economic effects of trade, resulting from economic advantages and other determinants, what are the effects of trade liberalization?

We shall answer this question supposing that first the economy opens, then the government modifies its intersectoral public policy according to the new structure of incentives, and finally the political structure changes.

Let us first assume that there are restrictions to capital flow. When the economy opens, the power of passive resistance ceases to be relevant in sector 2. If sector 2 is not very powerful, it is likely that before the opening of trade it had low levels of organized power, since the expense needed to maintain this kind of power was not warranted by the level of well-being which would accrue. We shall concentrate our analysis on this case, which is the interesting one. After the opening of trade, sector 2 must organize politically to sustain its levels of well-being, or its public capital will be allowed to deteriorate. But the steady state level of public capital was higher in the closed economy than the organized power of the sector now warrants. Therefore sector 2 may first suffer a negative growth rate, since the government will allow its public capital to depreciate. Later, as the level of public capital diminishes and if it is able to organize politically to achieve at least a minimum level of organized resistance, it will be able to achieve its potential growth rate. The level of international prices of its good determines wether this rate is higher or lower than sector 1's natural growth rate. Whichever sector has a larger growth rate has higher returns on its capital. The sectoral structure will tend to break down in some way.

Suppose now that there are no restrictions to capital flows. In this case the same conclusions hold, and sector 2's income will depend on its organized political power. However, the sectors will eventually grow at the same rates and the sectoral structure will be stable.

Until sector 2 is able to organize politically, public investment in its sector will be replaced by consumption in other sectors. Sector 2's utility will fall, and it will cease to invest, perhaps even consuming some of its capital. It may devote resources to becoming politically organized, since its economic size will depend on its political power. Sectors which were viable (and efficient) in a closed economy may be inviable in an open one because the cost of exercising political power is too high.

Thus, trade liberalization will be followed by a period of political adjustment in which previously weak sectors struggle to organize, and the politico-economic sectoral structure will be unstable if capital flows are restricted in some way.

Kenzo Abe, 1990, shows that in the presence of public inputs, ceteris paribus, the level of public input will strengthen the comparative advantage of the good enjoying the largest spillover from it. Hence we can state that the comparative advantage of sectors requiring public inputs will be a function of their political power.

Summarizing, trade liberalization can represent a formidable blow to sectors depending on public inputs but not counting with the political organization to obtain them, especially if capital flows towards them are hampered by diverse restrictions. For these sectors liberalization will change the political status quo. The income which could be previously based on market earnings and at most a strategy of passive resistance to avoid intersectoral transfers can only be attained now by a strategy of active political organization. If most public investment used to be allocated to such dependent, under-organized sectors, the resulting political change will be deep. What before could be decided by the leading sector on the basis of cost-benefit analyses now becomes the subject matter of intense power struggle based on political organization. Finally, if trade liberalization is accompanied by a laissez-faire policy seeking to leave adjustments to the markets and to diminish government participation in the economy (providing an excellent alibi for the selective cutting of public investment), the effects of opening trade on politics will be even stronger.

We believe this scenario to be of relevance for many viable sectors in economies changing from an import substitution to an open economy model of development, as in the case of Mexico. Although sectors whose growth has been inhibited may benefit from free trade and eventually even become engines of growth, these or other sectors requiring public inputs may face a need for political organization capable of reversing their growth and provoking an unstable political juncture which will have to be dealt with first. The smooth functioning of credit markets, which must reach all sectors, tends to have politically stabilizing effects in this respect. If the government is considered as a sector (as outlined above) it may survive by changing its system of alliances and priorities, identifying and becoming more responsive to the needs of the emerging sectors, which, however, includes meeting their politico-economic organizational requirements and may involve deeper changes in the nature of the government.

4.3. Increases in protectionism in sectoral economies

The differences in the importance of political organization according to the openness of trade imply that an increase in protectionism will tend to reduce organized resistance. Such a strategy has often been used.

Protectionism may also be used to change the engine of growth from one sector to another. The conflict between sectors involved in such changes may go as far as civil war.[1] Such an analysis may throw light, for example, on the American Civil War. The following quotations and account are from Olson, 1992, pages 106 and 32.

> Although historians in general have long seen the Civil War as the pivotal event in US political history, economic historians in particular see the Civil War as a key event in the evolution of the American political economy. In those terms, the federal government assumed a much broader role in American economic life than ever before. When Southern Democrats walked out of Congress in 1860 and 1861, the Republicans were finally able to enact fully Henry Clay's 'American System'. They passed the Morrill Tariff of 1861, and its revisions in 1862 and 1864 substantially raised tariff levels." Western settlement was also promoted, currency and financial markets centralized, and railroads authorized. The American System "advocated a comprehensive program of federal legislation designed to unite the various sections of the country. [...] a high tariff on foreign goods in order to stimulate American industry [...] vigorous federal development of roads, canals, and river system in order to be able to deliver the goods all over the country. Food would head from west to east and manufactured goods from east to west.

By closing the economy the Northern industrial sector (which otherwise had disadvantages with Europe) became a growth engine, changing the balance of power between North and South and involving the West with the North in an economic dependency relationship. The South had previously been growing in a system involving trade with a European engine of growth, from which it was cut off, transferring its dependence to the North.

[1] Professor Efraín Bringas Rábago suggested this line of enquiry.

4.4. A comment on the EC

"Many researchers have asked 'has the EC increased world protectionism?'" (Winters, 1994). Our results imply the following point: since the countries participating in the EC are open to each other, the incentives for political organization in each country for sectors requiring public inputs are high. Once political organization is achieved, though, these sectors lobby for trade tariffs with respect to countries outside the EC.

4.5. Effects of technical change on the political system

Historians often analyze transitions of political systems or changes in hegemony in terms of economic causes driven by technical change. One of the motivations behind the sectoral model is to be able to describe the economic logic behind such analysis. We have seen that the specification of the production functions implies relations of economic and political dependence, as well as defining the leading sectors in terms of growth. Transitions in which a dominated sector undergoes technical change and eventually obtains hegemony are described by our model by specifying the sectors and production functions before and after the technical transition. Some new or old sector which was not dominant may have a higher growth rate, maybe combined with a lower rate of dependence enabling it to resist transfers of wealth, which will ensure the growth of its capital and —by means unexplained by our model— its eventual access to power.

4.6. Fragmented perfect competition

The theoretical results of the fragmented perfect competition model are interesting in themselves. Fragmented competition means that the economy is divided into fragments each of which specializes in the production of some good, for which it has advantages with respect to the other fragments, but the fragments are large enough that there is perfect competition within each of them. The results show that this market structure achieves a different Pareto optimum than in perfect competition. Distribution does not occur according to the initial allocation of wealth but according to the economic potential of the fragments in question. *Within* each fragment, distribution will be proportional to initial wealth, but *aggregate* income in each fragment will depend on its economic potential. Wages could reflect this distribution structure, if there are barriers to the mobility of labor, or it could be restricted to capital income, with a uniform wage. The closed and open cases of sectoral economies ("sectoral" includes the political dimension while "fragmented" does not) give two examples of such a model, in which this economic potential is defined differently. In the closed case the relative demand for each fragment's product (including government demands, which may depend on political power) determines its aggregate wealth, while in the open case political power and economic independence determine aggregate wealth through the availability of public inputs. Income is distributed in this way because the productive advantages of any fragment inhibit capital flows from the others, *allowing it to capitalize on the basis of its own income*. In the case of the closed economy, or the open economy without restrictions on capital flow, the steady state is stable (in the face of competition) within a corridor of fluctuations whose width depends on the size of the advantages. Under these conditions, *in equilibrium every market will seem to be perfectly competitive*. In the open economy with restricted capital flows the structure is unstable since the fragments grow at different rates and profit differentials persist, so there may be a tendency to change. It is worth mentioning that if change does occur, it may follow a complex process. A sector may invade the domain of another sector in amoeba like fashion, covering the entry costs in certain regions only.

Considering that power often alters market structure, only the weaker effects of power on income distribution are modeled by fragmented perfect competition. Nevertheless this approach provides a way to represent these effects with the tools of perfect competition, modelling income distribution and capitalization schemes which are quite different from those implied by perfect competition.

5. Final remarks

The introduction of a sectoral structure in a model of endogenous growth has provided a rich structure full of intuitively appealing results relating the political properties of sectoral systems to their economic performance in income distribution, growth and stability. The objectives and restrictions of economic optimization relate naturally to aspects of political power which derive form collective action of different levels of complexity. We have called these the powers of passive and organized resistance, and socially organized power. This classification provides a point of entry for endogenizing the political system. That these levels of power are related to the costs of collective action also throws light on how limits are set to power.

The model of fragmented perfect competition has provided a means to treat some market imperfections with the tools of perfect competition. The model can yield Pareto equilibria different to those resulting from perfect competition but with the same efficiency conditions and prices. Distribution depends on the economic potential of each fragment, itself modified by the political power of the players, rather than exclusively on initial wealth. When there are restrictions to capital flow, the model also yields unbalanced growth. When there are no restrictions, in equilibrium every market will seem to be perfectly competitive.

The importance of whether a sectoral economy is open or closed is striking, *structurally determining* the mechanisms and incentives of the allocation of public goods, the economic potential yielding the income distribution between sectors, and the incentives for political organization. Trade liberalization leads to a period of political readjustment, and the comparative advantage of a sector depends on its political power.

For the present, our study has viewed the political system as exogenous to the economic system. However, the aim is to build a framework into which more structure can be introduced, such as games between sectors and the government in the political arena, which may affect growth, costs for collective action leading to political power and determining its exercise, and so on. Political transitions, income distribution and growth may then be modeled in terms of the incentives resulting from different power structures and the costs of achieving them for different sectors.

6. Appendix

Proof of Proposition 1. Given smooth functions p and τ_1, τ_2 the solution to the problem faced by the families in sector 1 exists, is unique and satisfies the first-order conditions of Pontriagyn's Maximum Principle (see Kamien and Schwartz [18]). The Hamiltonians are

$$\begin{aligned} H^1 &= u\,(a_1, b_1)\,e^{-\rho t} + \lambda_1\left[\varphi^1\,(k_1) - nk_1 - a_1 - pb_1 - \tau_1\right] \\ H^2 &= u\,(a_2, b_2)\,e^{-\rho t} + \lambda_2\left[\psi^2\,k_2 - nk_2 - (1/p)\,(a_2 - pb_2) - (1/p)\,\tau_2\right]. \end{aligned} \tag{51}$$

and the first order conditions (10) are derived as usual. Expressions (11) are obtained from the first two lines by substitution in the definition for u, and (14) from the third; (13) follows from (11). ■

Proof of Theorem 1. Equation (4) takes the form

$$pb_1\,N_1 = a_2\,N_2 + T_2 \tag{52}$$

Using equations (13), we obtain equations (18), from which (19) is derived as follows:

$$\frac{p\lambda_1}{\lambda_2} = \frac{u_a\,(a_1, b_1)}{u_a\,(a_2, b_2)} = \left[\frac{a_1}{a_2}\right]^{\alpha-1}\left[\frac{b_1}{b_2}\right]^{\beta} = \left[\left(1 + \frac{\tau_2}{a_2}\right)\frac{\alpha N_2}{\beta N_1}\right]^{-\sigma}. \tag{53}$$

Using the expression for c_1/c_2 in (18), (15) and (16) are obtained. Recalling equations (3) and using (52), we can verify that C_a and C_b represent aggregate physical consumption:

$$\begin{aligned} C_a &= C_1 - T_2 = (a_1 + pb_1)\,N_1 - T_2 = a_1\,N_1 + a_2\,N_2 \\ C_b &= \frac{C_2 + T_2}{p} = \frac{(a_2 + pb_2)\,N_2 + T_2}{p} = b_1\,N_1 + b_2\,N_2. \end{aligned} \tag{54}$$

By using substitutions such as

$$b_1 = \frac{b_1\,N_1}{b_1\,N_1 + b_2\,N_2}\,C_b = \frac{\vartheta}{\vartheta + 1}\,C_b$$

$$a_2 = \frac{a_2\,N_2}{a_1\,N_1 + a_2\,N_2}\,C_a = \frac{1}{\vartheta + 1}\,C_a$$

each family's utility can be written as in (20). Writing (52) in the form

$$p = \frac{\beta}{\alpha}\vartheta\frac{a_2 N_2}{b_1 N_1} \tag{55}$$

using equations (11) and (13), deriving logarithmically, and using the first order conditions (10) for λ_1 and λ_2, we get the dynamic equation (21) for p. The remaining equations in (21) are obtained by deriving

$$C_a = \frac{1+\vartheta}{1+\dfrac{\beta}{\alpha}} c_1 N_1 \quad \text{and} \quad C_b = \frac{\beta}{\alpha}\frac{1+\vartheta}{1+\dfrac{\beta}{\alpha}}\frac{1}{p} c_2 N_2. \blacksquare$$

Proof of Theorem 2. We first show that the solutions to the families' and government's problems can ae constructed from the solutions to Associated Problem A. The Lagrangian (from which the usual Hamiltonian is derived) is:

$$\begin{aligned} \mathcal{L}^A = &\int_0^\infty H^A\,dt + \Xi_a\left[K_1(0) - K_{10}\right]\underline{\Lambda}_1(0) \\ &+ \Xi_0\left[K_{10} + \frac{1}{\varepsilon_P}K_{P0} - K_1(0) - \frac{1}{\varepsilon_P}K_P(0)\right] \\ &+ \int_0^\infty [\Lambda_1' K_1 + \Lambda_2' K_2 + \Lambda_P' K_P + \Lambda_\vartheta' \vartheta]\,dt \\ &- [\Lambda_1 K_1 + \Lambda_2 K_2 + \Lambda_P K_P + \Lambda_\vartheta \vartheta]_0^\infty. \end{aligned} \tag{56}$$

Here

$$\begin{aligned} H^A = &\left[\chi u_1 + (1-\chi)u_2\right]e^{-\rho t} + \sum_{i=1,2}\eta^i\left(u_i e^{-\rho t} - \rho z_i\right) \\ &+ \Lambda_1\left[\varphi^1(K_1) - C_a - I_a\right] \\ &+ \Lambda_2\left[\varphi^2\left(\frac{K_P}{K_2}, 1\right)K_2 - C_b\right] + \Lambda_P\varepsilon_P I_a + \Lambda_\vartheta v \\ &+ \Xi_a\left[\frac{1}{1+\vartheta}\left(\overline{\omega}_a\vartheta + \frac{\beta}{\alpha}\vartheta - 1\right)\underline{C}_a - \underline{I}_a\right]\underline{\Lambda}_1 \\ &+ \Xi_b\left[\overline{\omega}_b - \frac{(\beta/\alpha)\vartheta - 1}{1+\vartheta}\right]\underline{C}_a\underline{\Lambda}_1 \end{aligned} \tag{57}$$

We need to consider the Lagrangian $\mathcal{L}^A$ because the restrictions on transfers involve integrals through time and also because there is an optimization in $K_1(0)$ and $K_P(0)$. Let

$$\begin{aligned} x_1 &= \chi + \eta^1 \\ x_2 &= 1 - \chi + \eta^2 \end{aligned} \tag{58}$$

Complementary slackness conditions aside, the first order conditions are

$$\left[\frac{1}{1+\vartheta}\right]^{1-\sigma}\left[\frac{x_1\vartheta^{1-\sigma}}{N_1^{1-\sigma}} + \frac{x_2}{N_2^{1-\sigma}}\right]u_a(C_a, C_b)\,e^{-\rho t} = \Lambda_1 \tag{59i}$$

$$\left[\frac{1}{1+\vartheta}\right]^{1-\sigma}\left[\frac{x_1\vartheta^{1-\sigma}}{N_1^{1-\sigma}} + \frac{x_2}{N_2^{1-\sigma}}\right]u_b(C_a, C_b)\,e^{-\rho t} = \Lambda_2 \tag{59ii}$$

$$[1 + \Xi_a]\,\Lambda_1 = \Lambda_P\,\varepsilon_P \tag{59iii}$$

$$\Lambda_\vartheta = 0 \tag{59iv}$$

$$\Lambda_1' = -\varphi^1_{K_1}\Lambda_1 \tag{59v}$$

$$\Lambda_2' = -\psi^2\Lambda_2 \tag{59vi}$$

$$\Lambda_P' = -\varphi^2_{K_P}\Lambda_2, \tag{59vii}$$

$$-\Lambda_\vartheta' = \frac{1}{[1+\vartheta]^{2-\sigma}}\left[\frac{x_1\,\vartheta^{-\sigma}}{N_1^{1-\sigma}} - \frac{x_2}{N_2^{1-\sigma}}\right] C_a^\alpha\, C_b^\beta\, e^{-\rho t}$$

$$+\frac{1}{(1+\vartheta)^2}\left[\Xi_a\,\overline{\omega}_a + (\Xi_a - \Xi_b)\left(1+\frac{\beta}{\alpha}\right)\right] C_a\,\Lambda_1 \tag{59viii}$$

$$[1+\Xi_a]\,\Lambda_1(0) = \Xi_0 = \Lambda_P(0)\,\varepsilon_P. \tag{59ix}$$

The concavity of the functions we are using usually implies that a unique solution exists and that conditions (59) are necessary and sufficient for there to exist a unique solution. The difference here is the use of the underlining technique. In this case the general proof of existence involves showing there is a maximum in functional space subject to the corresponding constraints $\underline{f} = f^*$ for the underlined functions, while the proof of the necessity and sufficiency of the first-order conditions is similar to the unconstrained case. A full proof of the underlining technique is beyond the scope of this paper and will appear in forth-coming work

We now define solutions to the families' problems in terms of the solution to Associated Problem A.

Condition (*ix*) (in equation (59)) governs the instantaneous transfer between K_1 (0) and K_P (0) at $t = 0$ (the families solve their problems after this transfer). This makes it possible to initiate positive time with condition (*iii*) valid. Conditions (*i*) and (*ii*) imply

$$\frac{x_1\,\vartheta^{1-\sigma}}{N_1^{1-\sigma}} + \frac{x_2}{N_2^{1-\sigma}} > 0. \tag{60}$$

Conditions (*iv*), (*viii*) and (*i*) imply

$$0 = \frac{x_1\,\vartheta^{1-\sigma}}{N_1^{1-\sigma}} - \frac{x_2}{N_2^{1-\sigma}} + \frac{1}{1+\vartheta}\left[\frac{x_1\,\vartheta^{1-\sigma}}{N_1^{1-\sigma}} + \frac{x_2}{N_2^{1-\sigma}}\right]\left[\left(1+\frac{\alpha\overline{\omega}_a}{1-\sigma}\right)\Xi_a - \Xi_b\right] \tag{61}$$

so ϑ is constant (and $v = 0$). Taking growth rates of conditions (*i*) and (*ii*),

$$(\alpha-1)\frac{C_a'}{C_a} + \beta\frac{C_b'}{C_b} - \rho = -\varphi_{K_1}^1$$

$$\alpha\frac{C_a'}{C_a} + (\beta-1)\frac{C_b'}{C_b} - \rho = -\psi^2. \tag{62}$$

Therefore C_a, C_b satisfy the differential equations

$$\frac{C_a'}{C_a} = \frac{(1-\beta)\,\varphi_{K_1}^1 + \beta\psi^2 - \rho}{\sigma}$$

$$\frac{C_b'}{C_b} = \frac{(1-\alpha)\,\psi^2 + \alpha\varphi_{k_1}^1 - \rho}{\sigma} \tag{63}$$

which in the case of constant ϑ are the same as (21). Define

$$(a_1, b_1) = \frac{\vartheta}{\mathrm{N}_1(1+\vartheta)}(C_a, C_b)$$

$$(a_2, b_2) = \frac{1}{N_2(1+\vartheta)}(C_a, C_b) \tag{64}$$

so that (18) holds and C_a, C_b represent aggregate consumption. Now we find that the first order conditions (10) hold if we define

$$\lambda_1 = u_a(a_1, b_1)\,e^{-\rho t} = u_a\left(\frac{\vartheta}{\mathrm{N}_1(1+\vartheta)}C_a, \frac{\vartheta}{\mathrm{N}_1(1+\vartheta)}C_b\right)e^{-\rho t}$$

$$= \left[\frac{\vartheta}{\mathrm{N}_1(1+\vartheta)}\right]^{-\sigma} u_a(C_a, C_b)\,e^{-\rho t} \tag{65}$$

$$= \Lambda_1\frac{N_1^\sigma}{\vartheta^\sigma(1+\vartheta)}\left[\frac{x_1\,\vartheta^{1-\sigma}}{N_1^{1-\sigma}} + \frac{x_2}{N_2^{1-\sigma}}\right]^{-1}$$

and similarly

$$\lambda_2 = u_b\,(a_2, b_2)\, e^{-\rho t} = \Lambda_2 \frac{N_2^{\sigma}}{1+\vartheta}\left[\frac{x_1 \vartheta^{1-\sigma}}{N_1^{1-\sigma}} + \frac{x_2}{N_2^{1-\sigma}}\right]^{-1}. \tag{66}$$

The remaining first order conditions in (10) are satisfied if

$$p = \frac{u_b\,(a_1, b_1)\, e^{-\rho t}}{\lambda_1} = \frac{\lambda_2}{u_a\,(a_2, b_2)\, e^{-\rho t}}$$

$$\Leftrightarrow p = \frac{(\beta a_1/\alpha b_1)\, u_a\,(a_1, b_1)\, e^{-\rho t}}{u_a\,(a_1, b_1)\, e^{-\rho t}} = \frac{u_b\,(a_2, b_2)\, e^{-\rho t}}{(\alpha b_2/\beta a_2)\, u_b\,(a_2, b_2)\, e^{-\rho t}} \tag{67}$$

$$\Leftrightarrow p = \frac{\beta a_1}{\alpha b_1} = \frac{\beta a_2}{\alpha b_2}$$

$$\Leftrightarrow p = \frac{\beta C_a}{\alpha C_b}$$

This defines p consistently with (17). Since a_1, a_2, b_1, b_2 also satisfy the boundary conditions and the state equations for $k_i = K_i/N_i$, $i = 1, 2$, $k_P = K_P/N_2$ hold, Associated Problem A reproduces the family and government decisions. By (63) the differential equation for p coincides with (21). From the first two conditions in (59),

$$\frac{p\Lambda_1}{\Lambda_2} = p\,\frac{\alpha C_b}{\beta C_a} = 1 \tag{68}$$

Using (58), (65), (66), we find consistency with (19). Taking growth rates of condition *(iii)* , by using the differential equations for Λ_1, Λ_2, Λ_P, (conditions *(v)*, *(vi)*, and *(vii)*), we get

$$\varphi^1_{K_1} = -\frac{\Lambda_1'}{\Lambda_1} = -\frac{\Lambda_P'}{\Lambda_P} = \varphi^2_{K_P}\frac{\Lambda_2}{\Lambda_1}\frac{\Lambda_1}{\Lambda_P} = \frac{p\varepsilon_P}{1+\Xi_a}\,\varphi^2_{K_P} = \frac{p\varepsilon_P}{1+\Xi_a}\,\psi^{2\prime}. \tag{69}$$

This is the efficiency condition for investment by the government. Thus

$$\frac{K_P}{K_2} = [\psi^{2\prime}]^{-1}\left(\frac{A_1(1+\Xi_a)}{p\varepsilon_P}\right) \tag{70}$$

and p follows the differential equation (34). Since $(\psi^2 \circ [\psi^{2\prime}]^{-1})'\,(x) = \dfrac{\psi^{2\prime} \circ [\psi^{2\prime}]^{-1}\,(x)}{\psi^{2\prime\prime}\,(x)} = \dfrac{x}{\psi^{2\prime\prime}\,(x)} < 0$, p converges to some value $\tilde{p}$ at which $\varphi^1_{K_1} = \psi^2$. When φ^2 is Cobb-Douglass we obtain

$$\frac{K_P}{K_2} = \left[\frac{(1-\theta)\,A_2\,\varepsilon_P}{(1+\Xi_a)\,A_1}\,p\right]^{\frac{1}{\theta}}$$

$$p' = A_1\,p - A_2^{1/\theta}\left[\frac{(1-\theta)\,\varepsilon_P}{(1+\Xi_a)\,A_1}\right]^{\frac{1-\theta}{\theta}} p^{1/\theta} \tag{71}$$

$$\tilde{p} = \left[\frac{A_1}{A_2}\right]^{\frac{1}{1-\theta}} \frac{1+\Xi_a}{(1-\theta)\,\varepsilon_P}$$

It also follows by differentiating the first equation in (71) that

$$\frac{\varepsilon_P\,(T_1+T_2)}{K_P} - \frac{\psi^2 K_2 - (1/p)\,C_2 - (1/p)\,T_2}{K_2} = \frac{1}{\vartheta}\left[A_1 - A_2\left[\frac{K_P}{K_2}\right]^{1-\theta}\right] \tag{72}$$

This equation gives T_1 once p, T_2, C_2, K_2, K_P are known.

We now show that the last two restrictions in (25) are the restrictions on taxes. Observe that $T_2 = \dfrac{(\beta/\alpha)\,\vartheta - 1}{1+\vartheta}\,C_a$, since

$$\vartheta C_a = \left(1 + \frac{\tau_2}{a_2}\right)\frac{\alpha}{\beta}\,C_a = (C_a + (1+\vartheta)\,T_2)\,\frac{\alpha}{\beta}$$

(see equation 64), and that

$$\frac{\tau_2}{c_2} \le \omega_2 \Leftrightarrow \vartheta \le \left(1 + \omega_2\left(1 + \frac{\beta}{\alpha}\right)\right)\frac{\alpha}{\beta}$$

$$\Leftrightarrow T_2 = \frac{(\beta/\alpha)\,\vartheta - 1}{1+\vartheta} C_a \le \frac{\omega_2\,(1+\beta/\alpha)}{(1+\omega_2)\,(1+\alpha/\beta)} C_a \le \frac{\beta}{\alpha}\frac{\omega_2}{1+\omega_2} C_a \Leftrightarrow T_2 = \overline{\omega}_b\, C_a.$$

Thus the third restriction in (25) means $\tau_2/c_2 \le \omega_2$. Now observe that

$$\frac{1}{\lambda_{10}}\int_0^\infty \tau_1\,\lambda_1\,dt = \frac{1}{\Lambda_{10}}\int_0^\infty T_1\,\Lambda_1\,dt = \frac{1}{\Lambda_{10}}\int_0^\infty (I_a - T_2)\,\Lambda_1\,dt$$

$$= \frac{1}{\Lambda_{10}}\int_0^\infty \left(I_a - \frac{(\beta/\alpha)\,(\vartheta-1)}{1+\vartheta} C_a\right)\Lambda_1\,dt\,,$$

and that

$$\frac{\omega_1}{\lambda_{10}}\int_0^\infty c_1\,\lambda_1\,dt = \frac{\omega_1}{\Lambda_{10}}\left(1+\frac{\beta}{\alpha}\right)\int_0^\infty \frac{\vartheta}{1+\vartheta} C_a\,\Lambda_1\,dt = \frac{\overline{\omega}_a}{\Lambda_{10}}\int_0^\infty \frac{\vartheta}{1+\vartheta}\,\underline{C_a}\,\underline{\Lambda_1}\,dt.$$

Thus the first restriction in (24) means $(\tau_1/c_1) \le \omega_1$. Ξ_a is the multiplier corresponding to these inequalities.

Summarizing, Associated Problem A maximizes the governments objective function subject to the political restrictions and the physical production restrictions (as they function *with* the distortions due to the fact that families in sector 2 perceive a higher yield on their capital K_2 then what is physically the case). This optimal value is *at least as great* as the optimal solution to the government's problem, which in principle has the additional restriction that prices are determined by the market. However, the solutions also generate the market behavior of the economy so the optimal value of the government problem is actually *equal* to the optimal value of associated Problem A. Thus we have solved the government's problem and also shown that the market optimizes physical production without restricting the government's problem (except for the distortions in K_2).

Consider Associated Problem B. In this problem $\vartheta \in \mathbb{R}$ is only a

parameter in the restriction on I_a, and the restrictions on $U_i \ge z_i$, and T_2, are not present. Let us write the multipliers of this problem with a tilde. The first order conditions are identical to conditions (59) (except for the absence of (*iv*) and (*viii*) once the factor preceding u_a and u_b in equations (*i*) and (*ii*) are replaced by 1. The solutions to Associated Problem A are the unique solutions to Associated Problem B once we make the change of variables

$$\tilde{\Lambda}_i = \left[\frac{1}{1+\vartheta}\right]^{-(1-\sigma)}\left[\frac{x_1\,\vartheta^{1-\sigma}}{N_1^{1-\sigma}} + \frac{x_2}{N_2^{1-\sigma}}\right]^{-1}\Lambda_i \tag{73}$$

Thus we can write the optimal utilities as in (29). Observe that when the restriction (28) is slack the aggregate physical flows C_a, C_b, K_1, K_2, K_P and therefore p, by (67), are independent of ϑ. Thus Cobb-Douglass preferences imply that the variable ϑ only affects distribution and not levels of production. As long as $\Xi_a = 0$ and the initial amounts of capital of both types, $K_{10} + (1/\varepsilon_P)\,K_{P0}$, K_{20}, are fixed, the aggregate physical flows are invariant. For these values of ϑ equations (29) give a convex parametric graph of possible outcomes in the (U_1, U_2) plane with negative slope.

Suppose the restriction $(\tau_1/c_1) \le \omega_1$ is slack for $\vartheta \ge \vartheta_{\text{crit}}$ but holds as an equality for $\vartheta \le \vartheta_{\text{crit}}$ (when sector 2 carries less of a burden). Then

$$\boldsymbol{t} = \frac{\partial}{\partial\vartheta}\left(\left[\frac{\vartheta}{N_1[1+\vartheta]}\right]^{1+\sigma}, \left[\frac{1}{N_2[1+\vartheta]}\right]^{1-\sigma}\right)\Bigg|_{\vartheta_{crit}} =$$

$$const \times \left(\frac{\vartheta^{-\sigma}}{N_1^{1-\sigma}}, \frac{-1}{N_2^{1-\sigma}}\right) \tag{74}$$

is a tangent vector to $((U_1(\vartheta), U_2(\vartheta))$ for $\vartheta \ge \vartheta_{\text{crit}}$ If we choose χ_{crit} so $(\chi_{\text{crit}}, 1-\chi_{\text{crit}})$ is normal to $\mathbf{t}$, the corresponding government problem has a maximum at ϑ_{crit}, and the corresponding value $\Xi_a \ge 0$ will be the slope of the objective function $\chi_{\text{crit}}U_1 + (1-\chi_{\text{crit}})U_2$ along the unrestricted (U_1, U_2) curve compared to the restricted curve for $\vartheta \le \vartheta_{\text{crit}}$. This shows that there may be a corner at $\vartheta = \vartheta_{\text{crit}}$. However, (60) and (61) imply that, if only the restriction on τ_1/c_1 holds at ϑ_{crit}, $x_1 = \chi_{\text{crit}}$, $x_2 = 1 - \chi_{\text{crit}}$, and therefore

$$\left[\frac{\alpha \overline{w}_a}{1-\sigma} + 1\right] \Xi_a = \Xi_b = 0,$$

so that $\Xi_a = 0$ and the restricted curve is tangent to the non-restricted curve at $\vartheta = \vartheta_{crit}$. If both restrictions hold, then there may be a non-trivial corner, but then ϑ_{crit} is the boundary of the government's feasible region. Similarly, by considering other values of χ making the level sets of the objective function tangent to the restricted curve at points at which it has a negative slope, the second order conditions of the optimization problem show that the restricted curve is convex.

We have reduced the first-order conditions of Associated Problem A to those defining $U_i(\vartheta)$ and to the first-order conditions of Associated Problem C. Thus the solutions U_1^*, U_2^*, ϑ^*, Ξ_a^* of both problems are equal. ■

Proof of Theorem 1. The Lagrangian of the government's problem is

$$\mathcal{L}^D = \chi U_1 + (1-\chi)\, U_2 + \sum_{i=1,2} \eta^i\,(U_i - z_i)$$

$$+ \Xi_a\left[\int_0^\infty \omega_1 \underline{c}_1\, \underline{\lambda}_1\, dt - \tau_{10}\int_0^\infty \Upsilon\, \underline{\lambda}_1\, dt\right]$$

$$+ \Xi_b\left[\int_0^\infty \frac{1}{p}\, \omega_2\, \underline{c_2}\, \underline{\lambda_2}\, dt - \tau_{20}\int_0^\infty \frac{1}{p}\, e^{\chi_2 t}\, \underline{\lambda_2}\, dt - \left(k_{20} - k_2(0)\right)\underline{\lambda_{20}}\right]$$

$$+ \Xi_0\left[\frac{1}{\varepsilon_P}\, k_{P0} + p(0)k_{20} - \frac{1}{\varepsilon_P}\, k_P(0) - p(0)k_2(0)\right].$$

Here the underlined quantities are functions of t which are equal to the values of the respective functions *after* optimization as before. Considering in the case of τ_2 a variation $\tau_2(s)$, the derivatives of U_i are:

$$\frac{\partial U_1}{\partial \tau_{10}} = -\int_0^\infty \lambda_1\, \Upsilon dt, \qquad \frac{\partial U_2}{\partial \tau_{10}} = \int_0^\infty \varepsilon_P \frac{N_1}{N_2}\lambda_P\, \Upsilon dt,$$

$$\frac{\partial U_1}{\partial s} = 0, \qquad \frac{\partial U_2}{\partial s} = \int_0^\infty \left(\varepsilon_P\lambda_P - (1/p)\,\lambda_2\right)\frac{\partial \tau_2}{\partial s}\, dt,$$

while $\partial U_2/\partial k_2(0) = \lambda_2(0)$ and $\partial U_2/\partial k_P(0) = \lambda_P(0)$. Observe that the derivative of the Lagrangian under the variation $\tau_2(s)$ is

$$\frac{\partial \mathcal{L}^D}{\partial s_2} = \int_0^\infty \left[x_2\, \varepsilon_P\, \lambda_P - (1/p)\left(x_2 + \Xi_b\right)\lambda_2\right]\frac{\partial \tau_2}{\partial s_2}\, dt, \tag{78}$$

defining x_i as in (58). When sector 2's tax restriction is lax, the variation $\tau_2(s)$ is unrestricted. When the restriction holds, the variations must satisfy the restriction $\int_0^\infty (1/p)\,(\partial\tau_2/\partial s_2)\,\lambda_2\, dt = 0$.Therefore

$$\Xi_b = 0 \Rightarrow (1/p)\, x_2\, \lambda_2 = x_2\, \varepsilon_P\, \lambda_P, \quad \Xi_b > 0 \Rightarrow x_2\, \varepsilon_P\, \lambda_P = \varsigma_\beta (1/p)\, \lambda_2 \tag{79}$$

where ς_b is a real number. The remaining first order conditions, aside from complementary slackness, are (after eliminating Ξ_0)

$$x_1\int_0^\infty e^{\chi_1 t}\, \lambda_1\, dt + \Xi_a\int_0^\infty \Upsilon\, \lambda_1\, dt = x_2\int_0^\infty \varepsilon_P \frac{N_1}{N_2} e^{\chi_1 t}\lambda_P\, dt \tag{80}$$

$$(x_2 + \Xi_b)\, \lambda_2(0) = p(0)\varepsilon_P\, x_2\, \lambda_P(0).$$

The last equality means that (79) holds at $t = 0$, and that $\varsigma_b = x_2 + \Xi_b$, so that (79) can be re-written as in (45). This condition implies, by logarithmic differentiation, the efficiency conditions (46) (and therefore a less than efficient ratio when there is a restriction in investment), which when φ^2 is Cobb-Douglass implies (47). A further logarithmic differentiation implies (23). The first condition in (80) implies $x_1 + \Xi_a$ and x_2 are both zero or both non-zero (since each of these quantities is non-negative). But their sum is $1 + \eta^2 + \eta^1 + \Xi_a \geq 1$. Hence they are both non-zero. In particular, when sector 1 is in power $\chi = 1$, so $x_2 = \eta^2 > 0$ and $U_2 = z_2$ while when sector 2 is in power $\chi = 0$, so $x_1 + \Xi_a = \eta^1 + \Xi_a > 0$ and either $U_1 = z_1$ or sector 1 is being taxed to its limit.

It is clear that Associated Problem D reproduces the first order conditions that define τ_2, $k_2(0)$, $k_P(0)$, so that U_2 takes the value $U_2(\tau_{10})$. The remaining first order condition corresponds to Associated Problem E, which therefore is a simplified representation of the problem. ■

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