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Effort Provision in Peer Groups

ISABEL MELGUIZO AND SERGIO TOVAR

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ORCID: 0000-0003-0463-9867 (Isabel Melguizo)

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Carretera México Toluca 3655, Col. Lomas de Santa Fe, 01210,
Álvaro Obregón, Ciudad de México, México.
www.cide.edu

𝕏@LibrosCIDE

Oficina de Coordinación Editorial
editorial@cide.edu
Tel. 5081 4003

Abstract

We study a model in which individuals, that are heterogeneous along a single dimension capturing productivity, choose which of two available groups to join and how much costly effort to exert within their chosen group. On the one hand, individuals like to be in groups in which others' average performance is high (global quality). On the other hand, individuals are concerned with their ranking with respect to their peers' average performance (local standing). Nash equilibrium efforts are such that the higher the individual's productivity the higher her private outcome. In contrast, it is not necessarily the case that highly productive individuals exert more effort. Nash equilibrium efforts are never efficient and whether they are higher or lower than efficient efforts, depends on the strength of global quality versus local standing concerns. Stable partitions of the society into groups may either resemble grouping by productivity or productivity mixing. In contrast, efficient partitions must always exhibit grouping by productivity.

Keywords: peer groups, segregation, mixing, effort choices, welfare.
JEL Codes: D61, D60, Z13

Resumen

Estudiamos un modelo en el que los individuos, que son heterogéneos a lo largo de una única dimensión que captura la productividad, eligen a cuál de los dos grupos disponibles unirse y cuánto esfuerzo costoso ejercer dentro de su grupo elegido. Por un lado, a los individuos les gusta estar en grupos en los que el rendimiento promedio de los demás es alto (calidad global). Por otro lado, a los individuos les preocupa su clasificación con respecto al rendimiento promedio de sus pares (posición local). Los esfuerzos de equilibrio de Nash son tales que cuanto mayor sea la productividad del individuo, mayor será su resultado privado. Por el contrario, no es necesariamente el caso de que los individuos altamente productivos realicen más esfuerzo. Los esfuerzos de equilibrio de Nash nunca son eficientes y el que sean mayores o menores que los esfuerzos eficientes depende de la fuerza de las preocupaciones de calidad global frente a la posición local. Las particiones estables de la sociedad en grupos pueden parecerse a la agrupación por productividad o a la mezcla de productividad. Por el contrario, las particiones eficientes siempre deben exhibir agrupación por productividad.

Palabras claves: Grupos de pares, segregación, mezcla, elección de esfuerzos, bienestar.

Códigos JEL: D61, D60, Z13.

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Effort Provision in Peer Groups *

Isabel Melguizo[a] Sergio Tovar[b]

[a]CIDE, Division of Economics, Carr. México-Toluca 3655, Santa Fe,
C.P. 01210, Mexico City, Mexico.
Corresponding Author: isabel.melguizo@cide.edu.
[b]CIDE, Carr. México-Toluca 3655, Santa Fe,
C.P. 01210, Mexico City, Mexico.
sergio.tovar@alumnos.cide.edu.

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Abstract

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1 Introduction

According to the theory of local comparisons by Festinger (1954) individuals have an innate desire to evaluate themselves, and they do so through comparisons with others. Nowadays, it is widely accepted that individuals compare themselves with others in their local reference group and that such comparisons determine, at least partially, their happiness.[1] Falling behind others is a source of pain. Choi et al. (2022) documents how the prevalent cultural norm of Singapore, Kiasu, commonly translated as the Fear of Losing Out, generates a constant concern among students about failing to keep up with others. Being above others quite often provides benefits. Apart from these local comparisons concerns, individuals also like to belong to high quality groups, where members are successful, because of the positive spillover effects that this may generate.

Heath (1993) emphasizes that one of the most important determinants of the selection of a university by students is the status or prestige of such institution. But also, students are concerned about how they stand academically within their circle of classmates and friends. The annual Survey of Admitted Students conducted by The National Research Center for College and University Admissions in the United States reveals that, for more than 60% of the students, school academic strength/quality is a key determinant of their enrollment decisions, and that concerns about poor performance is one of the top reasons as to why students decline enrollment offers.[2] In a similar spirit, Nolfi (1979) indicates that the attractiveness of educational alternatives first increases with the average quality of enrolled students and then decreases when such average quality is above the ability of the individual in question.

This paper examines how concerns for local standing and group quality determine the formation of groups by individuals and the provision of effort in such groups.

We analyze a full information game that is composed by two stages. In the first stage, individuals simultaneously decide which of the two available groups to join, with the restriction that each individual can belong to at most one of the groups. Individuals then learn how groups are composed and simultaneously decide how much (costly) effort to exert within their group. Effort then translates into a private outcome, that is, a grade in an exam or the number and quality of publications in a research department. Finally, payoffs are realized.

[1] See Frank (1985) and Frank (2013) for an illuminating analysis of the effect of status considerations on a wide range of economic and social dimensions such as salaries or health. See also Dijkstra et al. (2008) and Dumas et al. (2005) for evidence of how local comparisons take place in the classroom. Ferrer-i Carbonell (2005) shows that self-reported life satisfaction of West Germans is affected by the income of individuals in their reference group.

[2] See https://encoura.org/mind-gap-targeting-student-concerns-yield/ and https://encoura.org/mind-gap-targeting-student-concerns-yield/.

Individuals are heterogeneous. Individual heterogeneity materializes along a single dimension that reflects private productivity.

We assume that individuals' utility can be separated into two components: a private component and a social component. The private component reflects the utility that accrues to an individual due only to her productivity and to her chosen (costly) effort. The social component reflects the utility that accrues to an individual due to her social concerns. Such social component is itself composed of two elements: on the one hand, individuals care about the quality of the group they belong to. On the other hand, individuals care about how they rank relative to others' average performance within their own group.

The importance of group quality is justified by the positive effects that peers have on individuals' achievements. Ours is a framework where complementarities play a role. From the point of view of an individual, an increase in others' performance triggers an upward shift in that individual's effort. Prominent models of individual choice within social networks (Ballester et al., 2006; Ushchev and Zenou, 2020; Horváth, 2025) incorporate complementarities in analogous fashions. In these models, individuals' choices depend on the choices made by others to whom they are linked. We model groups instead of networks. In such groups, pairwise relations are not necessarily specified. Further, individuals do not necessarily rely on information regarding other's effort choices to make their own choices. We instead assume that individuals have some information regarding the quality of their group, specifically the others' average performance. We believe this is a natural assumption in many contexts where the emphasis is on group performance as a whole rather than on pairwise relations. Further, having specific information on each peer's effort choices may sometimes be very demanding.

Regarding local standing we simply posit that individuals are better off the higher they rank relative to others' average performance within their own group. In the area of education, there are several reasons for students to care about their relative position. It may be that better positions provide future benefits (Elsner and Isphording, 2017) or positively affect admissions at higher levels of education (Grau, 2018).[3]

We analyze partitions of the society into two groups. A partition is stable when no individual has unilateral incentives to deviate from her current group to join the alternative group. Notice that, as individuals' types are defined along a single dimension capturing productivity, groups can only consist of one (and thus consecutive) interval of productivity values or of the union of non-consecutive intervals of productivity values.[4]

[3] In a more anecdotal fashion, performing well in a research department may grant professionals with access to non-pecuniary benefits, as having access to non-corner offices or benefiting from the possibility of sabbatical periods.

[4] More formally, a group G is consecutive if for any pair of individuals' types belonging to it, all the types

As it is typical in a large proportion of group and network formation games, stable architectures, partitions in our case, are multiple in nature. Despite this potential multiplicity, we provide (Proposition 1 point 1) characteristics that groups in stable partitions must exhibit, thus limiting the number of such stable partitions. Moreover, for specific configurations of the model's primitives we are able to preclude specific partitions from being stable. In particular, for some parameter configurations, stable partitions may only exhibit productivity mixing, a situation in which partitions consist on non-consecutive groups (Proposition 1 point 3).

We consider effort choices that constitute a Nash equilibrium of the second stage of the game. Such equilibrium exists provided that individuals do not place excessive weight on others' effort choices. Intuitively, we require that the best reply of each individual to the effort made by others does not lead to an escalation of efforts (Lemma 1). In the Nash equilibrium, the private product individuals obtain, namely, productivity times effort, preserves the order of the exogenously given private productivities. In other words, more productive individuals produce more. However, it does not necessarily follow that more productive individuals exert more effort.

We also offer insights on the interesting question of how group configurations and efforts should be designed if we are interested in maximizing social welfare, that we define as the sum of individual utilities. A necessary condition to maximize social welfare is that within each group, individuals exert what we call efficient efforts. Efficient efforts and Nash equilibrium efforts never coincide (Lemma 5). In particular, effort choices made by individuals could be simultaneously efficient and a Nash equilibrium only if all the individuals would exert the same level of effort. This symmetric proposal, which has a particular form, is however never the solution of the system of best replies.

The conflict between efficiency and stability has its roots in the different ways social concerns are incorporated in individuals' decentralized effort choices versus efficient efforts. Nash equilibrium efforts incorporate both, concerns for local standing and group quality. In efficient efforts, concerns for local standing do not play a role. The reason is that in the aggregate the utility that some of the individuals derive from being above others compensates the utility losses of those who fall below. Thus, only group quality affects social welfare. From the societal point of view, the performance of each individual enters the utility function of all the remaining individuals in her group via average performance. Crucially, this effect is not internalized by individuals when they choose their desired level of effort in a decentralized way.

The aforementioned arguments lead to the conclusion that increasing the importance

in between these two individuals also belong to G.

individuals grant to group quality and decreasing the importance they grant to local standing raises efficient efforts with respect to Nash equilibrium efforts (Lemma 5).

With respect to the configuration of partitions, we show that social welfare could be maximal only if individuals are organized in consecutive groups (Proposition 3). This result follows from the fact that the sum of individual utilities is essentially the sum of individuals' private product (Proposition 2) and, crucially: (i) each individual's private product increases with others' productivity and (ii) the more productive individuals are, the more sensitive they are to others' productivity. The exact number of individuals that belong to each of the consecutive groups is however sensitive to the primitives of the model, as it is illustrated in Table 1 and Table 2, where we present our simulation results.

1.1 Our contribution

1.- We contribute to the literature that investigates the role of status concerns by naturally describing a process by which individuals form groups to engage in a strategic choice of effort, afterwards. Effort affects both the quality of the group, which confers global status, and the individuals' position relative to others' average performance, which confers local status. Global and local status are endogenous in our model. To the best of our knowledge, within the type of framework we are interested in, namely, status concerns in groups, previous literature (as it will be largely described below) either considers fixed social structures within which individuals make strategic choices, such as consumption or effort, or allow individuals to choose only their social circle. We aim to build in the direction of reconciling these two approaches.

2.- We also aim to contribute to the discussion on how status concerns affect group formation in environments other than firms.[5] We have in mind decisions such as accepting offers at universities or the formation of study groups by students. Our analysis may inform the understanding of how individuals' private productivities are the key drivers of the incentives to join groups of varying quality. In particular, we rationalize the emergence of groups featuring a mix of individuals' productivities, and not only consecutive groups.

Anecdotal evidence suggests that students do not always attend the most selective college that has admitted them. The desire to be a big fish in a small pond may guide this choice.[6] The empirical analysis by Cakir (2019) for the political system in Turkey for the period 1995-2014, suggests how resourceful politicians prefer to be part of small parties where they are more influential whereas politicians with little assets prefer to benefit from big parties.

[5] See Gola (2024) for a study on occupational sorting under social status concerns.
[6] See https://www.moorecollegedata.com/post/the-less-prestigious-college-choice.

These type of choices may create ability (productivity) mixing. Under specific configurations of parameters, the model offer results in line with this evidence.

Consider an individual evaluating whether to join a new group different from her current one. Despite the fact that individuals in this potential new group perform, on average, better than individuals in her current group, this better performance does not compensate the fact that she is among the worst performers if she joins the new group.[7] Our results thus shed light on the choice of being a big fish in a small pond or a small fish in a big pond, highlighting how these motives may result, in equilibrium, in consecutive (segregated) groups as well as in groups with ability mixing.

3.- We shed light on the relationship between effort choices and group formation in the presence of status concerns.

(i) more productive individuals obtain higher private outcomes, but it is not always the case that they exert more effort. The level of effort exerted depends on each individual's peers, specifically on those peers' productivity. In our model, efforts exerted by individuals are complements. A highly productive individual grouped with low-productivity others may end up exerting a low level of effort, while a low productivity individual grouped with highly productive others may end up exerting a high level of effort.

(ii) stable partitions may exhibit grouping by productivity but also mixing of productivities. That is in contrast with the pervasive emergence of segregative outcomes in group formation models within the strand of literature studying jurisdictions and the provision of public goods (see Baccara and Yariv (2013) and Jehiel and Scotchmer (2001) and the references therein).

The works by Pack and Pack (1977) for Pennsylvania and Persky (1990) for the Chicago metropolitan area conclude that communities appear to be more heterogeneous than the well-known Tiebout model predicts. Persky (1990) argues that Frank (1985)'s local status model is able to consistently explain such heterogeneity patterns. In a similar vein Stein (1987) documents little sorting across different dimensions (income, occupation, education) in the majority of the states in the United States. For the Boston metropolitan area Epple and Platt (1998) document how the incomes of the wealthiest households in a jurisdiction of low average income exceeds the incomes of the poorest households in a jurisdiction of high average income.[8]

The rest of the paper unfolds as follows. Section 2 provides further literature connections.

[7] This reasoning is behind the behaviour of individual 3 in Example 5, where a stable partition is formed by non-consecutive groups. In other cases, it is simply that the individual's productivity is so low that she has no incentives to join a group with higher average performance. This is the case of individual 5 in Example 5.

[8] See also Staab (2024) for a discussion on these lines.

Section 3 presents the model and the equilibrium concept. Section 4 presents the equilibrium analysis. Section 5 is devoted to the analysis of efficient outcomes. A discussion of the conflict between stability and efficiency then proceeds. Section 6 discusses about the existence of stable partitions. Section 7 concludes. The Appendix in Section 8 contains a discussion on additional aspects of the model (Subsection 8.1) and the technical proofs (Subsection 8.2).

2 Further literature connections

This paper is closely related to the literature that focuses on status concerns. To the best of our knowledge, Damiano et al. (2010) and Staab (2024) are the most closely related papers. Both papers incorporate individuals' concerns for group quality and local standing, although their models differ from the one we present here as we discuss below.

Damiano et al. (2010) consider a model in which individuals choose between two organizations of fixed capacity and derive utility from the mean quality of an organization as well as from their ranking within such organization. Contrary to our case, apart from choosing organization, no additional choice is made by individuals. Also in contrast, the authors consider a many-to-one matching model. The equilibrium consists of two overlapping intervals of individuals' types. In contrast to our results, perfect segregation of the individuals into groups is not necessarily a characteristic of efficient configurations.

Staab (2024) considers a model in which individuals observe prices for group membership and must decide group belonging and the level of engagement within their group. In contrast to our case, a key assumption in Staab (2024) is that higher types value group quality more. Additionally, the level of engagement determines how much and individual benefits from a group, but it has no strategic implications. In our case, efforts result from a strategic interaction of group members. Such efforts determine local standing and group quality. The research questions are also different. Our interest is in the relationship between strategic effort choices and group formation. Staab (2024) analyzes which groups can be formed and which might be offered by an institution, such as a monopolist or a competitive market. The author is also interested in the emergence of social exclusion.

Within the line of research studying the role of local standing of individuals embedded in networks, López-Pintado and Meléndez-Jiménez (2021) consider a dynamic model of random networks in which individuals derive extra utility when their performance in their group is above a comparison threshold that measures others' performance, as in our case. In contrast to our case, the authors primarily consider homogeneous agents and do not investigate the role of group quality. The authors' main research question, namely the role of competitiveness in large societies, is also different from ours.

Ghiglino and Goyal (2010) and Immorlica et al. (2017) analyze the impact of local comparisons in the choices made by agents embedded in exogenously given social networks. In contrast, in our case, group belonging is endogenously determined. The research questions in these two papers are also different from our main focus. Ghiglino and Goyal (2010) study the implications of allowing local comparisons in a general equilibrium model. Thus, they are concerned with how equilibrium prices and allocations are affected by local comparisons. Immorlica et al. (2017) consider, in contrast to our proposal, that only upward comparisons are of importance. They analyze the role of cohesion on the equilibrium outcomes.

Bramoullé and Ghiglino (2022) analyze the role of loss aversion in consumption in networks, a research question that greatly differs from our approach. They find that, in some circumstances, consumers choose the same level of consumption to avoid status losses. In our model, it is never the case that individuals' private products are the same within a group.

Ushchev and Zenou (2020) study a model in which individuals have preferences for conformity and interact on a fixed network. The authors characterize the Nash equilibrium of individual actions and also study efficient actions. In an extension of the model, the authors show that if individuals also have the option to choose their friends (endogenous network), then only pairwise Nash stable network is the complete one (when they study what they call the local aggregate model) or the homophilic network in which individuals relate only to others of the same type (in the local average model). In contrast, the current model allows for the possibility that extreme homophilic relations and more mixed group configurations arise.

Watts (2007) studies a model in which individuals, who care either about local comparisons or group quality, but not both simultaneously (contrary to our case), decide which preexisting group to join. The stability notion in Watts (2007) is less restrictive than Nash stability, as there is no free mobility, and more restrictive than individual stability since it is sufficient that a non-strict majority is against the admission of a new member. The author does not consider any additional decision made by the individuals, beyond which group to join. Some of the central research questions are also different. In particular, the author analyzes what happens to stable partitions when new locations are added.

Milchtaich and Winter (2002) also consider a model of group formation with fixed groups in which individuals have preferences for joining the group with individuals who are most similar to themselves. We do not consider such homophilic preferences. Their research question is also different from our main focus, as the authors are mainly concerned with the conflict between stability and efficiency.

Morelli and Park (2016) studies the formation of coalitions by heterogeneous agents through a cooperative game in which agents care about the power of their coalition and their individual ranking within it. Contrary to our case, the number of groups is endogenous to the model. Some of the research questions, as how the division of surplus determines the structure of coalitions, are different from our main focus.

As briefly advanced, the local public good literature is also related to our proposal as it analyzes the formation of jurisdictions where a local public good is to be produced (Wooders, 1980; Greenberg and Weber, 1986; Gravel and Thoron, 2007). The coalition formation literature (Bogomolnaia and Jackson, 2002; Banerjee et al., 2001) also shares some common aspects as it essentially studies group formation. The focus of these papers is on the role of different stability notions in the context of hedonic coalition formation games.

3 The model

Let $\mathcal{N}$ be a set with a population of N individuals. Each individual is labeled as $i \in \{1, 2, ..., N\}$ and is characterized by an exogenous productivity parameter $b_i \in (0, \infty)$ that defines her type. Without loss of generality, we assume that $b_1 > b_2 > ... > b_N$.

A partition $\mathcal{G}$ of the society is a specification of two groups, G_1 and G_2, such that each individual belongs to exactly one group. Thus, the number of groups is fixed, but the formation of these groups is endogenously determined. We have that $\mathcal{G} = \{G_1, G_2\}$.

The utility of an individual i who belongs to an arbitrary group G, consists of a private component and a social component. Regarding the private component, individual i enjoys the product generated when she exerts a costly effort $e_{i,G} \in (0, \infty)$ in G. The social component consists of two aspects: the individual's standing with respect to others' average performance within her group and the quality of the group.

The expression for others' average performance within individual $i's$ group G, is given by.

$$A_{i,G} = \frac{\sum_{j \neq i \in G} b_j e_{j,G}}{|G| - 1}. \tag{1}$$

We assume that the utility of individual $i \in G$ takes the form:

$$u_i(e_{i,G}, e_{-i,G}) = \underbrace{b_i\ e_{i,G} - \frac{1}{2}\ e_{i,G}^2}_{\textit{private component}} + \underbrace{\underbrace{\alpha[e_{i,G}\ A_{i,G}]}_{\textit{group quality}} - \underbrace{\beta[A_{i,G} - b_i\ e_{i,G}]}_{\textit{local standing}}}_{\textit{social component}}, \tag{2}$$

where $\alpha, \beta > 0$.

In Eq. (2), the private component consists of the private product minus the (convex) cost of effort. The social component includes concerns for both, group quality and local standing. In this component, others' average performance is computed following Eq. (1). Regarding Eq. (2):

(i) local standing materializes in that individuals compare their performance with the average performance of others within their group. They derive utility losses whenever they fall behind the average performance of others and utility gains when they stand above that average performance. Parameter $\beta \in (0, \infty)$ captures the relevance of local standing.[9]

(ii) group quality materializes in that there are complementarities between others' performance and own productivity.[10] From the point of view of a given individual, an increase in the performance of others in her group triggers an upward shift in her effort. Parameter $\alpha \in (0, \infty)$ captures the relevance of the group quality component.

For simplicity, we assume that the utility of an individual i who is the only member in her group is zero. In other words, no activity can be carried out when an individual is alone. It is natural to think it is difficult for individuals to benefit for school or research activities if they do not belong to a group or institution.

From Eq. (2) it follows that, everything else equal, higher average performance: (i) benefits individuals through improved group quality but (ii) hurts individuals via more disadvantageous local standing, thus, a trade-off emerges. As we will see below, in extreme scenarios in which an individual is highly productive only group quality matters for her decision of which group to join.

We study a two-stage game of full information with the following timing:

1.- Individuals simultaneously decide which group to join.

2.- Individuals learn the composition of groups.

3.- Individuals simultaneously choose the effort they exert in their own group.

4.- Payoffs are realized according to Eq. (2).

Since each individual belongs to only one of the two groups, the choice of groups by individuals induces a partition of the population into such groups. Recall that a particular partition is denoted $\mathcal{G}$. Let $\mathbb{G}$ the set of all possible partitions that can be formed by the population $\mathcal{N}$ of size N.

A strategy of an individual i consists of a pair $\{G_1, G_2\} \times e_i$. The first component refers to

[9] The study by Mujcic and Frijters (2013) analyses different dimensions of the trade-off between absolute and relative income by university students in Australia. The authors conclude that the relative comparison income model, in which individuals compare themselves to the average income in society is the one that best accounts for the data when predicting observed choices.

[10] As Damiano et al. (2010) state: "naturally, people desire to join organizations with high-quality members if being in the company of high-quality colleagues raises their own utility or productivity."

the group individual i wishes to belong to. The second component is a mapping $e_i : \mathbb{G} \to \mathbb{R}_+$, where $e_{i,G\in\mathcal{G}} \in \mathbb{R}_+$ is the effort made by individual i in a group $G \in \mathcal{G}$ she belongs to when a particular partition $\mathcal{G} \in \mathbb{G}$ is considered.

When there is no ambiguity, we simply use the shorthand notation $e_{i,G}$ to refer to the effort made by individual $i \in G \in \mathcal{G}$. A profile of effort strategies $e \equiv (e_{i,G})_{i=1,\dots,N} \in \mathbb{R}_+^{N\times|\mathbb{G}|}$ is a collection of efforts made by individuals for each particular partition $\mathcal{G}$ and each particular group $G \in \mathcal{G}$.

We look for effort choices and partitions that constitute a subgame perfect equilibrium of the proposed game. In particular, we require that partitions are immune to unilateral deviations and that, for each possible partition effort choices of individuals in their own groups constitute a Nash equilibrium. Definition 1 and Definition 2 help to formalize these ideas.

Definition 1. (**Effort-choice subgame -Nash equilibrium-**) Consider a partition $\mathcal{G}$ and a particular group $G \in \mathcal{G}$. Then, the effort choices $e_{i,G}$ for each $i \in G$ constitute a Nash equilibrium of the second stage of the game (effort choice subgame) whenever

$$u_i(e_{i,G}, e_{-i,G}) \geq u_i(e'_{i,G}, e_{-i,G}) \quad e'_{i,G} \neq e_{i,G}, \text{ for each } i \in G.$$

Definition 2. (**Stable partition**) A partition $\mathcal{G} = \{G_1, G_2\}$ is stable if there exists no individual $i \in G_s \in \mathcal{G}$, $s \in \{1,2\}$ such that

$$u_i(e_{i,G_{s'}\cup\{i\}}, e_{-i,G_{s'}\cup\{i\}}) > u_i(e_{i,G_s}, e_{-i,G_s}) \quad for \quad s' \in \{1,2\}, \quad s' \neq s.$$

Definition 1 simply captures the definition of Nash equilibrium efforts. Definition 2 states that a partition is stable when no individual has incentives to move to join the alternative group.

Two comments are in order: first, for the main results we are not considering situations in which the ability of individuals to move across groups is restricted by the consent of members in the group they wish to join.[11] We will discuss on this possibility to obtain further insights. Second, stability only relies on robustness to unilateral deviations and not on group deviations.

[11] We are basically considering the Nash stability notion in Bogomolnaia and Jackson (2002), Milchtaich and Winter (2002) and Bogomolnaia et al. (2008).

4 Equilibrium analysis

In this section we first analyze equilibrium efforts within a fixed group G. We then proceed to the analysis of stable partitions.

4.1 Exerting effort in a group

From the utility specification in Eq. (2) it is direct to assess that the optimal effort of an individual i that faces others' average performance $A_{i,G}$ in group G is given by $e_{i,G}(e_{-i,G}) = b_i(1+\beta) + \alpha A_{i,G}$.[12]

Let us focus on the case in which a group G is not a singleton, thus we have that $|G| > 1$. Let W be a square matrix of size $|G|$ such that an arbitrary row i consists of the following entries: $w_{ii} = 0$ and $w_{ij} = b_j/(|G|-1)$, $j \neq i$ and $\mu_1(W)$ the largest eigenvalue of W. Let I be the identity matrix of size $|G|$. The following result characterizes the Nash equilibrium of the effort choice subgame.

Lemma 1. The matrix $[I - \alpha W]^{-1}$ is well-defined and non-negative if and only if $1 > \alpha\mu_1(W)$. Then, the effort choice subgame has a unique Nash equilibrium. In such an equilibrium $b_i\, e_{i,G} > b_j\, e_{j,G}$ for each pair of individuals i, j such that $b_i > b_j$.

Intuitively, the largest eigenvalue modulus captures the extent to which a change is amplified within the group. When $1 > \alpha\mu_1(W)$, the impact of an individual's effort on that of her peers in a group is (eventually) less than one-for-one.

The result in Lemma 1 also states that individuals' abilities predict individual outcomes, which is consistent with the literature on students' performance and the relation between cognitive skills and wages (Murnane et al., 1995; Schmitt et al., 2007; Blázquez et al., 2018). We would like to emphasize in contrast, that it is not always the case that more productive individuals exert more effort.[13] That is consistent with the findings by Babcock and Betts (2009) which suggest that ability and effort are positively but not perfectly correlated.

Also, consistent with the findings by Hopkins and Kornienko (2004) is the result that the excess of efforts made by individuals within a given group when social concerns are present leaves them ranked equally, according to private products, than when social concerns are

[12] Notice that the best reply of an individual i in G would be the same regardless of the chosen indicator of others' performance in the local standing part. That is, the optimal effort would be the same if, instead of the average performance of others, we consider, for instance, the outcome of the best performer or the outcome of the worst performer among $i's$ colleagues. It is important to emphasize that if we change $A_{i,G}$ in the group quality part, the optimal effort rule will indeed change.

[13] As an example, in the case of three individuals of private productivities $b_1 = 2$, $b_2 = 0.4$ and $b_3 = 0.25$, and $\beta = \alpha = 1$ we would have that $e_{1,G} = 7 < e_{2,G} = 8.8 < e_{3,G} = 9.2$

absent. More specifically, consider that $\alpha = \beta = 0$ in Eq. (2). Then, each individual in a given group G will be exerting exactly $e_{i,G}(e_{-i,G}) = b_i$. Thus, individuals' private product will also preserve the order of individuals' productivities.

The following example illustrates the Nash equilibrium of the effort choice subgame induced in the case of a population of two individuals that form a group.

EXAMPLE 1. Consider that $\mathcal{N} = \{1, 2\}$. Consider a group G composed by these two individuals, 1 and 2. The Nash equilibrium of effort choices is

$$e_{1,G} = (1+\beta)\frac{b_1 + \alpha b_2^2}{1-\alpha^2 b_1 b_2}, \quad e_{2,G} = (1+\beta)\frac{b_2 + \alpha b_1^2}{1-\alpha^2 b_1 b_2}.$$

From this example it can be intuitively seen how the more important the social component, via larger β or larger α, the higher the efforts exerted by both individuals. Notice also that each individual's effort is increasing in her own and others' productivity. It can also be shown that the higher an individual's productivity, the higher the effect an increase in others' productivity has on her own effort.[14]

Consider that individuals 1 and 2 have private productivities $b_1 = 0.8 > b_2 = 0.4$, respectively. Let $\beta = 1$ and $\alpha = 0.5$. In this case $e_{1,G} = 1.9$ and $e_{2,G} = 1.56$. Thus, as established in Lemma 1, $b_1\, e_{1,G} = 1.52 > b_2\, e_{2,G} = 0.62$.

4.2 Stable partitions

The discussion around stable partitions may benefit from the introduction of some definitions.

DEFINITION 3. **Consecutive group.** A group G is consecutive if for any pair $i, j \in G$ such that $b_i < b_j$ it follows that $k \in G$ whenever $b_i < b_k < b_j$.[15]

Note that individuals can be organized in groups that are either consecutive or not consecutive. According to this basic idea we define below how the two groups relate to each other according to the productivities of the individuals that form them.

DEFINITION 4. **Absolute dominance.** A group G_s absolutely dominates another group $G_{s'}$, $s \neq s'$ whenever $\forall i \in G_s$ and $\forall j \in G_{s'}$ it follows that $b_i > b_j$.

This dominance relation in illustrated in Fig. 1.

[14] Formally, we have that $\partial^2 e_{i,G}/\partial b_i \partial b_j > 0$ for $i, j = 1, 2$. That is going to be a key element in designing efficient partitions, those that maximize social welfare.

[15] Baccara and Yariv (2013) and Bogomolnaia et al. (2008) also consider the notion of consecutive groups. See also Greenberg and Weber (1986) for a previous seminal related concept in the context of coalition formation in Tiebout economies.

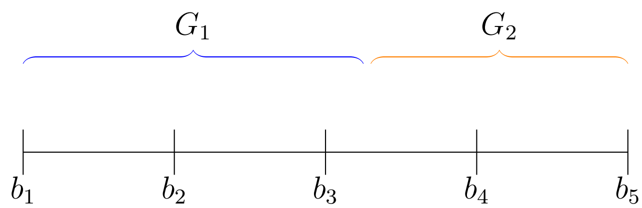

Figure 1 – A partition in which G_1 absolutely dominates G_2

Non-consecutive groups relate to each other in two alternative ways. The first case is illustrated in Fig. 2. For each individual in G_2 there is an individual in G_1 of higher productivity. Analogously, for each individual in G_1, there is an individual in G_2 of lower productivity. Definition 5 captures such relation.

Definition 5. **Relative dominance.** A group $G_s = \cup_{k=1}^{k'} \mathcal{I}_k$ relatively dominates another group $G_{s'} = \cup_{l=1}^{l'} \mathcal{J}_l$ whenever for each subinterval $k = l$ for each $i \in \mathcal{I}_k$ and for each $j \in \mathcal{I}_l$ it follows that $b_i > b_j$.

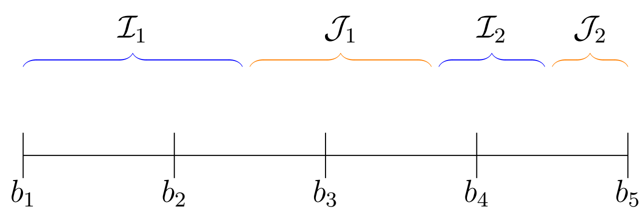

Figure 2 – A partition in which $G_1 = \mathcal{I}_1 \cup \mathcal{I}_2$ relatively dominates $G_2 = \mathcal{J}_1 \cup \mathcal{J}_2$

The second case arises when for some individuals in G_1 there is no individual in G_2 with lower productivity. Then, we may instead have the type of structure shown in Fig. 3.

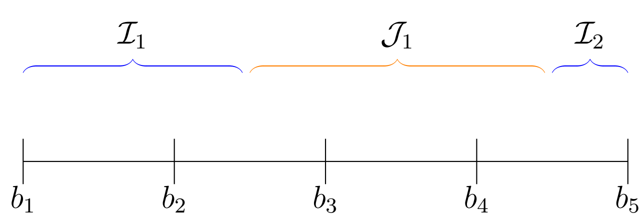

Figure 3 – A partition in which G_1 neither absolutely nor relatively dominates G_2

The characteristic of this structure is that group G_1 contains the highest productivity individuals and also lowest productivity individuals, relative to G_2. In this case no group relatively nor absolutely dominates the other.

Notice that the partition presented in Fig. 1 is the only one formed by consecutive groups. In contrast, the partitions in Fig. 2 and Fig. 3 require that (some of the) groups are non-consecutive, and, in a sense, more heterogeneous in their compositions than if they both were consecutive.

We first highlight how sufficiently productive individuals never want to be isolated. Then, we provide some of the characteristics that stable partitions must exhibit.

Lemma 2. There exists a threshold $\underline{b}(\alpha, \beta) = \beta/2\alpha(1+\beta)$ for the individuals' private productivity such that each individual i whose productivity satisfies $b_i > \underline{b}(\alpha, \beta)$ prefers to be a member of a non-singleton group G, for any value of $A_{i,G}$, than being the only member of a group.

Proposition 1. In a stable partition $\mathcal{G}$ composed by non-singleton groups:

1. For each individual $i \in G_s$, $s, s' \in \{1,2\}$, $s' \neq s$, it must hold that

   (i) $A_{i,G_s} > A_{i,G_{s'} \cup \{i\}} \Rightarrow e_{i,G_s} + e_{i,G_{s'} \cup \{i\}} > \dfrac{2\beta}{\alpha}$;

   or

   (ii) $A_{i,G_s} < A_{i,G_{s'} \cup \{i\}} \Rightarrow e_{i,G_s} + e_{i,G_{s'} \cup \{i\}} < \dfrac{2\beta}{\alpha}$;

2. If individual $i \in G_s$, $s \in \{1,2\}$ is such that $b_i > 2\underline{b}(\alpha, \beta)$, then it must be that $A_{i,G_s} > A_{i,G_{s'} \cup \{i\}}$, $s, s' \in \{1,2\}$, $s' \neq s$.

3. If $b_i > 2\underline{b}(\alpha, \beta)$ holds for each individual $i \in \mathcal{N}$ then:

   (a) all the individuals can be always organized in just one group and the other group is therefore empty.

   (b) two non-empty groups emerge if and only if for the most productive individual $i \in G_s$, $A_{i,G_s} > A_{i,G_{s'} \cup \{i\}}$, $s, s' \in \{1,2\}$, for each s and $s' \neq s$. Thus, these two groups cannot be ranked according to absolute dominance. That implies that at least one of them is necessarily non-consecutive.

Conditions (i) and (ii) in point 1 summarize the requirements to be met for an individual not to have incentives to join a group different from her current one. These conditions depend on whether in her current group she faces higher (case (i)) or lower (case (ii)) average performance than in the group she is considering to move. Such conditions involve the sum of equilibrium efforts in the current and the alternative group an individual i might join. This seems intuitive, in case (i) gains and costs of belonging to a particular group depend on whether individuals can benefit from average performance, and they do benefit if they are productive enough and therefore able to exert sufficiently high efforts. The intuition is analogous (in the natural opposite direction) for case (ii).

More formally, it can be directly stated from Eq. (2), that the utility of an individual $i \in G$ is increasing in others' average performance if and only if $e_{i,G} > \beta/\alpha$.[16] Condition 1.(i) then states that an individual does not have incentives to join the alternative group, in which the average performance would be smaller than the one she faces in her current group, whenever efforts in both her current group and the alternative group would be sufficiently high. For that to happen, it is necessarily the case that the effort exerted by such individual in her current group is larger than β/α, because otherwise condition (i) could not be satisfied. That is so because the effort exerted in the alternative group would be (using the best reply of effort choice) necessarily smaller than the effort exerted in her current group. The intuition for condition 1.(ii) is analogous.

Point 2 tells that if an individual is sufficiently productive, according to the threshold $2\underline{b}(\alpha, \beta) = \beta/\alpha(1+\beta)$, then she prefers the group with the higher average performance, as she benefits for group quality and local comparisons are not so disadvantageous. Technically, sufficiently productive individuals exert equilibrium efforts above β/α, thus the condition in (i) is satisfied for them.[17] In consequence, stability requires that no group is available to such individuals in which others' average performance increases if she deviates to join it.

An immediate consequence of the statement made in the previous point 2 and Lemma 2 is summarized in point 3. When an individual is productive enough, according to the aforementioned threshold, she does not want to be alone in a group and a partition with a single group involving all the individuals and the other group being empty is always stable.

It is important to notice that if all the individuals are sufficiently productive, again according to the aforementioned threshold, any other stable partition in which no group is empty necessarily features some mixing of abilities, as at least one of the groups should be non-consecutive. If both groups were consecutive (see Fig. 1) individuals in one of the groups will profitably deviate to the alternative group in which they experience higher average performance.

[16] In particular, the derivative of the equilibrium utility $2^{-1}e_{i,G}^2 - \beta A_{i,G}$, where $e_{i,G} = b_i(1+\beta) + \alpha A_{i,G}$, is increasing in others' average performance if and only if $e_{i,G} > \beta/\alpha$.

[17] That can be directly assessed by looking at the value of the best reply of effort choices $e_{i,G} = b_i(1+\beta) + \alpha A_{i,G}$ when it is the case that $b_i > 2\underline{b}(\alpha, \beta)$.

The necessary and sufficient condition for partitions involving two non-empty groups to be stable is that the most productive individual of each group does not face higher average performance in the alternative group. If that is the case for the most productive individual in a group, the condition also holds for any other individual who is less productive. The reason is that the less productive an individual is, the higher the average performance she faces in her own group and the lower the average performance she faces in the alternative group

Finally, it thus follows that having individuals whose private productivities do not exceed the threshold $2\underline{b}(\alpha, \beta)$ becomes a necessary condition for the stability of a partition in which a group absolutely dominates the other (see Fig. 1). From now on such partition is denoted an AD-partition. The reason is that, if all the individuals are sufficiently productive, they have incentives to join the group in which average performance is going to be the highest and such a group always exist for some individuals in an AD-partition. That is illustrated in Example 2 below.

Example 2. Consider a population $\mathcal{N} = \{1, 2, 3, 4\}$ organized into the partition $\mathcal{G} = \{G_1, G_2\}$, with $G_1 = \{1, 2\}$ and $G_2 = \{3, 4\}$. Let $\alpha = \beta = 1$ and assume that for each individual $i \in \mathcal{N}$, $b_i > 2\underline{b}(\alpha, \beta) = \beta/\alpha(1 + \beta) = 0.5$. In particular let $b_i = 1 - i/10$ for each i. As dictated by point 2 of Proposition 1 such partition cannot be stable because G_1 absolutely dominates G_2. Direct computations lead to that: $e_{3,G_2} = 3.65$ and $e_{3,G_1\cup\{3\}} = 8.09$ and thus $e_{3,G_2} + e_{3,G_1\cup\{3\}} > 2 = \beta/\alpha$. Thus, individual 3 prefers to deviate from G_2 to join G_1. More specifically, the utility of individual 3 in group G_2 is $u_3(e_{3,G_2}, e_{-3,G_2}) = 8.75 < u_3(e_{3,G_1\cup\{3\}}, e_{-3,G_1\cup\{3\}}) = 58.7$.

To gain intuition, notice that each individual in $G_1 \cup \{3\}$ has (weakly) higher productivity than any other individual in G_2. This leads to higher equilibrium efforts by each individual in $G_1 \cup \{3\}$ than in G_2 and hence to higher private outcomes. Thus, from the point of view of individual $3 \in G_2$, others' average performance is the highest in $G_1 \cup \{3\}$. That individual therefore has incentives to join G_1. Individual 3 is productive enough so that the effect of eventually disadvantageous local comparisons is overcome.

Observe also that for each $i \in G$, $e_{i,G}(e_{-i,G}) = b_i(1 + \beta) + \alpha A_{i,G} > b_i$ holds, so that, in equilibrium, whenever for some $i \in G$, $b_i \geq 2\underline{b}(\alpha, \beta)$ equilibrium effort is sufficiently high, in particular, higher than β/α. That observation, in combination with the statement in Proposition 1, give rise to the following conclusions.

Corollary 1. For an arbitrary partition $\mathcal{G}$:

1. $b_i > \beta/\alpha(1 + \beta)$, for each $i \in G_s$ such that $A_{i,G_s} > A_{i,G_{s'}\cup\{i\}}$, $s' \neq s$, is sufficient for 1.(i) of Proposition 1 to hold.

2. $b_i < \beta/\alpha(1+\beta)$, for each $i \in G_s$ such that $A_{i,G_s} < A_{i,G_{s'}\cup\{i\}}$, $s' \neq s$, is necessary for 1.(ii) of Proposition 1 to hold.

The statements made in Corollary 1 are useful because they inform about the possibilities of stability, in terms of the primitives of the model. Also they are useful in situations in which we know, because of the structure of the groups forming the partition, the way in which A_{i,G_s} and $A_{i,G_{s'}\cup\{i\}}$ relate to each other, from the point of view of an individual i. Then it is enough to look at how productive an individual is to assess, up to a certain extent, the absence of incentives to switch groups.[18]

The following Lemma 3 provides the conditions for an AD-partition $\mathcal{G} = \{G_1, G_2\}$ to arise as stable. In an AD-partition, the individuals in G_1 face a lower average performance if they deviate to join G_2 and vice versa, the individuals in G_2 face a higher average performance if they deviate to join G_1. Thus, the conditions that stable partitions must feature (see Proposition 1) materialized as established in the following result.

Lemma 3. Let $b_i < 2\underline{b}(\alpha,\beta)$ for some $i \in \mathcal{N}$. Then an AD-partition $\mathcal{G} = \{G_1, G_2\}$ where $|G_1|, |G_2| > 1$ is stable if and only if

(i) $e_{i,G_1} + e_{i,G_2\cup\{i\}} \geq \dfrac{2\beta}{\alpha}$ for each $i \in G_1$;

(ii) $e_{j,G_2} + e_{j,G_1\cup\{i\}} \leq \dfrac{2\beta}{\alpha}$ for each $j \in G_2$;

Condition (i) of Lemma 3 tells that no individual in group G_1, consisting on the individuals with the highest productivities, has incentives to join G_2, which is the group formed by the individuals with the smallest productivities. An analogous interpretation follows for condition (ii), describing the incentives of an individual in G_2.

Condition (ii) also implies that the productivity of any individual in G_2, who does not have incentives to join G_1, must be sufficiently low. In particular, it must hold that $\max_{j\in G_2} b_j < 2\underline{b}(\alpha,\beta)$. Such restriction on the magnitude of private productivities thus defines an upper bound on the cardinality of G_2 and a lower bound on the cardinality of G_1.

In the class of stable partitions characterized in Lemma 3 above, whenever two types belong to the same group, so do all types ordered between them. In such stable configurations, individuals who belong to the same group thus exhibit certain similarity.

The following example illustrates a stable AD-partition.

[18] That is, for instance, the case in the partitions studied in Lemma 3 and Observation 1 together with Lemma 4.

Example 3. Consider a population $\mathcal{N} = \{1,2,3,4\}$ organized into the partition $\mathcal{G} = \{G_1, G_2\}$, with $G_1 = \{1,2\}$ and $G_2 = \{3,4\}$. Let $\alpha = \beta = 1$ and assume that for each individual $i \in G_1$, $b_i \geq 2\underline{b}(\alpha,\beta) = \beta/\alpha(1+\beta) = 0.5$. In particular let $(b_1,b_2,b_3,b_4) = (0.6, 0.5, 0.05, 0.025)$. Direct computations lead to that $e_{3,G_1\cup\{3\}} = 1.65$ and $e_{3,G_2} = 0.11$. Therefore, for individual 3, the right hand side of condition (ii) in Lemma 3 equals 2 and the left hand side equals 1.76. Therefore, 3 does not have incentives to join G_1. It can be shown that this is also the case for individual 4. Additionally, no individual in G_1 has incentives to abandon her group. For her it directly holds that the left hand side of condition (i) is above $2\beta/\alpha$. Thus the exemplified AD-partition is stable.

The following Observation 1 describes a particular partition consisting of two groups, one that relatively dominates other. From now on, we refer to such a partition as a RD-partition. Lemma 4 provides the conditions for such partition to be stable. We also provide an example where the described RD-partition is stable.

Observation 1. Let $N \geq 5$ be an odd integer and consider the following RD-partition $\mathcal{G}' = \{G'_1, G'_2\}$ satisfying $|G'_1| = |G'_2| + 1$:

a) $G'_1 = \mathcal{I}'_1 \cup \mathcal{I}'_2$ such that $|G'_1| = 2^{-1}(N-1)+1$. Furthermore

- $|\mathcal{I}'_1| = |\mathcal{I}'_2|$ whenever $2^{-1}(N-1)+1$ is even.
- $|\mathcal{I}'_1| = |\mathcal{I}'_2| + 1$ whenever $2^{-1}(N-1)+1$ is odd.

b) $G'_2 = \mathcal{J}'_1 \cup \mathcal{J}'_2$ such that $|G'_2| = 2^{-1}(N-1)$. Furthermore

- $|\mathcal{J}'_1| = 1$;
- $|\mathcal{J}'_2| = 2^{-1}(N-1) - 1$.

Example 4. Consider a population $\mathcal{N} = \{1,2,3,4,5\}$ organized into two groups according to the specification in Observation 1. Thus we have $\mathcal{G}' = \{G'_1, G'_2\}$ such that $G'_1 = \mathcal{I}'_1 \cup \mathcal{I}'_2 = \{1,2\} \cup \{4\}$ and $G'_2 = \mathcal{J}'_1 \cup \mathcal{J}'_2 = \{3\} \cup \{5\}$.

In the partition specified in Observation 1 each individual $i \in G'_1$ would face a smaller average performance in $G'_2 \cup \{i\}$ than in her current group. The contrary happens to $j \in \mathcal{J}'_1 \in G'_2$. For such individual, others' average performance is higher in $G'_1 \cup \{j\}$. Individuals in $\mathcal{J}'_2 \in G'_2$ they may either face higher or lower average performance if they join the alternative group, that very much depends on the parameter values. According to these specifications, the following result provides the conditions under which the partition in Observation 1 emerges as stable.

Lemma 4. Consider the RD-partition described in Observation 1 and let $b_i \leq 2\underline{b}(\alpha,\beta)$ for each $i \notin \mathcal{I'}_1$. Such RD-partition is stable if and only if:

(i) $e_{i,G'_1} + e_{i,G'_2\cup\{i\}} \geq \dfrac{2\beta}{\alpha}$ for each $i \in G'_1$;

(ii) $e_{-j,G'_2} + e_{-j,G'_1\cup\{j\}} \leq \dfrac{2\beta}{\alpha}$ for $j \in \mathcal{J'}_1$;

(iii) For each $j \in \mathcal{J'}_2$;

  (a) If $A_{j,G'_2} > A_{j,G'_1\cup\{j\}}$ then (i) must hold.

  (b) If $A_{j,G'_2} < A_{j,G'_1\cup\{j\}}$ then (ii) must hold.

Moreover, if $b_i > 2\underline{b}(\alpha,\beta)$ for some $i \in \mathcal{I'}_1 \Rightarrow$ then such $i \in \mathcal{I'}_1$ does not have incentives to deviate.

Example 5. Use the environment of Example 4 and let $\alpha = \beta = 1$, $(b_1, b_2, b_3, b_4, b_5) = (0.6, 0.5, 0.27, 0.26, 0.25)$.

Individuals who belong to G'_1 enjoy a higher average performance in their group than if they join G'_2. The reason is that when comparing average performance in their current group versus average performance in the alternative group, they basically replace highly productive individuals by low productivity counterparts.[19] It thus directly follows that individuals 1 and 2 do not have incentives to join G'_2 since there are productive enough. For each of them the left hand side of the inequality in (i) in Lemma 4 is higher than $2\beta/\alpha = 2$, as $b_i \geq 2\underline{b}(\alpha,\beta) = \beta/\alpha(1+\beta) = 0.5$, $i = 1, 2$. Thus (i) directly holds. For individual 4 we have that $e_{4,G'_1} = 1.48$ and $e_{4,G'_2\cup\{4\}} = 0.68$. As $e_{4,G'_1} + e_{4,G'_2\cup\{4\}} > 2\beta/\alpha = 2$, (ii) holds for her and therefore she does not have incentives to deviate to join G'_2.

Consider now the incentives of the individuals in G'_2. Individual $3 \in \mathcal{J'}_1$ faces a higher average performance if she joins G'_1 as she replaces low productivity individuals by highly productive counterparts. We have that $e_{3,G'_1\cup\{3\}} = 1.24$ and $e_{3,G'_2} = 0.718$. Thus the left hand side of the inequality (ii) amounts to less than $2\beta/\alpha = 2$. She thus does not have incentives to join G'_1.

However it is not the case for individual 5 that she replaces low productivity individuals by highly productive counterparts. When individual $5 \in \mathcal{J'}_2$ considers joining G'_1, she replaces individual 3 by individuals 1, 2 and 4. Thus, some are more productive than 3 and some are less productive. For individual 5, it is the case that $A_{5,G'_2} = 0.194$ and $A_{5,G'_1\cup\{5\}} = 0.713$. As

[19] For a formal statement see the proofs of Lemma 3 and Lemma 4.

individual 5 faces the highest average performance in $G_1' \cup \{5\}$ hence (ii) must hold for her.[20] The left hand side of (ii) is $e_{5,G_2'} + e_{-5,G_1'\cup\{5\}} = 0.713 + 1.2 < 2\beta/\alpha = 2$. Thus, (ii) indeed holds for individual 5.

4.3 Increasing the importance of group quality ($\uparrow \alpha$), or of local standing ($\uparrow \beta$)

An interesting question is how stable partitions are shaped when group quality or local standing conerns are of increasing importance. Given the potential multiplicity of equilibria present in our model, we analyze changes in parameters α or β when we depart for a particular partition that is initially assumed to be stable.

The comparative statics exercise is not obvious. That is due to the fact that effort choices are endogenously determined and that average performance enters through two different channels, namely, group quality and local standing, into the individuals' utility function (see Eq. (2)). As an illustration, consider that group quality is on increasing importance, ($\uparrow \alpha$). In this case equilibrium effort choices also increase (recall the effort best reply), and this in turn boosts group quality. Local standing of individuals might instead be deteriorated. All these effects are quite sensitive to the values of the private productivities.

Despite of the difficulties highlighted above, we are able to predict the direction in which a stable AD-partitions reacts to changes in parameters, in specific scenarios.

Observation 2. Consider that for the primitives $(b_i)_{i\in\mathcal{N}}$, α and β an AD-partition $\mathcal{G} = \{G_1, G_2\}$ is stable.

1. Let α increase to $\alpha' > \alpha$ such that for the most productive individual $i \in G_2$ it holds that
$$\alpha' \geq \frac{\beta}{b_i(1+\beta)} > \alpha. \tag{3}$$
Then, $\mathcal{G}$ ceases to be stable, and departing from it, any stable partition is an AD-partition $\mathcal{G}' = \{G_1', G_2'\}$ such that $|G_1'| > |G_1|$ and $|G_2'| < |G_2|$.

2. Let β increase to $\beta' > \beta$ and $b_i \geq \beta'/\alpha(1+\beta')$ for each $i \in G_1$. Then $\mathcal{G}$ may cease to be stable. In such case, departing from $\mathcal{G}$ any stable partition is an AD-partition $\mathcal{G}' = \{G_1', G_2'\}$ such that $|G_1'| > |G_1|$ and $|G_2'| < |G_2|$.

[20] When G_1' relatively dominates G_2', it is not always the case that individuals in G_2' face a higher average performance if they deviate to join G_1'. For our particular RD-partition, either condition (i) or (ii) in Lemma 4 must hold for individuals in $\mathcal{J}_2'$ depending on the average performance they face if they deviate to join G_1. In contrast to the case of individual 5 in Example 5 above, consider now that private productivities are $(b_1, b_2, b_3, b_4, b_5) = (0.4, 0.39, 0.38, 0, 2, 0.19)$. It turns out that $A_{5,G_2'} = 0.341 > A_{5,G_1'\cup\{5\}} = 0.331$. In this case, it is (i), and not (ii), the condition that should be met for this individual not to have incentives to switch groups.

Point 1 of Observation 2 illustrates the case in which group quality concerns become stronger. In particular, consider an increase in α that precludes condition (ii) in Lemma 3 to hold for some individuals. Such change incentivizes, at least, the most productive individual in $i \in G_2$ to join G_1.[21] Notice that no individual in $j \in G_1 \cup \{i\}$ has incentives to move to $G_2 \setminus \{i\}$ as for them the condition in Eq. (3) automatically holds since they are, by definition, more productive than any individual in G_2. That means that condition (i) in Lemma 3 necessarily holds for individuals in G_1.

If after the departure of the most productive individual $i \in G_2$ to join G_1, condition (ii) in Lemma 3 holds for each individual $j \in G_2 \setminus \{i\}$, we have already reached a new AD-Partition, otherwise, such j joins G_1 and so on.[22] Thus, basically a sufficiently strong concern for group quality will cause the movement of low productivity individuals to the group in which average performance is higher.

An analogous analysis also applies to a situation in which instead of having common group quality concerns, individuals differ in the importance they grant to group quality. Thus, we move from having common α to a setup in which $\alpha_i \neq \alpha_j$ for some pairs $i \neq j$. We keep β constant. Suppose that for common α the AD-partition $\mathcal{G} = \{G_1, G_2\}$ is stable. When we move to a situation in which $\alpha_i > \alpha$ for each $i \in G_2$, then any AD-partition $\mathcal{G}' = \{G'_1, G'_2\}$ is such that $|G'_1| > |G_1|$ and $|G'_2| < |G_2|$.

Point 2 in Observation 2 illustrates the case in which local standing concerns become stronger. In this case, a sufficiently high α favors that the right hand side of condition (ii) Lemma 3 is not that high. Additionally, an increase in β boosts efforts sufficiently so that some individuals may be now be willing to move to the group in which they face the higher average performance, that is, condition (ii) may end up be violated for them. A more technical argument emerges by taking a look at such condition (ii) of Lemma 3. The left hand side of such condition (ii) reveals that the larger α the larger the effect of an increase in β on the average performance of individuals in a group, and thus the larger the effect on effort choices. In contrast, in the right hand side of expression (ii), the larger α the smaller the effect of an increase in β.[23]

[21] Recall that the utility of an individual i such that $b_i \geq 2\underline{b}(\alpha, \beta)$ is strictly increasing in the average performance she faces.

[22] Notice that once the most productive individual $i \in G_2$ joins G_1, from the point of view of any other individual $j \in G_2 \setminus \{i\}$, the average performance of others in $G_1 \cup \{i\} \cup \{j\}$ and in $G_2 \notin \{i\}$ are both smaller than in the case in which i would not have previously switched her group G_2 to join G_1.

[23] Formally for each $i \in G$ we have $\partial e_{i,G}/\partial\alpha\partial\beta = \partial A_{i,G}/\partial\alpha\partial\beta > 0$ and $\partial^2(\beta/\alpha)\partial\alpha\partial\beta = -1/\alpha^2 < 0$.

5 Socially optimal efforts and partitions

In this section we analyze the relevant question of how efforts and partitions should be designed in order to maximize social welfare. For this purpose, we define social welfare as the sum of individuals' utilities, that is

$$\mathcal{W} \equiv \sum_{i \in \mathcal{N}} u_i(e_{i,G}, e_{-i,G}).$$

We say that an outcome pair composed by a partition $\mathcal{G}$ and individual efforts $e_{\mathcal{G}} \equiv (e_{i,G \in \mathcal{G}})_{i=1,\dots,N} \in \mathbb{R}_+^N$ is socially optimal, or efficient, when it maximizes $\mathcal{W}$.

We start by characterizing the efforts that maximize the sum of utilities within a given group G, namely the efficient efforts. Let $e_{i,G}^E$ denote the efficient effort of individual $i \in G$ and e_G^E be the vector of efficient efforts of all the individuals in G. Let $A_{i,G}^E$ be the average performance, from the point of view of i, when all the individuals in G exert the efficient efforts.

Proposition 2. Consider a given group $G \in \mathcal{G}$. Then, for each $i \in G$ the efficient effort satisfies

$$e_{i,G}^E = b_i \left[1 + \frac{\alpha \sum_{j \neq i} e_{j,G}^E}{|G| - 1} \right] + \alpha A_{i,G}^E. \tag{4}$$

Furthermore,

1. $\sum_{i \in G} u_i(e_{i,G}^E, e_{-i,G}^E) = 2^{-1} \sum_{i \in G} b_i e_{i,G}^E$

   and thus

2. $\sum_{i \in \mathcal{N}} u_i(e_{i,G}, e_{-i,G}) = 2^{-1} \sum_{G \in \mathcal{G}} \sum_{i \in G} b_i e_{i,G}^E$.

From Eq. (4) it is direct to observe that efficient efforts do not incorporate concerns for local standing, that recall, are modulated by β. In the aggregate the gains in utility that individuals who stand above others' average performance enjoy, compensate the utility losses that individuals that stand below others' average performance suffer, thus the local standing effect cancels out. Further, as the effort exerted by an individual i (positively) affects all others' utility via group quality, efficiency requires the inclusion of such positive externality. That effect is captured through the term $\alpha[|G| - 1]^{-1} \sum_{j \neq i} e_{j,G}^E$.

Efficient efforts described by Eq. (4) are different from Nash efforts, described by the best reply $e_{i,G}(e_{-i,,G}) = b_i(1+\beta) + \alpha A_{i,G}$. Only the effort profile such that all the individuals exert the same effort level, $e_{i,G} = \beta/\alpha$ for each $i \in G$, would be a candidate to be simultaneously efficient and a Nash equilibrium. It actually follows that such symmetric proposal cannot be neither a solution of the system of best replies nor a solution to the system describing efficient efforts, thus the set of efficient and Nash efforts is empty. This result, together with some additional aspects, is stated below.

LEMMA 5. No effort profile can be simultaneously efficient and a Nash equilibrium. Specifically, we have that $e_G^E - e_G$ has, at least one entry different from zero. Further, the higher α and the smaller β the higher $e_{i,G}^E - e_{i,G}$, for each i.

The direction of the discrepancy between efficient efforts and Nash efforts depends on the importance of local standing, modulated by β and group quality, modulated by α. If group quality is of great importance, efficient efforts tend to be the highest. The reason is that in the aggregate higher efforts by individuals positively affects all others via high average performance. If on the contrary local standing is of great importance Nash equilibrium efforts tend to be the highest because in the aggregate local standing is not relevant.

The relation between efficient and Nash efforts may also vary by individual. For some individuals efficient efforts are higher than Nash efforts while for others the ranking may be reversed.[24]

EXAMPLE 6. As a follow-up of Example 1 recall that the Nash efforts are $e_{1,G} = 1.9$ and $e_{2,G} = 1.56$. For the same parameter values as in Example 1, efficient efforts are

$$e_{1,G}^E = \frac{\alpha(b_1 + b_2)b_2 + b_1}{1 - \alpha^2(b_1 + b_2)^2} = \frac{1.04}{0.64} = 1.62; \quad e_{2,G}^E = \frac{\alpha(b_1 + b_2)b_1 + b_2}{1 - \alpha^2(b_1 + b_2)^2} = \frac{0.88}{0.64} = 1.37.$$

Suppose now group quality concerns become stronger. That is, suppose there is an increase from $\alpha = 0.5$ to $\alpha' = 0.57$. In this case we would have that $e_{1,G} = 1.97$ and $e_{2,G} = 1.68$ and $e_{1,G}^E = 1,94$ and $e_{2,G}^E = 1.70$. Thus now, the efficient effort for individual 2 is above the Nash equilibrium effort. More specifically, the difference $e_{2,G}^E - e_{2,G}$ has increased for her, with a change of sign included. The difference $e_{1,G}^E - e_{1,G}$ has also increased for individual 1, from -0.28 to -0.03.

An efficient profile in which the two individuals exert the same level of effort does not exist. More specifically, requiring that $e_{1,G}^E = e_{2,G}^E$ is equivalent to demand that $1 = \alpha(b_1 + b_2)$. Notice how in this case, the expressions above for $e_{1,G}^E$ and $e_{2,G}^E$ do not take a well-defined value, as the denominator equals zero. More specifically, under the proposed parametric restriction the system of equations described by Eq. (4) does not have a solution. Perhaps

[24] As a related aspect, see the discussion in Subsection 8.1 for an analysis of how to restore efficient efforts.

more intuitively, when $1 = \alpha(b_1 + b_2)$ we have that $e^E_{1,G} = b_1 + e^E_{2,G}$ and analogously, $e^E_{2,G} = b_2 + e^E_{1,G}$. That means that each individual over-reacts to the effort made by her peer, precluding the system to yield a solution. In fact, the higher the efforts made by individuals the higher the aggregate utility.

The previous Example 6 puts attention on the need to discuss about the existence of efficient efforts, as we did for Nash equilibrium efforts (see Lemma 1). The following Lemma 6 serves this purpose.

For a group G of cardinality $|G|$, let V be a square matrix of size $|G|$ such that an arbitrary row i consists on the following entries: $v_{ii} = 0$ and $v_{ij} = (b_i + b_j)/(|G| - 1)$, $j \neq i$. Let $\delta_1(V)$ the largest eigenvalue of V. The result is as follows:

Lemma 6. The matrix $[I - \alpha V]^{-1}$ is well-defined and non-negative if and only if $1 > \alpha\delta_1(V)$. Then, efficient efforts are uniquely characterized by:

$$e^E_G = (I - \alpha V)^{-1} b.$$

Efficient efforts are such that $e^E_{i,G} > e^E_{j,G}$ for each pair of individuals $i, j \in G$ satisfying $b_i > b_j$. Thus, it follows that $b_i\, e_{i,G} > b_j\, e_{j,G}$.

In studying efficient partitions, it is important to keep in mind that in order to maximize social welfare we require that individuals exert efficient efforts, described in Eq. (4), within their groups.

The utility specification in Eq. (2) implies that individuals' productivities complement each other. More specifically, the reaction of one's individual effort to an increase in the productivity of another individual is stronger the higher the individual's own productivity.

This prediction is consistent with the findings by Ding and Lehrer (2007) that high-ability students benefit more from having higher-achieving schoolmates, than students of lower ability do. This type of complementarity in abilities naturally leads to that an efficient partition must avoid ability mixing and promote the grouping of individuals according to their productivities. That is the content of the following result.

Proposition 3. Efficient partitions consist on consecutive groups.

An additional implication is that for a partition in which some of the groups is a singleton to be efficient, such singleton group must be composed by the least productive individual.

The specific architecture of the consecutive groups inducing efficient partitions is however sensitive to the primitives of the model. The following tables show the percentage of times each of the possible partitions consisting on consecutive groups are efficient in the simple case in which $N = 4$ and $N = 6$. It is perhaps important to emphasize that the planner is also restricted by the existence of two groups. Thus his only task is to assign individuals to groups in order to maximize welfare.

We consider simulations (1000 iterations) in which we compute efficient efforts to calculate aggregate utility according to the expression in point 2 of Proposition 2. In each iteration, a vector of productivities is drawn such that efficient efforts exist. In other words, we guarantee that the statement in Lemma 6 regarding existence of efficient efforts, holds. Further, we consider different values for α.

In Table 1 and Table 2 the cases $N = 4$ and $N = 6$ are respectively presented. The partition in which one of the groups is empty is never efficient (see first row in Table 1 and Table 2). Also, partitions in which the group containing the most productive individuals is as small as possible, that is, of cardinality two, are efficient with the disproportionately highest frequency (last row in Table 1 and Table 2). For the case $N = 6$, as the group of the most productive individuals increases in size, the frequency with which such group is efficient decreases.[25]

Partition	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$	$\alpha = 1.25$	$\alpha = 1.5$
(1,2,3,4) ($\emptyset$)	0	0	0	0	0	0
(1,2,3) (4)	9.9	6.7	5.4	5.9	4.7	5
(1,2) (3,4)	90.1	93.3	94.6	94.1	95.3	95

Table 1 – Efficient partitions (%) with $N = 4$

Partition	$\alpha = 0.25$	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 1$	$\alpha = 1.25$	$\alpha = 1.5$
(1,2,3,4,5,6)($\emptyset$)	0	0	0	0	0	0
(1,2,3,4,5)(6)	0	0	0	0	0	0
(1,2,3,4) (5,6)	5.6	4.8	4.8	4.8	7	4.9
(1,2,3) (4,5,6)	19	17	15.2	17	15	17.4
(1,2) (3,4,5,6)	75.4	78.2	80.5	78.2	78	77.7

Table 2 – Efficient Partitions (%) with $N = 6$

6 Existence of stable partitions

In this section we discuss aspects related to the existence of stable partitions in our model.

[25] The codes for these simulations are available as supplementary material.

First, in the extreme case in which local standing becomes unimportant, $\beta \to 0$, or group quality becomes increasingly important, $\alpha \to \infty$, then the set of primitives $(b_i)_{i\in\mathcal{N}}$ under which the partition consisting of just one group involving all the individuals becomes bigger (in the inclusion sense). Notice that such partition is always stable if $b_i > \underline{b}(\alpha, \beta)$ for each $i \in \mathcal{N}$. If individuals' productivities satisfy the previous requirement, existence is guaranteed.

Second, the impossibility to guarantee existence for any set of parameters is (in part) due to the free mobility of individuals among groups, which may be seen as a rather demanding assumption. In this respect we discuss about the possibility of restricting free mobility.

Then, consider that a partition $\mathcal{G}$ is stable if a) there is no individual who is willing to switch her group and b) a given number of individuals in the group she pretends to join permit the adhesion of such new member. More formally:

Definition 6. In a stable partition $\mathcal{G}$ there do not exist:

a) an individual $i \in G_s$, $s \in \{1, 2\}$, such that:

$$u_i(e_{i,G_{s'}\cup\{i\}}, e_{-i,G_{s'}\cup\{i\}}) > u_i(e_{i,G_s}, e_{-i,G_s}) \quad for \quad s' \in \{1, 2\}, \quad s' \neq s,$$

and

b) at least a number $\lceil |G_{s'}|/2 \rceil$ of individuals in $G_{s'}$ (each of them j), such that

$$u_j(e_{j,G_{s'}\cup\{i\}}, e_{-j,G_{s'}\cup\{i\}}) \geq u_j(e_{j,G_{s'}}, e_{-j,G_{s'}}).$$

Thus, for a partition to be stable when an individual is willing to switch her group, at most $|G_{s'}| - \lceil |G_{s'}|/2 \rceil$ individuals in the new group she pretends to join allow this adhesion. This stability notion is very much in the spirit of Watts (2007). Notice how with this alternative definition:

1. A stable AD-partition $\mathcal{G} = \{G_1, G_2\}$ exists for any parameter configuration $(b_i)_{i\in\mathcal{N}}$, α and β such that $b_i > 2\underline{b}(\alpha, \beta)$ for individuals in G_1. In this case the number of individuals that would allow the adhesion of a new member to G_1 is not sufficient since in fact all of them suffer utility losses when others' average performance decreases. Recall that whenever an individual $j \in G_2$ moves to G_1 the average performance from the point of view of any individual i currently in G_1 decreases because j is less productive than any individual originally in G_1.

2. Consider an RD-partition $\mathcal{G} = \{G'_1, G'_2\}$, as the one described in Observation 1. Notice that $|\mathcal{I}'_1| = \lceil |G'_1|/2 \rceil$. Let $(b_i)_{i\in\mathcal{N}}$, α and β be such that $b_i > 2\underline{b}(\alpha, \beta)$ for each $i \in \mathcal{I}'_1$. In this case, the movement to G'_1 of any $i \in \mathcal{J}'_2$ is vetoed by all the individuals in $\mathcal{I}'_1$. The reason is the one presented above: this adhesion lowers the average performance that each of the individuals in $\mathcal{I}'_1$ faces. It then follows that such a RD-partition would be stable

when condition (i) only for the individuals in $\mathcal{I}_2'$ is satisfied and also, either condition (ii) of Lemma 4 is satisfied or such condition (ii) is not satisfied but still individuals in $\mathcal{I}_1'$ veto the adhesion of individual $j \in \mathcal{J}'_1$.

Overall, when $b_i > 2\underline{b}(\alpha, \beta)$ for each $i \in \mathcal{I}_1'$, we need to just analyze the behavior individuals in $\mathcal{I}_2'$ and of individuals in $\mathcal{J}_1'$.

First, it is intuitive that when individual $j \in \mathcal{J}_1'$ is of sufficiently low productivity, that is, sufficiently similar to individuals in $\mathcal{I}_2'$, the average performance from the point of view of individuals in $\mathcal{I}_1'$ would go down if they admit $j \in \mathcal{J}_1'$ into G_1'. We illustrate this point in the following example.

Example 7. Consider the same environment as in Example 4 and Example 5. That is, a population $\mathcal{N} = \{1, 2, 3, 4, 5\}$ and a partition $\mathcal{G}' = \{G_1', G_2'\}$ such that $G_1' = \mathcal{I}_1' \cup \mathcal{I}_2' = \{1, 2\} \cup \{4\}$ and $G_2' = \mathcal{J}_1' \cup \mathcal{J}_2' = \{3\} \cup \{5\}$. Let $\alpha = \beta = 1$ and $(b_1, b_2, b_3, b_4, b_5) = (0.6, 0.5, 0.27, 0.26, 0.25)$. In this case, for individuals 1 and 2 who belong to G_1', $A_{1,G_1'} = 0.62 > A_{1,G_1' \cup \{3\}} = 0.47$ and $A_{2,G_1'} = 0.74 > A_{2,G_1' \cup \{3\}} = 0.55$. Thus, they both will veto that 3 joins G_1'. In this case, stability only requires that individual $4 \in \mathcal{I}_2'$, prefers to stay in her own group. As shown above that is already the case.

In line with the discussion above, the following result establishes how equilibrium efforts are shaped when an individual enters a group. With this information, we can keep track of how average performance is affected and then offer sharper insights regarding the stability of a RD-partition, as it will be made clearer below. For this purpose let e^{+j} the vector of equilibrium efforts of individuals in $G_1' \cup \{j\}$ and e the augmented vector of equilibrium efforts of individuals in G_1' where an entry equal to zero is introduced in the position that individual j would occupy, according to the ranking of productivities.

Lemma 7. Consider a RD-partition as the as the one described in Observation 1. Define $\Delta_j e = e^{+j} - e$. It then follows that

$$\frac{(\partial \Delta_j e)_i}{\partial b_j} > 0$$

for each $i \in G_1' \cup \{j\}$.

The equilibrium efforts of individuals in $G_1' \cup \{j\}$ are increasing in b_j because on the one hand the efforts contained in e, that are computed before individual j joins the group, are not affected by productivity b_j and on the other hand, the efforts contained in e^{+j} all increase when the productivity of j increases. Then, the smaller b_j the easier is that the average performance from the point of view of individuals in $\mathcal{I}_1'$ decreases when j joins the group. That is consistent with Example 7. Hence, such individuals will veto the adhesion of individual j.

In what follows we illustrate this intuition for a RD-partition in the case in which $N = 5$. In Table 3, the RD-partition described in Observation 1 is stable for the parameters considered, when $b_3 = \{0.21, 0.4, 0.5\}$. Individual 4 does not have incentives to join G'_2 as $e_{4,G'_1} = 2.5 > 2\beta/\alpha = 2$, thus condition in (i) of Lemma 5 is satisfied for her. Further, individuals 1 and 2 are sufficiently productive and thus they do not have incentives to deviate to join G_2. They do not allow the adhesion of 3 to their group G'_1, since this lowers all the efforts and hence average performance from the point of view of both individuals 1 and 2. They neither allow that 5 joins G'_1.

$\Delta_3 e$	$b_3 = 0.21$	$b_3 = 0.4$	$b_3 = 0.5$	$b_3 = 0.6$	$b_3 = 0.69$
$e_{1,G'_1\cup\{3\}} - e_{1,G'_1}$	-0.46	-0.2	-0.02	0.19	0.4
$e_{2,G'_1\cup\{3\}} - e_{2,G'_1}$	-0.52	-0.25	-0.06	0.16	0.37
$e_{3,G'_1\cup\{3\}} - e_{3,G'_1}$	1.68	2.21	2.52	2.85	3.13
$e_{4,G'_1\cup\{3\}} - e_{4,G'_1}$	-0.82	-0.51	-0.3	-0.03	0.2
e_{4,G'_1}	2.5	2.5	2.5	2.5	2.5

Table 3 – Differences in equilibrium efforts for increasing values of b_3, $(b_1, b_2, b_3, b_4, b_5) = (0.8, 0.7, b_3, 0.2, 0.1)$ and $\alpha = \beta = 1$

Second, we provide a sufficient condition for individuals in $\mathcal{I}'_2$ not to have incentive to abandon their own group.

LEMMA 8. Consider a RD-partition described in Observation 1 and let $b_j \geq 2\underline{b}(\alpha, \beta)$ for each $j \in \mathcal{I}'_1$. Then, if for the least productive individual $k \in \mathcal{I}'_2$ it holds that

$$b_k(1+\beta) \geq \frac{\beta}{\alpha}\left[2 - \frac{\alpha \sum_{i\in\mathcal{I}'_1} b_j}{|G'_1| - 1}\right] \tag{5}$$

then, no individual $j \in \mathcal{I}'_2 \in G'_1$ has incentives to join G'_2.

If group quality is sufficiently relevant with respect to local standing (low β/α), individuals in $\mathcal{I}'_2$ would have less incentives to abandon her own group. The same happens in individuals in $\mathcal{I}'_1$ are sufficiently productive. In both situations the right hand side of Eq. (5) tends to be small.

The results in Lemma 7 and Lemma 8 allow us to conclude that (i) the higher the productivities of the individuals in $\mathcal{I}'_1$, who are also the most productive individuals in the society, and (ii) the closer $j \in \mathcal{J}'_1$ is to the most productive individual in $\mathcal{I}'_2$, the easier is that the conditions that ensure the stability of the proposed RD-partition are met.

7 Conclusions

This paper studies a model in which individuals are concerned with the quality of their group and with their standing within such group. In contrast with the majority of papers within this literature, both group configurations and effort choices are endogenously determined within the model.

The purpose of the paper is to shed light on the relation between effort choices and group formation in environments other than firms. Despite the potential multiplicity of equilibria, we provide properties that stable partitions must satisfy. Further, for specific parameter configurations we are able to exclude some of the configurations as possible stable outcomes. Finally, we rationalize the emergence of group configuration resembling segregation but also mixing of productivities, which is consistent with anecdotal evidence.

In our model, equilibrium efforts are never efficient. In light of this result, we discuss on the tax/subsidy scheme that a social planner would implement to recover efficiency. How the fact that individuals anticipate the introduction of tax and subsidies affects the formation of groups in the first stage is an interesting question that is left for further research.

Regarding group configurations, efficiency requires that groups are consecutive. We would like to emphasize how this result is due, of course, to the functional form of the utility of individuals but also to our choice of the social welfare function. In particular, we care about the sum of individual utilities and we assume that all the individuals are equally important. In this case the complementary nature of individual productivities naturally leads to the result stated above.

If alternatively we would consider that some, maybe the less productive individuals, are more important from the societal point of view, the result would probably change in favor of mixing productivities. More specifically, and in the context of the important debate on whether schools should introduce ability tracking or not, we would like to clearly state that we are not advocating that ability tracking is better than ability mixing. This is a mainly empirical question, very sensitive to the characteristics of the environment in question. A model that incorporates other aspects more specific to the analysis of educational policies would probably be needed.

Finally, and in relation to the local standing part of the utility function, an interesting venue of research entails considering individuals that are more affected by losses, being below others, than by gains, being above others (Tversky and Kahneman, 1991).

8 Appendix

8.1 Further discussion

In this section we discuss additional aspects that may be of interest, namely recovering efficiency in efforts and allowing for the number of groups to be higher than two.

1.- Restoring efficient efforts. We comment on the possibility of introducing per unit taxes/subsidies to restore efficient efforts. Suppose there is a social planner interested in restoring such efforts. In particular, suppose we add a stage before the effort game is played, in which a planner announces a per unit of effort tax/subsidy. In this case individuals choose effort by internalizing such tax/subsidy scheme, and that would induce the choice of efficient efforts as a result. We show how such tax/subsidy scheme would look like.

Lemma 9. The efficient efforts are restored if the social planner gives to each agent $i \in G$ the following tax/subsidy per unit of effort:

$$S_i^E = b_i \left[\frac{\alpha \sum_{j \neq i} e_{j,G}^E}{|G| - 1} - \beta \right]. \tag{6}$$

According to Eq. (6) it is necessary to subsidize individuals when others' in their own group exert average efforts above, β/α and tax individuals' effort in the opposite case.

Example 8. As a follow-up to Example 6, with $\beta = 1$ and $\alpha = 0.5$, $b_1 = 0.8$ and $b_2 = 0.4$ the Nash equilibrium efforts are $e_{1,G} = 1.9$ and $e_{2,G} = 1.56$ whereas the efficient efforts are smaller for each individual and equal to $e_{1,G}^E = 1,62$ and $e_{2,G}^E = 1.37$ respectively. Notice that both individuals are over-investing with respect to the efficient efforts. Overall externalities are negative (α is sufficiently small and β is sufficiently high). In this case, both individuals should be taxed to restore the efficient efforts. In particular, $S_1^E = -0.25 < 0$ and $S_2^E = -0.076 < 0$.

Consider now an increase from $\alpha = 0.5$ to $\alpha' = 0.57$. This change in turn yields, $e_{1,G} = 1.97$ and $e_{2,G} = 1.68$ and $e_{1,G}^E = 1,94$ and $e_{2,G}^E = 1.70$. Thus now, the efficient effort for individual 2, is above the Nash equilibrium effort (the difference $e_{2,G}^E - e_{2,G}$ has increased for her, with a change of sign included). The difference $e_{1,G}^E - e_{1,G}$ has also increased for individual 1, from -0.28 to -0.03. In this case, In particular $S_1^E = -0.025 < 0$ and $S_2^E = 0.042 > 0$.

The result in Lemma 5 and Lemma 9 as well as the illustration in Example 8 suggest that there are situations in which to increase welfare, efforts should be even taxed. That may happen in particular when local standing concerns are very important and group quality concerns, on the contrary, are not.[26]

[26] See and Helsley and Zenou (2014) and Ushchev and Zenou (2020) for an analysis of taxes/subsidies on

Notice that if it is know before groups are formed, that the social planner is going to tax/subsidy efforts in the way proposed above, then individuals will in turn modify their behavior in the group formation stage of the game. We leave this part of the analysis for further research.

2.- More than two groups. We comment here on the case in which there are more than two groups to which individuals may join. Regarding the results in Proposition 1, notice that for an arbitrary individual i conditions (i) and (ii) in point 1 of Proposition 1 are incompatible. In words, if an individual is not willing to join a group in which others' average performance is smaller than in her current group then she will in fact have incentives to join a group in which others' average performance is higher than in her current group. The implication of this result is that the maximum number of groups all of which can be ranked according to absolute dominance is two. More generally, a partition is stable only if groups are such that each individual faces either higher or lower average performance, but not both at the same time, in all the groups she evaluates whether to join.

Finally notice that when a partition consists on more than two groups but some of the groups absolutely dominates another, then conditions (i)-(ii) in Lemma 3 would become only necessary for stability.

8.2 Proofs

Proof of Lemma 1. Consider a group G of size $|G| > 1$. The best reply $e_{i,G}(e_{-i,G}) = b_i(1+\beta) + \alpha A_{i,G}$ of each $i \in G$ can be expressed in matrix form as

$$e_G = Bb + \alpha W e_G. \tag{7}$$

In expression in Eq. (7), $\alpha > 0$ is a scalar, e_G is the $|G| \times 1$ vector of efforts, b is the $|G| \times 1$ vector of productivities, B is a square diagonal matrix of size $|G|$ such that entry $b_{jj} = 1+\beta$. Finally, W is the square matrix of size $|G|$ such that an arbitrary row i consists of entries: $w_{ii} = 0$ and $w_{ij} = b_j/(|G|-1)$, for each $j \neq i$. By Theorem 6.2.24 in Horn and Johnson (2013) and Theorem III* in Debreu and Herstein (1953), the system of equations specified in Eq. (7) has a unique solution described by

$$e_G = (I - \alpha W)^{-1} Bb \tag{8}$$

if and only if $1 > \mu_1(W)$ where $\mu_1(W)$ is the largest eigenvalue of W.

effort choices. See also Langtry (2023) for somewhat related intuitions, in the context of consumption choices and local comparisons.

We show that for any pair $i, j \in G$ such that $b_i > b_j$, $b_i e_{i,G} > b_j e_{j,G}$ must hold. We proceed by contradiction. Consider

$$e_{j,G}(e_{-j,G}) = b_j(1+\beta) + (|G|-1)^{-1}\alpha\big[b_i e_{i,G} + \sum_{k\neq i,j} b_k e_{k.G}\big],$$

and

$$e_{i,G}(e_{-i,G}) = b_i(1+\beta) + (|G|-1)^{-1}\alpha\big[b_j e_{j,G} + \sum_{k\neq i,j} b_k e_{k.G}\big].$$

It directly follows that $e_{j,G} - e_{i,G} = (b_j - b_i)(1+\beta) + \alpha(|G|-1)^{-1}[b_i e_{i,G} - b_j e_{j,G}]$.

Recall that $b_j < b_i$. Consider, contrary to our previous statement, that $b_j e_{j,G} \geq b_i e_{i,G}$. In this case the right hand side of the previous equality is negative. For the left-hand side to be negative $e_{i,G} > e_{j,G}$ must hold as well, but then we have that $b_j e_{j,G} < b_i e_{i,G}$, which is a contradiction. Thus, it must be the case that $b_i e_{i,G} > b_j e_{j,G}$. ∎

Proof of Lemma 2. Consider a non-singleton group G. Using the best reply of an individual $i \in G$, we write the equilibrium utility (taking the group as fixed) of i as $2^{-1}[b_i(1+\beta) + \alpha A_{i,G}]^2 - \beta A_{i,G}$. Consider then the function $f(x) = 2^{-1}[b_i(1+\beta) + \alpha x]^2 - \beta x$. We look for the zeros of $f(x)$. Direct algebra leads to that $f(x) = 0$ for

$$x = \frac{-[\alpha b_i(1+\beta) - \beta] \pm \sqrt{\beta[\beta - 2\alpha b_i(1+\beta)]}}{\alpha^2}.$$

Thus $f(x) > 0$ for all x whenever $b_i \geq \beta/(1+\beta)2\alpha$. Thus $b_i \geq \beta/(1+\beta)2\alpha$ is sufficient for individual i not being willing to be the unique member of a group.

∎

Proof of Proposition 1. Consider a partition $\mathcal{G}$ and an arbitrary group $G_s \in \mathcal{G}$, $s \in \{1, 2\}$.

Points 1 and 2. The utility of individual i in $G_s \in \mathcal{G}$ when she plays her best reply $e_{i,G_s} = b_i(1+\beta) + \alpha A_{i,G_s}$ can be rewritten, using Eq. (2), as

$$2^{-1} e^2_{i,G_s} - \beta A_{i,G_s}. \tag{9}$$

Analogously, the payoff of individual i if she joins group $G_{s'} \in \mathcal{G}$, $s' \in \{1, 2\}$, $s' \neq s$ is

$$2^{-1} e^2_{i,G_{s'}\cup\{i\}} - \beta A_{-i,G_{s'}\cup\{i\}}. \tag{10}$$

Let $A_{i,G_{s'}\cup\{i\}} \leq A_{i,G_s}$. It then follows that $e_{i,G_{s'}\cup\{i\}} < e_{i,G}$. In this case i does not have incentives to join $G_{s'}$ if and only if Eq. (9) $\geq$ Eq. (10), that is, $e^2_{i,G_s} - e^2_{i,G_{s'}\cup\{i\}} \geq 2\beta[A_{i,G} - A_{-i,G_{s'}\cup\{i\}}]$. This expression is equivalently rewritten as

$$e_{i,G} + e_{i,G_{s'}\cup\{i\}} \geq \frac{2\beta}{\alpha}. \tag{11}$$

By an analogous reasoning, an individual j in G_s is not willing to join a group $G_{s'}$ such that $A_{j,G_s} \leq A_{j,G_{s'}\cup\{j\}}$ if and only if

$$e_{j,G_s} + e_{j,G_{s'}\cup\{j\}} \leq \frac{2\beta}{\alpha}. \tag{12}$$

Notice that Eq. (12) is violated for any j such that $b_j > \beta/\alpha(1+\beta)$, since in this case we have that $e_{j,G_s} > \beta/\alpha$ and $e_{-j,G_{s'}\cup\{j\}} > \beta/\alpha$. Thus, such an individual can only face smaller average performance in the group different from her own.

Point 3. let $b_i > \beta/\alpha(1+\beta)$ hold for each individual $i \in \mathcal{N}$, then:

(i) it directly follows from the arguments in Lemma 2 that the partition consisting on a unique group containing all the individuals and the other group being empty is stable since unilateral deviations are not profitable.

(ii) elaborating on the arguments in point 1 above, in a stable partition each individual i must be facing a higher average performance in her current group than the one she faces if she joins the alternative group, otherwise she has incentives to deviate to join the latter group, as Eq. (12) would be violated for her. Thus, the two groups cannot be ranked according to absolute dominance, because each individual in the dominated group has incentives to join the dominant group. That implies that at least one of the groups must be non-consecutive.

(iii) a necessary and sufficient condition for a partition to be stable is that the most productive individual in each group faces a higher average performance in her current group than the one she faces if she joins the alternative group. The necessary part is direct, if such an individual faces a higher average performance had she join the alternative group then she would have incentives to move as Eq. (12) is violated for her. For sufficiency, notice that if the most productive individual faces higher average performance in her current group than in the alternative group, that is also the case any other individual who is less productive. The reason is twofold: first, it is directly implied by the proof of Lemma 1 that the average performance an individual faces in a group increases in the descending order of productivities. Second, the average performance an individual faces in a given group is smaller the smaller is, ceteris paribus, her own productivity. That can be argued by considering **??** in the proof of Lemma 1, the entries in W and b decrease when the private productivity of a given individual decreases, thus that is also the case for all the equilibrium efforts. Thus, less productive individuals face higher (respectively, smaller) average performance in their own group (respectively, the alternative group) than more productive individuals. ∎

Proof of Lemma 3. Consider a partition composed by non-singleton groups $\mathcal{G} = \{G_1, G_2\}$ where G_1 absolutely dominates G_2. In this case any individual in G_1 faces a smaller average performance if she deviates to join G_2. The contrary happens to an individual in G_2 that deviates to join G_1. We prove the general statement below.

According to the proof of Lemma 1 we have that $e_G = (I - \alpha W)^{-1} Bb$. For $\alpha\mu_1(W) < 1$, we argued that $(I - \alpha W)^{-1}$ is well-defined. Note that $(I - \alpha W)^{-1}$ can be equivalently written, using the Neumann series expansion, as $T \equiv \sum_{k=0}^{\infty} \alpha^k W^k$. Let t_{ij} be an arbitrary ij entry of T. We have that $t_{ij} = \sum_{k=0}^{\infty} \alpha^k w_{ij}^{[k]}$, where $w_{ij}^{[k]}$ is ij the entry of W^k.

Consider two groups, $\overline{G}$ of cardinality n and $\underline{G}$ of cardinality m, where n and m are not necessarily equal. Set, for simplicity, $|\overline{G}|, |\underline{G}| > 2$.[27]

We first prove that when each non-zero entry in a $n \times n$ productivity matrix $\overline{W}$ associated to $\overline{G}$ is strictly higher than any other non-zero entry in a $m \times m$ productivity matrix $\underline{W}$ associated to $\underline{G}$, then each entry $\overline{w}_{ij}^{[k]}$ of $\overline{W}^k$ is (weakly) higher than any other entry $\underline{w}_{hl}^{[k]}$ in $\underline{W}^k$. We then use this result to conclude that equilibrium efforts are the highest in $\overline{G}$. In doing so, it is important to recall the expression above for Neumann series expansion of $(I - \alpha W)^{-1}$, upon which equilibrium efforts are characterized. Consider that a productivity matrix $\overline{W} = (n-1)^{-1}\overline{S}$, where $\overline{S}$ has each non-zero generic entry $\overline{s}_{ij}$ (weakly) higher than any other non-zero entry $\underline{s}_{ij}$ of a matrix $\underline{S}$ such that $\underline{W} = (m-1)^{-1}\underline{S}$. We have that $\overline{w}_{ij}^{[2]} = 1/(n-1)^2 \sum_{k=1}^{n} \overline{s}_{ik}\overline{s}_{kj}$.[28] Analogously, for $\underline{\mathrm{W}}$ consider $\underline{w}_{hl}^{[2]} = 1/(m-1)^2 \sum_{k=1}^{m} \underline{s}_{hk}\underline{s}_{kl}$. Notice then that each entry in $\sum_{k=1}^{n} \overline{s}_{ik}\overline{s}_{kj}$ is (weakly) higher than each entry in $\sum_{k=1}^{m} \underline{s}_{hk}\underline{s}_{kl}$. That implies that $\overline{w}_{ij}^{[2]} > \underline{w}_{hl}^{[2]}$ $\forall i, j, h, l$. Consider now $\overline{W}^k$ for $k > 1$. We have that $\overline{w}_{ij}^{[k]} = 1/(n-1)^k \sum_{k=1}^{n} \overline{s}_{ik}^{[k-1]}\overline{s}_{kj}$.[29] Analogously, for $\underline{W}^k$ we have $\underline{w}_{hl}^{[k]} = 1/(m-1)^k \sum_{k=1}^{n} \underline{s}_{hk}^{[k-1]}\underline{s}_{kl}$. Let each element in $\sum_{k=1}^{n} \overline{s}_{ik}^{[k-1]}\overline{s}_{kj}$ be higher than each of the elements in $\sum_{k=1}^{n} \underline{s}_{hk}^{[k-1]}\underline{s}_{kl}$. Thus, by the same reasoning as above, each entry ij in $\overline{W}^{k+1}$ is higher than any other entry hl in $\underline{W}^{k+1}$.

Let $\overline{G}$ a group that is defined by the above productivity matrix $\overline{W}$. By previous arguments, each entry in $\overline{W}^k$ is higher than any other entry $\underline{W}^k$ for each k. As the vector $\overline{b}$ of productivities associated to $\overline{G}$ has each entry weakly higher than the vector $\underline{b}$ of

[27] Below we comment on the trivial case in which some group has cardinality two.

[28] Notice that there are $(n-1)^2$ elements in such sum, since each of the $(n-1)$ elements in row i multiplies each of the $(n-1)$ elements in column j.

[29] There are $(n-1)^k$ elements in such sum, $(n-1)^{k-1}$ for column i and row i and $(n-2)(n-1)^{k-1}$ for row i and column $j \neq i$.

productivities associated to $\underline{G}$, we then conclude that for any $i \in \overline{G}$ the equilibrium effort $e_{i,\overline{G}} = \sum_j((I-\alpha\overline{W})^{-1}\beta\bar{b})_{ij}$ (see Eq. (8)) is higher than the equilibrium effort $e_{j,\overline{G}} = \sum_j((I-\alpha\underline{W})^{-1}\beta\underline{b})_{ij}$ of any $j \in \underline{G} \cup \{i\}$. Thus the private product of each individual in $\overline{G}$ is higher than the private product of any other individual in $\underline{G} \cup \{i\}$. Therefore from the point of view of $i \in \overline{G}$ average performance is the highest in her current group. The case in which an individual $i \in \underline{G}$ is considering to join $\overline{G}$ operates in an analogous way (in the natural opposite direction).[30]

Using the result in point 1 of Proposition 1 together with the insights above, it is direct to assess that the partition $\mathcal{G}$ defined above, is stable if and only if: (i) Eq. (11) holds for each $i \in G_1$ and (ii) Eq. (12) holds for each $j \in G_2$. ∎

Proof of PROPOSITION 2. Consider a group G. The utility of each $i \in G$ is given by the expression in Eq. (2). Using such an expression, we have that

$$\sum_{i\in G} u_i(e_{i,G}, e_{-i,G}) = \sum_{i\in G}\left[b_i\, e_{i,G} - \frac{1}{2}\, e^2_{i,G} + \alpha[e_{i,G}\, A_{i,G}] - \beta[A_{i,G} - b_i\, e_{i,G}]\right]. \tag{13}$$

In Eq. (13) note that $\sum_{i\in G}[A_{i,G} - b_i\, e_{i,G}] = 0$. Thus,

$$\sum_{i\in G} u_i(e_{i,G}, e_{-i,G}) = \sum_{i\in G} e_{i,G}\left[b_i - \frac{1}{2}\, e_{i,G} + \alpha A_{i,G}\right]. \tag{14}$$

The efforts that maximize the sum individual utilities is such that for each individual i

$$e^E_{i,G} = b_i\left[1 + \frac{\alpha\sum_{j\neq i} e^E_{j,G}}{|G|-1}\right] + \alpha A^E_{i,G}. \tag{15}$$

Plugging Eq. (15) into Eq. (14) we get

$$2^{-1}\sum_{i\in G} e^E_{i,G}\left[b_i + \alpha\frac{\sum_{j\neq i}(b_j - b_i)e^E_{j,G}}{|G|-1}\right]. \tag{16}$$

In Eq. (16), $\sum_{j\neq i}(b_j - b_i)e^E_{j,G} = 0$. The reason is that for each pair $i, j \in G$, the expression $-b_i e^E_{i,G} e^E_{j,G}$ which is negative from the point of view of i, enters with positive sign for individual j. Thus the sum of individuals utilities in group G amounts to $2^{-1}\sum_{i\in G} b_i e^E_{i,G}$ and therefore the sum of individuals utilities in the society is the sum of the utilities of the two groups. ∎

[30] When a group, say $\overline{G}$, has cardinality two, we need to take into account that some of the entries of the productivity matrix (and its subsequent powers of order k) are zero -these are the off-diagonal entries when k is odd and the diagonal entries when k is even-. In this case the proof follows as well. The result simply states that in the power of the productivity matrix associated to a group $\overline{G}$, each non-zero entry is higher than any other non-zero entry in the corresponding power of the productivity matrix of the group $\underline{G}$.

Proof of Lemma 4. By analogous reasoning than in the proof of Lemma 3 it follows that for two arbitrary groups, G and G', of equal cardinality, if each entry ij in the productivity matrix associated to G is weakly higher than its counterpart entry ij in G' then the equilibrium efforts in G are higher than the equilibrium efforts in G'.

Consider the RD-partition described in Observation 1. For each individual $i \in G'_1$ it follows that $|G'_1| = |G'_2 + 1|$. Notice that $|G'_2 + 1|$ is the cardinality of $G'_2 \cup \{i\}$. Moreover, each individual in G'_1 has (weakly) higher productivity than the individual that occupies the same position in G'_2. Thus, the individuals who belong to G'_1 enjoy a higher average performance in their group, than if they join G'_2. In this case (i) in Lemma 4 must hold for each of these individuals.

For $j \in \mathcal{J}'_1 \in G'_2$ we have that $|G'_1 + 1| > |G'_2|$. Consider the $|G'_2|$ least productive individuals in $G'_1 \cup \{j\}$. It follows that for each, relative to each individual in G'_2, the individual that occupies the same position among the least $|G'_2|$ productive individuals in $G'_1 \cup \{j\}$, is (weakly) more productive. Thus, relying exclusively on this comparison, we are able to conclude, using the same arguments as above, that $j \in \mathcal{J}'_1 \in G'_2$ faces a higher average performance in $G'_1 \cup \{j\}$ than in her current group G'_2.

Moreover, there are $|G'_1+1|-|G'_2|$ who are more productive than the $|G'_2|$ least productive individuals in $G'_1 \cup \{j\}$. It is directly implied by the proof of Lemma 3 that when we add to a group G, individuals that are more productive than the ones already in G, then efforts, and therefore private product, of the individuals originally in G, increase. Thus, definitely, $j \in \mathcal{J}'_1 \in G'_2$ faces a higher average performance in $G'_1 \cup \{j\}$ than in G'_2. In this case (ii) in Lemma 4 must hold for her.

Finally, for individuals in $\mathcal{J}'_2$ we cannot make the same type of claims above regarding the pairwise comparisons of productivities. Thus, either (i) or (ii) in Lemma 4 must hold for each individual in such group depending of whether she is facing lower or higher average performance in the alternative group. ∎

Proof of Lemma 5. Efficient efforts in Eq. (4) of Proposition 2 can be described in matrix form as

$$e^E_G = b + \alpha V e^E_G, \tag{17}$$

where $\alpha > 0$ is a scalar, e^E_G is the $|G| \times 1$ vector of efforts, b is the $|G| \times 1$ vector of productivities and V is the square matrix of size $|G|$ such that an arbitrary row i consists on the entries: $v_{ii} = 0$ and $v_{ij} = (b_i + b_j)/(|G| - 1)$, for each $j \neq i$.

Let O be a square matrix of size $|G|$ such that an arbitrary row i consists on the entries:

$o_{ii} = 0$ and $o_{ij} = b_i/(|G| - 1)$, for each $j \neq i$. Recall that $O = W'$. Notice that $V = W + W'$ where W is the square matrix defined in Lemma 1.

It is direct to observe that the expression in Eq. (7) and the expression in Eq. (17) coincide, and therefore the vector e of efficient and Nash equilibrium efforts coincide, whenever $b + \alpha Oe = Bb$. Using such equality, for each $i \in G$ we have

$$\frac{\alpha \sum_{j \neq i} e_j}{|G| - 1} = \beta. \tag{18}$$

Eq. (18) simultaneously holds for each $i \in G$ whenever all individuals exert the same effort β/α.

Suppose that there is an effort profile that is simultaneously efficient and a Nash equilibrium of the effort choice subgame. Then, using the expression for the efficient efforts in Eq. (4) or the best reply of efforts, it must hold that for each $i \in G$:

$$\frac{\beta}{\alpha} = b_i[1 + \beta] + \beta \frac{\sum_{j \neq i} b_j}{|G| - 1}, \tag{19}$$

or equivalently

$$\frac{\beta}{1 + \beta} = \frac{\alpha b_i}{1 - \alpha \dfrac{\sum_{j \neq i} b_j}{|G| - 1}}. \tag{20}$$

As the left-hand side is a constant, it follows that for each pair $i, j \in G$ we have

$$\frac{\alpha b_i}{1 - \alpha \dfrac{\sum_{m \neq i} b_m}{|G| - 1}} = \frac{\alpha b_j}{1 - \alpha \dfrac{\sum_{k \neq j} b_k}{|G| - 1}}. \tag{21}$$

The expression in Eq. (21) implies that

$$b_i[|G| - 1 - \alpha \sum_{k \neq j} b_k] = b_j[|G| - 1 - \alpha \sum_{m \neq i} b_m].$$

$$(b_i - b_j)[|G| - 1] = \alpha[b_i \sum_{k \neq j} b_k - b_j \sum_{m \neq i} b_m].$$

$$(b_i - b_j)[|G| - 1] = \alpha[b_i \sum_{k \neq j} b_k - b_j \sum_{m \neq i} b_m].$$

$$(b_i - b_j)[|G| - 1] = \alpha[b_i^2 - b_j^2) + \alpha(b_i - b_j) \sum_{s \neq i,j} \sum_{m \neq i} b_s.$$

When then have that

$$|G| - 1 - \alpha(b_i + b_j) = \alpha \sum_{s \neq i,j} b_s$$

or, for an arbitrary s, we equivalently write

$$|G| - 1 - \alpha \sum_{i \neq s} b_s = \alpha b_s. \tag{22}$$

According to Eq. (22) the right hand side of Eq. (20) equals $|G| - 1 \geq 1$. That is clearly a contradiction since the left-hand side of Eq. (20) is smaller than one. Thus, we conclude that no strategy profile in which all efforts are the same and equal to β/α can be efficient and a Nash equilibrium.

Using expressions Eq. (7) and Eq. (17), it has been already stated above that both efficient and Nash efforts are be the same if and only if $b + Oe = Bb$. We also argued that this equality led to an impossibility. Thus, for at least one individual, the efficient effort and the Nash efforts cannot be the same. The difference $Oe_G^E - (B - I)b$ measure the discrepancy between two such vectors of efforts. Using $e_G^E = (I - \alpha V)^{-1} b$ we have that the difference between the two vectors of efforts, $e_G^E - e_G$, must amount to

$$O(I - \alpha V)^{-1} b - (B - I)b.$$

Using the Newman series decomposition, we can simply rewrite

$$O(I - V)^{-1} b = O \sum_{k=0}^{\infty} \alpha^k V^k b. \tag{23}$$

Thus, the higher α and the smaller β, the more positive the difference $e_G^E - e_G$ is. ∎

Proof of Lemma 6. By Theorem 6.2.24 in Horn and Johnson (2013) and Theorem III* in Debreu and Herstein (1953), the system of equations described in Eq. (17) has a unique solution if and only if $1 > \gamma_1(V)$ where $\gamma_1(V)$ is the largest eigenvalue of V.

We show that for any pair $i, j \in G$ such that $b_i > b_j$, $e_{i,G}^E > e_{j,G}^E$. For this purpose let $\gamma_1 = (|G| - 1)^{-1} \sum_{k \neq i,j} b_k e_{k.G}$ and $\gamma_2 = (|G| - 1)^{-1} \sum_{k \neq i,j} e_{k.G}$

$$e_{j,G}^E = b_j(1 + \alpha\gamma_2) + \alpha\gamma_1 + (|G| - 1)^{-1} \alpha(b_j + b_i) e_{i,G}^E$$

and

$$e^E_{i,G} = b_i(1+\alpha\gamma_2) + \alpha\gamma_1 + (|G|-1)^{-1}\alpha(b_j+b_i)e^E_{j,G}.$$

It directly follows that $[e^E_{i,G} - e^E_{j,G}][1+\alpha(|G|-1)^{-1}(b_i+b_j)] = (b_i - b_j)(1+\alpha\gamma_2)$.

As $b_j < b_i$, the right hand side is positive, so is the left hand side. That implies that $e^E_{i,G} > e^E_{j,G}$ and thus it directly holds that private product preserve the order or exogenous productivities, that is, $b_i e^E_{i,G} > b_j e^E_{j,G}$. ∎

Proof of Lemma 7. We analyze how equilibrium efforts change when a new individual joins a given group G

Let Δ_i the operator that maps a vector or matrix into the values of this vector or matrix before and after an individual j joins a group. Thus $\Delta_j e = e^{+j} - e$ captures changes in equilibrium efforts and $\Delta_j W = W^{+j} - W$ captures changes in the matrix of weights changes after after i joins G. note that $\Delta_j b$ is a vector of cardinality $|G|$ in which the only non-zero entry is the one corresponding to individual j, thus it captures the inclusion of this new member in group G.

Recall that $e = \alpha We + Bb$. As individual j is not present initially in G, the entry corresponding to this individual in e takes value zero. Similarly, W is an augmented matrix of cardinality $|G|$ (instead of $|G|-1$) such that row j is full of zeros). We need to augment the cardinality of e and W in order to have consistent dimensions to operate with these matrices and vectors. We therefore write

$$\Delta_j e = \alpha[W^{+j}e^{+j} - We] + B\Delta_j b.$$

We rewrite the expression above, adding and subtracting the term $W^{+j}e$, as

$$\Delta_j e = \alpha[W^{+j}e^{+j} - W^{+j}e + W^{+j}e - We] + B\Delta_j b,$$

$$\Delta_j e = \alpha[W^{+j}\Delta_i e + \Delta_j We] + B\Delta_j,$$

$$\Delta_j e = \alpha[(W + \Delta_j W)\Delta_j e + \Delta_j We] + B\Delta_j b,$$

$$\Delta_j e = \alpha W\Delta_j e + \alpha\Delta_j W\Delta_j e + \alpha\Delta_j We + B\Delta_j b.$$

Using the fact that $\Delta_j W = W^{+j} - W$, we end up with the expression

$$\Delta_j e = \alpha W^{+j}\Delta_j e + \alpha\Delta_j W e + B\Delta_j b,$$

which is equivalently written as

$$\Delta_j e = (I - \alpha W^{+j})^{-1}[\alpha\Delta_j W e + B\Delta_j b]. \tag{24}$$

The derivative of $\Delta_j e$ in Eq. (24) with respect to b_j is

$$\frac{\partial\Delta_j e}{\partial b_j} = (I - \alpha W^{+j})^{-1}\left[\frac{1}{b_j}B\Delta_j b\right] + \frac{\partial(I - \alpha W^{+j})^{-1}}{\partial b_j}[\alpha\Delta_j W e + B\Delta_j b]$$

.

This expression is equivalent to

$$\frac{\partial\Delta_j e}{\partial b_j} = (I - \alpha W^{+j})^{-1}\left[\frac{1}{b_j}B\Delta_j b - \frac{\partial(I - \alpha W^{+j})}{\partial b_j}(I - \alpha W^{+j})^{-1}[\alpha\Delta_j W e + B\Delta_i b]\right].$$

Note that the last term $\alpha\Delta_j W e + B\Delta_i b$ is precisely $\Delta_j e$ in Eq. (24). Thus we have

$$\frac{\partial\Delta_j e}{\partial b_j} = (I - \alpha W^{+j})^{-1}\left[\frac{1}{b_j}B\Delta_j b - \frac{\partial(I - \alpha W^{+j})}{\partial b_j}\Delta_j e\right]. \tag{25}$$

To assess the sign of such derivative, we compute $\Delta_j W e$. To do so note that

$$\Delta_j W = \begin{bmatrix}
0 & \frac{-b_2}{|G|(|G|-1)} & \cdots \frac{b_j}{|G|} \cdots & \frac{-b_k}{|G|(|G|-1)} & \cdots\cdots \\
\frac{-b_1}{|G|(|G|-1)} & 0 & \cdots \frac{b_j}{|G|} \cdots & \frac{-b_k}{|G|(|G|-1)} & \cdots\cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{b_1}{|G|} & \frac{b_2}{|G|} & \cdots\ 0\ \cdots & \frac{b_k}{|G|} & \cdots\cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{-b_1}{|G|(|G|-1)} & \frac{-b_2}{|G|(|G|-1)} & \cdots\ 0\ \cdots & \cdots & \cdots\cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{-b_1}{|G|(|G|-1)} & \frac{-b_2}{|G|(|G|-1)} & \cdots \frac{b_j}{|G|} \cdots & \cdots\cdots & 0
\end{bmatrix}$$

Thus, we have that

$$\Delta_j We = \begin{bmatrix} A_1 & \\ A_2 & \\ \vdots & \vdots \\ A_{|G|} & \end{bmatrix} \text{ where for } i \neq j,\ A_i = -\frac{\sum_{m\neq i,j} b_m e_m}{|G|(|G|-1)} \text{ and } A_j = \frac{\sum_{m\neq j} b_m e_m}{|G|}.$$

Note then that $(\Delta_j e)_j > 0$, individual j exerts positive effort in $G'_1 \cup \{j\}$. Thus using Eq. (25) we conclude that

$$\frac{(\partial \Delta_j e)_i}{\partial b_j} > 0$$

for each $i \in G \cup \{j\}$, since $(I - \alpha W^{+j})^{-1}$ is a matrix of positive entries and $\left[\frac{1}{b_j} B\Delta_j b - \frac{\partial (I-\alpha W^{+j})}{\partial b_j}\Delta_j e\right]$ in Eq. (25) is a vector of positive entries. In particular, entry j is $(1+\beta)$ and each entry $i \neq j$ is $\alpha A_j/|G|$. ∎

Proof of Lemma 8. Consider a RD-partition described in Observation 1 and let $b_i \geq 2\underline{b}(\alpha,\beta)$ for each $i \in \mathcal{I}'_1$. For each individual $j \in \mathcal{I}'_2$ not to have incentives to join G'_2, condition in (i) in Lemma 4 must hold for her. For such condition to hold it is sufficient that $e_{j,G'_1} \geq 2\beta/\alpha$. It follows that:

$$e_{j,G'_1} = b_j(1+\beta) + \frac{\alpha \sum_{i\in\mathcal{I}'_1} b_i e_{i,G'_1}}{|G'_1|-1} + \frac{\alpha \sum_{i\neq j\in\mathcal{I}'_2} b_i e_{i,G'_1}}{|G'_1|-1}. \tag{26}$$

Since $b_i \geq 2\underline{b}(\alpha,\beta)$ for each $i \in \mathcal{I}'_1$, each of these individuals exerts, at least, an effort of β/α in G'_1. Thus, a lower bound for Eq. (26) is

$$b_j(1+\beta) + \frac{\beta \sum_{i\in\mathcal{I}'_1} b_i}{|G'_1|-1}.$$

Thus, a sufficient condition for (i) in Lemma 4 to hold for each $j \in \mathcal{I}'_2$ is that

$$b_j(1+\beta) \geq \frac{\beta}{\alpha}\left[2 - \frac{\alpha \sum_{i\neq j\in\mathcal{I}'_1} b_i}{|G'_1|-1}\right].$$

If this expression hold for the least productive individual $k \in \mathcal{I}'_2$, it holds for everyone else.

∎

Proof of Lemma 9. Consider the expression

$$u_i(e_{i,G}, e_{-i,G}) = [b_i + S_i^E]e_{i,G} - \frac{1}{2}e_{i,G}^2 + \alpha[e_{i,G}\, A_{i,G}] - \beta[A_{i,G} - b_i e_{i,G}], \tag{27}$$

where S_i^E is a per-unit of effort tax/subsidy that i faces. Then individual i chooses her effort level to maximize Eq. (27). It directly follows that S_i^E, as specified in Lemma 9, induces the choice of the efficient efforts by individuals.

∎

Proof of Proposition 3. Consider an arbitrary group G and use the expression in Lemma 6 describing efficient efforts in G in matrix form

$$e_G^E = (I - \alpha V)^{-1} b. \tag{28}$$

We have that

$$\frac{\partial e_G^E}{\partial b_i} = \frac{\partial (I - \alpha V)^{-1}}{\partial b_i} b + (I - \alpha V)^{-1} \frac{\partial b}{\partial b_i}. \tag{29}$$

In the expression above $\partial b / \partial b_i$ is a vector in which each entry $j \neq i$ is zero and entry i is equal to one. Thus, the second component of Eq. (29), that is, $(I - \alpha V)^{-1} \partial b / \partial b_i$, is a vector of positive entries. Regarding the first component of Eq. (29) we have that

$$\frac{\partial (I - \alpha V)^{-1}}{\partial b_i} = -(I - \alpha V)^{-1} \frac{\partial (I - \alpha V)}{\partial b_i} (I - \alpha V)^{-1}, \tag{30}$$

where for i and each $j \neq i$:

$$-\left[\frac{\partial (I - \alpha V)}{\partial b_i}\right]_{ij} = -\left[\frac{\partial (I - \alpha V)}{\partial b_i}\right]_{ji} = \frac{\alpha}{|G| - 1} \tag{31}$$

and for $j, k \neq i$:

$$\left[\frac{\partial (I - \alpha V)}{\partial b_i}\right]_{kj} = \left[\frac{\partial (I - \alpha V)}{\partial b_i}\right]_{jk} = 0. \tag{32}$$

Thus, $[\partial (I - \alpha V)^{-1} / \partial b_i] b$ in Eq. (29) results in a vector of positive entries. Therefore, $\partial e_G^E / \partial b_i$ in Eq. (29) has each entry positive. Using Eq. (29) it follows that

$$\frac{\partial}{\partial b_j}\left[\frac{\partial e_G^E}{\partial b_i}\right] = \frac{\partial}{\partial b_j}\left[\frac{\partial (I - \alpha V)^{-1}}{\partial b_i} b + (I - \alpha V)^{-1} \frac{\partial b}{\partial b_i}\right]. \tag{33}$$

Consider a group of cardinality higher than two. Using the Neumann series expansion we have that $(I - \alpha V)^{-1} = \sum_{k=0}^{\infty} \alpha^k V^k$, we are able to conclude that for each i, each entry in $(I - \alpha V)^{-1}$ includes the element b_j for each $j \neq i$. Thus, the second component in Eq. (33)

$$\frac{\partial}{\partial b_j}\left[(I - \alpha V)^{-1} \frac{\partial b}{\partial b_i}\right] = \frac{\partial (I - \alpha V)^{-1}}{\partial b_j} \frac{\partial b}{\partial b_i}, \tag{34}$$

is a vector of positives entries.[31] Additionally, the first component in Eq. (33)

$$\frac{\partial}{\partial b_j}\left[\frac{\partial (I-\alpha V)^{-1}}{\partial b_i}b\right] = \frac{\partial^2 (I-\alpha V)^{-1}}{\partial b_i \partial b_j}b + \frac{\partial (I-\alpha V)^{-1}}{\partial b_i}\frac{\partial b}{\partial b_j}.$$

is a vector of non-negative entries. Thus, for each pair of individuals $i, j \in G$ it follows that

(i) $\partial e^E_G / \partial b_i > 0$;

and

(ii) $\partial / \partial b_j \left[\partial e^E_G / \partial b_i\right] > 0$.

Thus, (i) each individual's effort is increasing in others' productivity and (ii) the more productive each individual is, the more sensitive she is to an increase in others' productivity.

Given that by the proof of Proposition 2 aggregate utility of individuals within a group is essentially the sum of individuals' private product and private product is productivity times effort, in an efficient partition groups should be consecutive.

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[31] The case of groups of cardinality two is analogous. The only difference is that the main diagonal of V^K is composed of zeros, for any value of k. Despite this fact, the above statement regarding the derivative in Eq. (34) follows.

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