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Diverse Opinions and Obfuscation through Hard Evidence in Voting Environments

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Abstract

When it is mandatory for leaders in voting processes to acquire and provide hard evidence, can they still obfuscate voters? How are such leaders affected by majority rules and by external sources of information in the hands of voters? How are voters affected by the leaders' obfuscation strategies? How do obfuscation strategies, and their welfare implications, depend on whether leaders are moderate or radicals? To answer these questions we investigate a model where leaders must conduct research to obtain evidence and yet such research efforts may be unsuccessful. Leaders can take advantage of this possibility that evidence be not finally obtained to conceal pieces of evidence that would harm them. In turn, voters react skeptically when leaders do not disclose any piece of evidence, which influences the optimal obfuscation strategies by leaders. Leaders want to obfuscate those voters who are closer to them within the spectrum of opinions. Moderate leaders have only weak incentives to conceal evidence and, in some circumstances, they may end up revealing all successfully obtained evidence. In contrast, radical leaders have strong incentives to conceal evidence. Radical leaders prefer that external means of information not be in the hands of voters with opinions similar to the leaders' opinions, whereas moderate leaders do not care much about which voters have external sources of information. When leaders are moderate, voters prefer that external means of information be in the hands of those voters who are closer to the leaders in their opinions.

Keywords: Information Acquisition; Strategic Obfuscation; Persuasion; Voting JEL

Classification: C72; D72; D83; D84

Resumen

¿Pueden los líderes de procesos de votación ofuscar a los votantes cuando es obligatorio adquirir y proveer evidencia? ¿Cómo son esos líderes afectados por reglas de mayoría y por medios de información externos en poder de los votantes? ¿Cómo son los votantes afectados por las estrategias de ofuscación de los líderes? ¿Cómo varían las estrategias de ofuscación, y sus implicaciones sobre bienestar, según los líderes sean moderados o radicales? Para responder estas preguntas, investigamos un modelo donde los líderes deben investigar para obtener evidencia y, aún así, esos esfuerzos de investigación pueden no dar resultados. Los líderes se aprovechan de esta posibilidad de que la evidencia no sea finalmente obtenida para ocultar evidencia que les dañaría. Los votantes reaccionan con escepticismo cuando los líderes no revelan evidencia, lo que da forma a las estrategias óptimas de ofuscación de los líderes. Los líderes quieren ofuscar a los votantes más cercanos a ellos mismos dentro del espectro de opiniones. Los líderes moderados tienen sólo incentivos débiles para ofuscar y, en algunos casos, pueden revelar toda la evidencia obtenida. Por contra, los líderes radicales tienen incentivos fuertes para ocultar evidencia. Los líderes radicales prefieren que medios externos de información no estén en poder de votantes con opiniones similares a ellos mismos, mientras que los líderes moderados no se preocupan mucho por qué votantes tienen medios de información externos. Cuando los líderes son moderados, los votantes prefieren que los medios externos de información estén en poder de los votantes con opiniones más similares a los líderes.

Palabras Clave: Adquisición de Evidencia; Ofuscación Estratégica; Persuasión; Votaciones

Clasificación JEL: C72; D72; D83; D84

Diverse Opinions and Obfuscation through Hard Evidence in Voting Environments*

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November, 2021

Abstract

When it is mandatory for leaders in voting processes to acquire and provide hard evidence, can they still obfuscate voters? How are such leaders affected by majority rules and by external sources of information in the hands of voters? How are voters affected by the leaders' obfuscation strategies? How do obfuscation strategies, and their welfare implications, depend on whether leaders are moderate or radicals? To anwer these questions we investigate a model where leaders must conduct research to obtain evidence and yet such research efforts may be unsuccessful. Leaders can take advantage of this possibility that evidence be not finally obtained to conceal pieces of evidence that would harm them. In turn, voters react skeptically when leaders do not disclose any piece of evidence, which influences the optimal obfuscation strategies by leaders. Leaders want to obfuscate those voters who are closer to them within the spectrum of opinions. Moderate leaders have only weak incentives to conceal evidence and, in some circumstances, they may end up revealing all successfully obtained evidence. In contrast, radical leaders have strong incentives to conceal evidence. Radical leaders prefer that external means of information not be in the hands of voters with opinions similar to the leaders' opinions, whereas moderate leaders do not care much about which voters have external sources of information. When leaders are moderate, voters prefer that external means of information be in the hands of those voters who are closer to the leaders in their opinions.

Keywords: Informacion Acquisition; Strategic Obfuscation; Persuasion; Voting

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1 Introduction

In voting environments, such as political elections or committees, information provision is central to gauge the available alternatives. By listening to informed leaders, voters may modify their opinions about relevant variables and, in consequence, their preferred voting outcomes. Economists and political scientists have abundantly investigated how the traditional machineries of strategic information disclosure can be fruitfully applied to voting situations. In particular, information provision by leaders to voters has been studied using the settings of multi-dimensional cheap-talk (Schnakenberg, 2015), Bayesian persuasion (Alonso and Camara, 2016), hard evidence provision when voters can be privately targeted (Titova, 2021), vote-buying screening mechanisms (Eguia and Xefteris, 2021), or behavioral approaches (Bonomi et al., 2021).

In this paper, we are concerned with public disclosure of "hard evidence." Political and committee leaders are often enforced (by institutional and/or legal mandates) to make investigation choices and to disclose verifiable information obtained from such efforts. Government agencies must seek for information relevant to their citizens (e.g., on a wide variety of health, natural phenomena, economic, or social issues relevant to the population). Hiring committees' leaders are in charge of obtaining additional information about job candidates before submitting proposals for voting. In many countries, state-regulated media outlets must investigate and disclose information relevant in elections. Campaign leaders are enforced to disclose information, which is subject to fact-checking by the media and by state agencies. In some legal systems, board leaders must conduct research and disclose information about prospective mergers before submitting the merging proposal for the approval of the shareholders.

In most practical situations, nonetheless, leaders are also interested in the outcome of the voting process, and their opinions about the most suitable course of action might not coincide with those of the (required majority of) voters. In addition, there is a prevalent view among political scientists (Carmines and Stimson, 1980; Sniderman et al., 1991; Carpini and Keeter, 1996) that having access to external means of information, such as education, make voters rely less on their initial opinions (e.g., opinions attached to ideological positions).

In these scenarios, some fundamental questions remain open when there are conflicting opinions about the most suitable course of action. Can a leader that is legally enforced to investigate and communicate *only through hard evidence* still strategically obfuscate voters? If so, in which ways? How does the majority rule influence the investigation efforts of the leader? Which type of majority rule would the leader prefer? How does the private information that the voters may obtain from external sources affect the leader's well-being? Which voters would the leader prefer to have external sources of information, voters with opinions more similar or dissimilar to his own's? Which majority rules and which distributions of external sources of information would the group of voters prefer?

To answer those questions, we investigate a model where a leader must acquire evidence about a relevant variable before being able to publicly disclose it to a group of voters. We formalize the way in which the leader can "manipulate" voters through evidence by adapting key features of the model proposed by Che and Kartik (2009). A central consideration is that the leader's investigation effort may be unsuccessful, which enables him to act strategically when he obtains evidence that would harm him if disclosed. The possibility of not obtaining completely accurate information is thus employed as a resource to obfuscate voters. In turn, voters react skeptically when they are reported that the investigation conducted to deliver evidence has been unsuccessful. In addition to their information acquisition and disclosure machinery, we also incorporate Che and Kartik (2009)'s approach of "diverse opinions" about the relevant variable.

We restrict attention to voting environments where people are confronted with a binary choice, either accept a new initiative or remain in the status quo, such as it is the case in referendums (e.g., to hire a certain job candidate or to leave the post open, to remain or to leave the EU, to issue or not company shares). Acceptance of the new initiative requires a minimum number of votes in support of it, which specifies the *majority rule* in our model. We will be interested in allowing majority rules to vary generally, ranging from more "dictatorial" to more "unanimous" ones.[1] Also, based on the spectrum of opinions, we propose a classification of possible leaders that distinguishes between moderate and radical leaders. Radical leaders are strongly biased for either one alternative or the other. Moderate leaders hold a "centrist" view of the most suitable alternative. Radical leaders remain biased towards one alternative even if they receive immense amounts of information in favor of the other alternative. Unlike this, small changes in what a moderate leader learns about the relevant variable makes him switch his preferred alternative. As to the role played by possible external means of information in the hands of the voters, we consider that a fraction of voters may fully learn the relevant variable by themselves, according to a certain probability. Thus, following the insights of the aforementioned view in political science, external means of information may drastically change the initial opinions of voters in our model.

Our insights can be summarized as follows.

1. Leaders have incentives (that may be either weak or strong) to conceal evidence, seeking to obfuscate precisely those voters who have opinions more similar to the leader's own opinion. Leaders care less about obfuscating voters that are far away from their own opinions. Radical leaders have *always strong incentives* to obfuscate those voters that are closer to them within the opinion spectrum. Moderate leaders, however, *have only weak incentives to obfuscate, and they may even end up disclosing all successfully obtained evidence in equilibrium*.

2. In equilibrium, radical leaders in favor of accepting the new initiative prefer more

[1] By more dictatorial, we simply mean that lower numbers of votes in favor or the new initiative are required to approve it. Accordingly, by more unanimous, we mean that higher numbers of votes in favor of the new initiative are required to approve it.

dictatorial majority rules. Such radical leaders wish to obfuscate larger sets of voters as the majority rule becomes more unanimous. Analogously, radical leaders in favor of remaining in the status quo prefer more unanimous majority rules. Such radical leaders want to obfuscate larger sets of voters as the majority rule becomes more dictatorial.

3. Moderate leaders are incentivized to *invest more in obtaining evidence and to obfuscate less when majority rules become closer to make voters with moderate opinions to be decisive for the election outcome*.

4. *Radical leaders prefer that external means not be provided to those voters that are closer to the leader within the opinion spectrum*. Unlike this, *moderate leaders care little about whether or not external means of information are in the hands of voters that are similar in their opinions to the leader*.

5. When leaders are moderate, in equilibrium, *the group of voters prefer that the educated voters be those with opinions closer to the leaders' opinions.*

Recent history has provided anecdotal, together with more systematic, evidence that may help interpret our model's implications. The 2016 Brexit referendum in the UK was a simple majority rule voting process—with full commitment as to its outcome. Our model would suggest that radical leaders in favor of any of the two options would seek to obfuscate voters with similar views by concealing pieces of evidence. According to media coverage, the campaign director of the Leave option, Dominic Cummings, spent months doing detailed evidence-based research into the relationships between the UK and the EU. However, in his public disclosure and campaign speeches, he revealed a narrow set of pieces of evidence. During the campaign, it became famous his display of a figure (mostly on buses) that said "Let's give our NHS the £350 million the EU takes every week." Shortly after, the Office for National Statistics, concluded that such a £350 amount "did not take into account the rebate or other flows from the EU to the UK public sector (or flows to non-public sector bodies), alongside the suggestion that this could be spent elsewhere, without further explanation, was potentially misleading."[2] Thus, the available evidence that full research on the flows from the EU to the UK could have gathered should have included also rebates and other flows. Conceivably, obtaining all the available evidence is costly and such particular pieces of evidence were not disclosed to the public by Mr. Cummings. This suggests a concealment strategy, under a logic of imperfect evidence acquisition, that is at the heart of our theoretical proposal. At least to the extent that it was voiced out only through Leave campaign events and speeches,[3] the disclosure of

[2] Also, in an interview with BBC's journalist Andrew Marr, NHS chief executive questioned the veracity of the £350 million figure.

[3] Since the UK government supported the Remain option, most prominent Remain campaigners, including David Cameron, could use official government channels—sometimes even echoed through international meetings or institutions such as the IMF—to disclose information. For instance, US president Barack

this incomplete piece of evidence was presumably addressed to voters who were already followers of the Leave campaign and thus with opinions favorable to leaving the EU.

On the side of the Remain campaign, it became also famous the extensive reporting of the BBC on a statement by the Confederation of British Industry (CBI). "The CBI says that all the trade, investment, jobs and lower prices that come from our economic partnership with Europe is worth £3000 per year to every household." However, UK in a Changing Europe Fellow Jonathan Portes subsequently detailed that this was not a complete disclosure estimate.[4] Here again, full research could have gathered also key qualifications to complete the potentially available evidence on the topic. Also, this incomplete piece of evidence on the Remain side was addressed to voters that paid attention to the BBC's reporting, thus conceivably to voters whose options were already closer to the Remain option.

Arguably, each of the two campaigns sought to obfuscate—by disclosing only part of all the evidence that full investigation on the topics could potentially gather—those voters who were more similar to their own views.[5] Trying to make voters who are far away in the opinion spectrum to change their minds up requires the leader to conceal larger amounts of evidence. This raises skepticism on the voters' side, which could be harmful for the leader. On the other hand, "less obfuscation"—formally described in this paper as smaller sets, in the set inclusion order, of concealed bits of evidence—can more easily change the minds up of those voters who are already very close to the leader's opinion. These considerations are central to the sort of obfuscation strategies suggested in this paper.

On the role played by external sources of information, such as education, data on the Brexit referendum showed a distribution of education levels (Fig. 1) in which the Remain option was strongly supported by highly educated voters. Inclination for the Remain option decreased dramatically among less educated voters (in particular, among voters with higher schooling or less levels of education). One might be tempted to think that sociological or income factors—which are usually correlated with education levels—could account as well for the inclination of highly educated voters towards the Remain option.

Obama used government press conferences to campaign in favor of the Remain option. Leave campaigners, on the other hand, needed to resort to non-official channels that, consequently, required a certain degree of involvement by the attendees and followers. Presumably, such an audience was mainly composed of voters closer to the Leave option.

[4] According to Mr. Porter, such an estimate was "based on a selection of studies produced at different times (some date back well over a decade), with different methodologies, and designed to answer different questions. Some looked at the economic impact of EU membership to date, and some at the future impact of a vote to leave. Some are not even specific to the UK."

[5] The disclosure of evidence by Leave leaders was aimed at persuading those voters who were already biased to thinking that leaving the EU would reduce immigration, and improve their social and income prospects. Similarly, the disclosure by Remain leaders was addressed to persuade voters who were already more inclined to considering that leaving the EU would damage the size and productivity of the UK's economy. Incidentally, a survey by Ipsos Mori shortly before the referendum, found that while most people (70 percent to 17 percent) did not believe a claim that British people would be significantly poorer outside the EU, they were more likely to accept (by 47 percent to 39 percent) the £350 million a week figure.

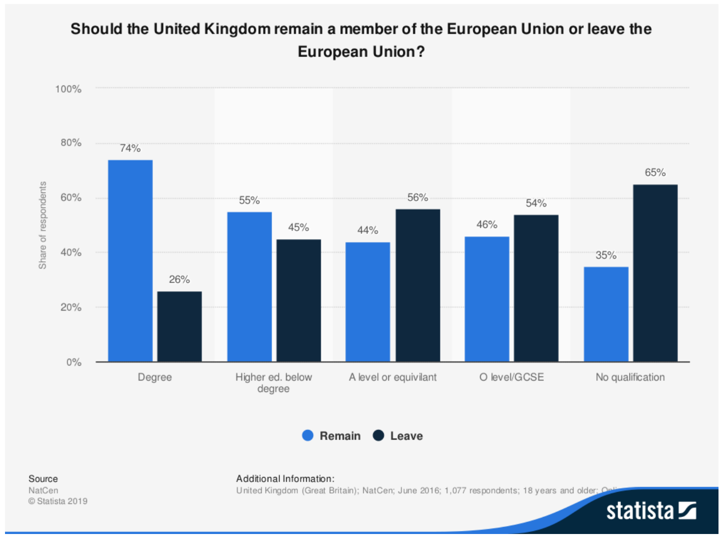

Figure 1 – Brexit Vote and Education Levels

However, Fig. 2 shows that the relative inclination of voters from the upper social class in favor of the Remain option stood far below that of highly educated voters. While the margin of support for the Remain option was 14 points for voters that belong to the upper social class, the corresponding margin of support was 48 points for highly educated voters. This bit of evidence suggests that the informative content of education did play a role in voting in the referendum. Therefore, under the premise that education has informative content and provides analytical tools (independent of the investigation efforts and disclosure of either campaign) to gauge the convenience of either leaving or staying, our model's insights would suggest the following. First, it would convey the message that it was in the interest of radical leaders in favor of the Leave option that voters inclined to leaving were relatively less educated than voters whose opinions were more aligned with remaining. Analogously, radical leaders in favor of the Remain option would be better off if education, or other means of external information about the Brexit, be in the hands of voters with opinions more aligned with leaving the EU.

We turn now to comment on some literature connections. The canonical model of verifiable information disclosure (Grossman, 1981; Milgrom, 1981) features an "unravelling" mechanism that typically leads to full revelation. Some recent contributions, however, have proposed realistic twists to the classical framework that are able to break down the unravelling mechanism. Our model builds upon one of such contributions, Che and Kartik (2009), where the expert's research technology allows for the possibility that the investigation efforts be unsuccessful. In this case, the expert ends up with no evidence to release, which conflicts with the incentives of his different types to separate between themselves. The sort of questions investigated here, though, are quite different as we are interested in exploring how voting systems, and external sources of information in the hands of the public, affect the leader's strategy to obfuscate, and the implications on the well-beings of

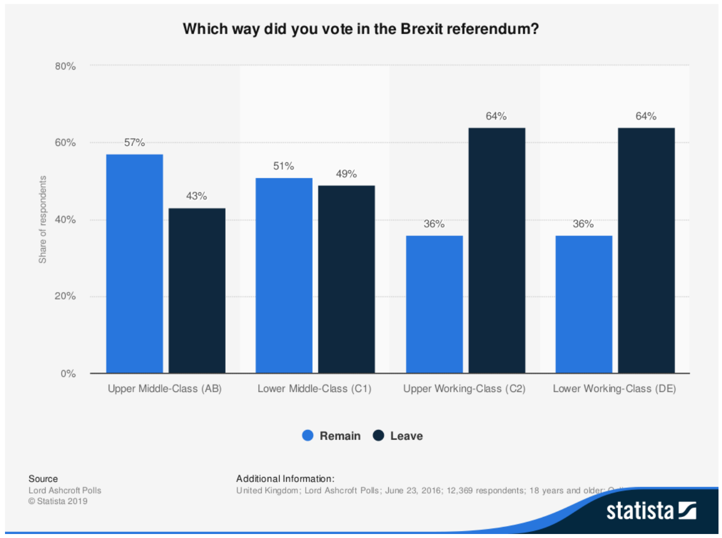

Figure 2 – Brexit Vote and Social Class

the leader and the voters. Key ingredients of the two setups are also different.[6] In another paper where the unravelling mechanism fails, Dziuda (2011) considers a model in which the expert may provide a number of bits of evidence in support of one alternative or the other. In this case, it is the assumption that receivers are uncertain about the total number of bits available for disclosure what breaks down the unravelling mechanism.

Our interest in exploring a logic for obfuscation by leaders connects with Dewan and Myatt (2008). Using a model where individuals want to take actions that be both suitable to a variable of interest and to the actions of the entire group, Dewan and Myatt (2008) provide a rationale for obfuscation when leaders compete for audiences and are able to choose the clarity of their information disclosure. The idea here is that the incentives of the leaders to attract the attention of the audience for longer periods makes them lower strategically the clarity of their speeches, therefore, obfuscating their followers. The fundamental questions explored, as well as the main ingredients of the two models, are quite different. For instance, Dewan and Myatt (2008) do not investigate the role of voting processes, neither do they consider verifiable information, whereas we do not consider the implications of agents acting in consonance with others.

Manipulative behavior from informed experts is also connected to media biased reporting. Using a bias confirmatory approach where listeners wish to see their own opinions confirmed by new information, Mullainathan and Shleifer (2005) investigate slanting in media reporting. Also, exploiting reputation concerns of media firms to signal high qualities, Gentzkow and Shapiro (2006) provide a rationale for such sources of information strategically adjusting their reporting to the listeners' opinions.

[6] Che and Kartik (2009) considers a continuum of possible actions which involves a totally different approach to explore equilibrium, relative to the one considered in the current paper.

Finally, our exploration of optimal obfuscation strategies has also some connections to the empirical research of Kono (2006) on the transparency of trade policies. Using data from voting on trade tariffs in 75 countries during the 1990s, his investigation concludes that simple majority rules, instead or more dictatorial roles, give leaders incentives to follow particular forms of obfuscation strategies to influence voters.

The paper is organized as follows. In Section 2 we present the model. Section 3 offers preliminaries to the equilibrium analysis, which we develop fully in Section 4. Section 5 and Section 6 provide insights about the well-beings of the leader and of the group of voters, respectively. Section 7 comments on further empirical evidence to illustrate the paper's insights. The formal arguments omitted in the main text are relegated to Appendices A,B.

2 Model Setup

A political, or committee, leader $i = l$ (he) and a group of voters $i \in N \equiv \{1, \ldots, n\}$ (each of them, she) are interested in some underlying *state of the world* ω, such as the state of the economy, the profitability of a merger, or the quality of a job market candidate. The leader must (by institutional and/or legal mandates) investigate and publicly disclose "hard evidence" (e.g., data, scientific reports, convincing insights,...) about ω.

The state of the world ω is distributed $N(\mu_i, 1)$ *from the perspective of player* $i \in \{l\} \cup N$. Thus, we assume that all players agree on the underlying distribution (and variance) of the relevant state, but they disagree on its mean. This approach crudely captures the idea that the players have different (prior) opinions[7] about the unknown variable of interest. Such diverse opinions are common knowledge though, so that players "agree to disagree".[8] Throughout the paper, we will use $P_i[\cdot]$ and $E_i[\cdot]$ to denote the probability and expectation operators, respectively, from the perspective of player i. In some cases where all players agree on the underlying probability space we will switch to notation $\Pr[\cdot]$ to indicate the corresponding probability operator.[9]

[7] In some practical situations, we may regard such opinions μ_i as stemming from ideological positions.

[8] Thus, such assumptions challenge the commonly accepted view in game-theoretic models—known as the *Harsanyi doctrine*—that heterogenous priors cannot persist if fully rational players have common knowledge either of such priors or of the learning processes of others. Nevertheless, we follow some recent efforts to understand the practical implications of individuals having different opinions—e.g., Che and Kartik (2009), Mullainathan and Shleifer (2005). An alternative, more complex, formulation to allow for different opinions could consider that the players have instead common priors but receive private uncorrelated signals about such priors. Then, if we assume that how and what the players learn from their signals remains their private information, these considerations would provide a setup with common priors that yet captures the idea proposed in this paper that the players begin with heterogeneous opinions before making any strategic choice. Acemoglu et al. (2016) have recently proposed an interesting approach to justify the persistence of different opinions in game-theoretical models by introducing uncertainty on learning processes.

[9] In particular, all players will agree on the probability of the outcome of the voting process even though they disagree about the state of the world. In short, the players are aware of the opinions of others and incorporate such different opinions to assess, in a common manner, which will be the outcome from voting.

Each voter $i \in N$ must simultaneously cast a vote $v_i \in \{A, R\}$, in favor of either accepting (A) or rejecting (R) a certain proposal. Acceptance is interpreted as approving a new initiative, whereas rejection is interpreted as remaining in the "status quo." The proposal is accepted by means of the voting process if at least a certain number $k \in N$ of voters vote in favor of it. Otherwise, the proposal is rejected. Thus, the minimum number k of votes required to switch from the status quo to approving the new initiative parameterizes the *majority rule*. Given a voting profile $v = (v_1, \ldots, v_n)$, the outcome of the voting process is given by $o(v) = A$ if $|\{i \in N \mid v_i = A\}| \geq k$, and $o(v) = R$ otherwise.[10] The two extreme majority rules $k = 1$ and $k = n$ correspond, respectively, to *dictatorship* and to *unanimity*. For values of $k \leq n/2$, we will say that the rule becomes "more dictatorial" as k lowers and, similarly, for values of $k > n/2$, we will say that the rule becomes "more unanimous" as k raises.

Players $i \in \{l\} \cup N$ have a common utility function $u(o(v), \omega)$, described by

$$\begin{aligned} &\text{if } \ \omega < 0, \ \text{ then } \ u(R, \omega) = 0 \ \text{ and } \ u(A, \omega) = -1; \\ &\text{if } \ \omega \geq 0, \ \text{ then } \ u(R, \omega) = -1 \ \text{ and } \ u(A, \omega) = 0. \end{aligned}$$

Therefore, each player (strictly) prefers rejection if $\omega < 0$, and acceptance if $\omega \geq 0$. Notably, there is no conflict of interests regarding the most desirable course of action *conditional on the actual realization of the state*. The disagreement takes place only at the level of opinions.

Without loss of generality, we consider that there is an even number n of voters. Furthermore, we assume that opinions are heterogenous across players, with the particular form: $\mu_l \in \mathbb{R}$, and $\mu_n < \cdots < \mu_{(n/2)+1} < 0 < \mu_{n/2} < \cdots < \mu_1$.[11]

We will consider three (qualitatively different) categories for the leader, based on his own position relative to the spectrum of opinions. A *(centrist) moderate* leader will be captured by considering that $\mu_l = 0$, a radical leader biased towards approving the new initiative will be described by considering that $\mu_l = \overline{\mu} > \mu_1$, and a *radical* leader biased towards remaining in the status quo will be captured by considering that $\mu_l = \underline{\mu} < \mu_n$. Furthermore, merely for technical reasons, in most of the analysis we will additionally consider that $\overline{\mu} \to +\infty$ and $\underline{\mu} \to -\infty$.

Notice that, based only on his own opinion, the moderate leader is (ex ante) indifferent between the two possible outcomes of the election process.[12] In the absence of further information about the state, the moderate leader (ex ante) utility is $-1/2$. Similarly, it can be verified that if there were no additional information about the state, then the (ex ante)

[10] Therefore, without loss of generality, we consider that ties are broken by having the proposal accepted in the case of a tie.

[11] Thus, without any further information, half of the voters prefer acceptance of the new initiative and half of them prefer rejection. Based on the opinions of the voters, the proposal would be accepted under the simple majority rule, $k = n/2$, or under more dictatorial rules, $k < n/2$.

[12] Based on his mean $\mu_l = 0$, he assigns probability $1/2$ to the event $\omega < 0$ and probability $1/2$ to the event $\omega \geq 0$.

utility of a radical leader in favor of the new initiative, with $\overline{\mu} \to +\infty$, would be 0, whereas the (ex ante) utility of a radical leader in favor of the status quo, with $\underline{\mu} \to -\infty$, would be -1. Nonetheless, we emphasize that we will consider that the leader *must* invest in acquiring additional information about the state and, therefore, that there will be additional information available about the relevant state.

2.1 Voters' Private Information

Voters may possess some private information by themselves about the underlying state. We interpreted this as information that the voter can obtain from any source that is external to the leader's information disclosure. For simplicity, we encompass all possible forms of private information that a voter may have under the label *education*.[13] In particular, each voter $i \in N$ may be either uneducated, $x_i = ne$, or educated, $x_i = e$. Let $x = (x_1, \dots, x_n) \in \{ne, e\}^n$ be a *profile of education levels*. Educated voters are endowed with a (common) probability $\varepsilon \in (0, 1)$ of learning the true state of the world. Conditional on obtaining education, the distribution according to which voters learn the true state is independent across voters. The outcome of an education level $x_i \in \{ne, e\}$ remains voter i's private information. Thus, we consider that there are no *educational spillovers* across voters, neither from the voters to the leader.

2.2 Leader's Information Acquisition and Disclosure

The leader has an *institutional role as a researcher of additional information* about the relevant state ω. Although he cares about the outcome of the election as well, there is a legal role (and obligation) for the leader to acquire further information about the relevant state and to communicate publicly with the voters about his findings.

The analysis of the leader's information acquisition and disclosure follows closely the approach that Che and Kartik (2009) propose for the case of a single decision-maker that receives information from an expert. In particular, the leader acquires information by choosing the probability $\lambda \in (0, \bar{\lambda}]$, with $\bar{\lambda} < 1$, of obtaining a noisy signal s about ω, at a cost $c(\lambda)$. By doing so, the leader chooses the likelihood of his investigation being successful. The cost function $c(\cdot)$ is smooth, increasing, convex, and it satisfies the typical Inada conditions $\lim_{\lambda \to 0} c'(\lambda) = 0$ and $\lim_{\lambda \to \bar{\lambda}} c'(\lambda) = +\infty$. Notably, because of his institutional role, the leader does not have the option of not acquiring any information about the state. Therefore, the leader must choose positive efforts $\lambda > 0$ to learn about the variable of interest. Voters can verify the investigation effort λ exerted by the leader. While

[13] In modern democracies, voting systems, or committee voting environments, our "education" label thus captures as well any other sources external to the leader's efforts in political power, such as independent media, social networks, information accessible through the Internet, and so on. In practice, a political power restricting access to the Internet or social networks, or not providing cheap quality education to some voters can be regarded in our model as making such voters uneducated.

we consider that the maximum investigation effort that the leader can exert is bounded (by $\overline{\lambda}$), we impose no specific minimum required level of investigation effort.[14]

Then, with probability λ, the leader obtains a signal $s \equiv \omega + \eta$, where η is a "noise term" distributed $N(0, 1)$, uncorrelated with the state.[15] Under such normality assumptions, the information structure that relates the state with the signal is captured by a normally distributed random pair (ω, s) such that, for each player $i \in \{l\} \cup N$, it follows that $E_i[s] = E_i[\omega] = \mu_i$, $var[\omega] = 1$, $var[s] = 2$, and $cov[\omega, s] = 1$. With the complementary probability $1 - \lambda$, the leader's investigation is unsuccessful and he obtains nothing from his investigation efforts, which we denote as obtaining signal $s = \emptyset$.

After learning privately the outcome of his investigation, the leader chooses whether or not to disclose such findings *publicly to all* voters. The information contained in signal s is (verifiable) "hard evidence" and it cannot be modified or falsified. Thus, if the leader obtains the signal and chooses to disclose it, he is constrained to transmitting true information to the voters. In addition to the intensity of his investigation effort, the only other strategic choice of the leader, therefore, is whether to disclose or to conceal the signal when his investigation is successful. Since the leader can choose to conceal signals, in the event that the voters are reported signal $s = \emptyset$, they update their beliefs (in a Bayesian way) to assess whether the leader's investigation has indeed been unsuccessful, or he is instead hiding evidence. Although the leader is restricted to making investigation efforts and to providing evidence about such efforts, notice that the suggested mechanism enables him to act strategically about how he communicates with the voters. In particular, we will see that the leader has incentives to obfuscate voters by concealing pieces of evidence obtained from his research.

Given an investigation effort λ and a set of signals $C \subseteq \mathbb{R}$ that the leader conceals from all voters, let $v_i(\cdot \mid C, \lambda) : \mathbb{R} \cup \{\emptyset\} \to \{A, R\}$ be a *voting rule* for voter i. Specifically, $v_i(s \mid C, \lambda)$ gives us the vote cast by voter i upon observing signal $s \in \mathbb{R} \cup \{\emptyset\}$. Also, let $v(s \mid C, \lambda) \equiv (v_1(s \mid C, \lambda), \dots, v_n(s \mid C, \lambda))$ be a *voting profile* conditional on the observed signal $s \in \mathbb{R} \cup \{\emptyset\}$.

2.3 Time Line

The timing of the game played by the leader and the voters is as follows. First, nature chooses the state of the world ω and the profile of education levels x. While ω remains unknown to everyone, the profile x becomes publicly known. In a second stage, without any further information about the underlying state ω, the leader chooses his investigation effort λ. The leader's investigation effort (or, alternatively, the investigation cost $c(\lambda)$ incurred) becomes commonly known to all voters. The leader observes the outcome of

[14] The ideas behind these requirements are intuitive, yet we choose the above stated forms for such restrictions for technical reasons.

[15] Our particular Gaussian approach aims at keeping the proposed information structure as simple as possible.

his investigation, which is unobservable to the voters. Then, the leader chooses whether to disclose or to conceal the successful outcome of his investigation. In a third stage, according to probability ε, each educated voter i ($x_i = e$) either learns or not the true state of the world. What an educated voter learns from her education level remains her private information. Voters cannot communicate among them what they learn from their education levels, neither can they communicate with the leader. Finally, each voter i casts a vote v_i either in favor or against the proposal raised for the election. Based on the considered majority rule k, an outcome $o(v)$ is then obtained from the votes cast.

The equilibrium notion that we use to study the proposed game is that of perfect Bayes equilibrium—to which we will simply refer as *equilibrium*. Furthermore, to avoid uninteresting equilibria, we will restrict attention to equilibria in which the voters vote according to their preferred alternative, regardless of whether any given voter may consider that her vote would be inconsequential to the outcome.[16]

2.4 Interim Information

Given our normality assumptions, if a player $i \in \{l\} \cup N$ observes a signal s and has no further information about the underlying state, then i considers that $\omega \mid s$ follows a normal distribution with posterior mean $E_i[\omega \mid s] = [\mu_i + s]/2$. Similarly, when voter $i \in N$ is educated, she considers that $\omega \mid s, e$ follows a normal distribution with posterior mean $E_i[\omega \mid s, e] = \varepsilon\omega + (1 - \varepsilon)[\mu_i + s]/2$.[17]

3 Preliminaries: Persuading a Single Voter

Before exploring the logic behind the (optimal) behavior of the leader in the proposed voting setup, let us first consider what the leader would do if he had to persuade only *a single voter* i (to vote for his preferred alternative).[18] Consider the case in which the investigation effort of the leader is successful so that he obtains a certain signal $s \in \mathbb{R}$.

Suppose first that voter i is uneducated ($x_i = ne$). This voter can improve her information about the state only by observing the signal s obtained by the leader. Then, the expected utility $E_i[u(o(v), \omega) \mid ne, s]$ that such a voter receives from a voting outcome $o(v)$

[16] Although our model does not consider an abstention alternative, we wish to guarantee that the analysis focuses on meaningful equilibria that avoid the sort of "swing voter's curse" implications (see, e.g., the seminal paper by Feddersen and Pesendorfer (1996), and a subsequent body of literature both in economics and in political science).

[17] Obviously, when voter $i \in N$ is not educated, we simply consider $E_i[\omega \mid s, ne] = E_i[\omega \mid s] = [\mu_i + s]/2$.

[18] The leader might be interested in persuading a single voter because such a voter could be decisive, or *pivotal*, to achieve one outcome or the other through the election process. In particular, we would like to focus at this point on (hypothetical) situations where $o(v)$ coincides with $v_i(s \mid C, \lambda)$ for the given voter i. In other words, conditional on a set of signals C concealed by the leader and on observing a signal realization s, the preferred alternative of voter i is the one that determines the outcome of the election.

satisfies $E_i[u(A,\omega) \mid ne, s] > E_i[u(R,\omega) \mid ne, s]$ if and only if $E_i[\omega \mid s] \geq 0$. Clearly, such a voter i strictly prefers acceptance over rejection if and only if $s \geq -\mu_i$.

Secondly, suppose that voter i is educated ($x_i = e$). Then, in a manner totally analogous to the case of the uneducated voter, we can derive the conditions that ensure that, conditional on observing a signal realization s, such an educated voter (strictly) prefers that the outcome of the election be acceptance of the initiative rather than rejection. In particular, observe that

$$E_i[u(A,\omega) \mid e, s] > E_i[u(R,\omega) \mid e, s] \;\Leftrightarrow\; E_i[\omega \mid e, s] \geq 0$$
$$\Leftrightarrow\; s \geq -\mu_i - \left(\frac{2\varepsilon}{1-\varepsilon}\right)\omega.$$

Recall that education gives the voter the opportunity of learning the true realization ω of the state. However, conditional on his investigation efforts being successful and on receiving a signal s, the leader does not learn ω. Instead, the leader learns only $E_l[\omega \mid s]$ and, accordingly, considers that voter i votes for acceptance if and only if

$$s \geq -\mu_i - \left(\frac{2\varepsilon}{1-\varepsilon}\right) E_l[\omega \mid s] \;\Leftrightarrow\; s \geq -\big[(1-\varepsilon)\mu_i + \varepsilon\mu_l\big].$$

Notice that the leader anticipates that, with probability ε, the voter will be better informed than himself.

The conditions derived above for both cases, those of an uneducated and of an educated voter, allow us to define the *critical signal realization* $\bar{s}_i(x;\varepsilon,\mu_l)$ (for any voter $i \in N$) as

$$\bar{s}_i(x;\varepsilon,\mu_l) \equiv \begin{cases} -\mu_i & \text{if } x_i = ne; \\ -\big[(1-\varepsilon)\mu_i + \varepsilon\mu_l\big] & \text{if } x_i = e. \end{cases} \tag{1}$$

Given the profile of education levels x, the probability ε of education being fruitful, and the opinion μ_l of the leader, the critical signal realization $\bar{s}_i(x;\varepsilon,\mu_l)$ determines then a cutoff value for observed signals such that voter i prefers the outcome of the election to be rejection whenever she observes $s < \bar{s}_i(x;\varepsilon,\mu_l)$ and acceptance whenever she observes $s \geq \bar{s}_i(x;\varepsilon,\mu_l)$.

On the other hand, the leader himself can only obtain additional information about the underlying state through his investigation. Therefore, he finds optimal acceptance if and only if he observes a signal $s \geq -\mu_l$. Analogously to the case of voters, this allows us to set the critical signal realization for the leader as $\bar{s}_l \equiv -\mu_l$.

Then, given the discrepancies between the critical signal realizations $\bar{s}_l$ and $\bar{s}_i(x;\varepsilon,\mu_l)$, it will be convenient to pay attention to subsets of signals $C_i = C_i(\mu_l;x)$ with the following forms: $C_i(\mu_l;x) \equiv [\bar{s}_l, \bar{s}_i(x;\varepsilon,\mu_l))$ if $\bar{s}_l < \bar{s}_i(x;\varepsilon,\mu_l)$, or $C_i(\mu_l;x) \equiv [\bar{s}_i(x;\varepsilon,\mu_l), \bar{s}_l)$ if $\bar{s}_l > \bar{s}_i(x;\varepsilon,\mu_l)$. For those signals $s \notin C_i$, the leader's preferred voting outcome coincides with that of voter i. Conditional on $s \notin C_i$, the optimal vote for voter i is

$$v_i^*(s \mid C_i, \lambda) = \begin{cases} A & \text{if } s \geq \bar{s}_i(x; \varepsilon, \mu_l); \\ R & \text{if } s < \bar{s}_i(x; \varepsilon, \mu_l). \end{cases} \tag{2}$$

However, for signals $s \in C_i$, the leader and the voter disagree completely on the best course of action. We thus interpret the interval $C_i = C_i(\mu_l; x)$ as the set of signals under which the leader and voter i disagree about the suitability of the two alternatives.[19] Given the assumed preferences, the leader will have certain incentives to conceal signals $s \in C_i$. By concealing signals that belong to the set C_i, the leader would seek to *obfuscate* voter i. On one side, if the leader discloses the signals $s \in C_i$, then he gets a negative payoff with probability one. On the other side, through obfuscation, the leader is able (at least in some cases, as we will see) to induce a voting response that leaves him with a such negative payoff, yet with probability less than one.

We turn now to consider how the single voter i processes (and best responds to) the information disclosed when the leader reports that his investigation efforts have been unsuccessful, so that the voter observes $s = \emptyset$. From the previous arguments, for the given voter i, it will be convenient to restrict attention to the family $\mathcal{C}_i$ of all subsets of the interval $C_i = C_i(\mu_l; x)$. Of course, the set $C_i(\mu_l; x)$ will give us the largest set (according to the set inclusion order) within the family $\mathcal{C}_i$ of plausible sets of concealed signals.

Suppose then that the leader chooses a set C of concealed signals from the suggested family $\mathcal{C}_i$ of subsets. Then, voter i assigns probability λ to the leader's investigation having been successful or, equivalently, to the event that $s \in C$. In this case, the voter places herself in the leader's position and uses the signals $s \in C$, yet combined with her own prior information about the state, in order to determine her optimal vote. Intuitively, the voter would in this case use the leader's "technology" but according to her own opinion. In addition, voter i assigns probability $1 - \lambda$ to the leader's investigation having indeed been unsuccessful. In this case, the voter is left only with her own prior opinion μ_i about the state to determine her optimal vote.

We shall use $f(\cdot; \mu_i)$ and $F(\cdot; \mu_i)$ to indicate the density and the (cumulative) distribution function of the signal s from player i's perspective, which distributes $N(\mu_i, 2)$.[20] Given a subset of signals $C \in \mathcal{C}_i$ that the leader may conceal and an investigation effort λ, notice that the optimal voting behavior $v_i^*(\emptyset \mid C, \lambda)$ of voter i, when the leader discloses

[19] In particular, conditional on observing any signal $s \in C_i(\mu_l; x)$, the disagreement takes the following particular form: (a) if $\bar{s}_l < \bar{s}_i$, then the leader prefers acceptance whereas the voter prefers rejection and (b) if $\bar{s}_l > \bar{s}_i$, then the leader prefers rejection whereas the voter prefers acceptance.

[20] For instance, given a subset of signals $C \in \mathcal{C}_i$, in some parts of the paper we will be particularly interested in computing the probabilities $P_i[s \in C] = \int_C f(s; \mu_i) ds$ and $P_i[s \notin C] = 1 - \int_C f(s; \mu_i) ds$.

signal $s = \emptyset$, takes the form of a *mixed strategy*. In particular, with probability

$$\pi_i(C; \lambda) \equiv \frac{\lambda P_i[s \in C]}{\lambda P_i[s \in C] + (1 - \lambda)} = \frac{\lambda P_i[s \in C]}{1 - \lambda P_i[s \notin C]}, \tag{3}$$

voter i optimally votes according to

$$v_i^*(\emptyset \mid C, \lambda) = \begin{cases} A & \text{if } E_i[s \mid s \in C] \geq \bar{s}_i(x; \varepsilon, \mu_l); \\ R & \text{if } E_i[s \mid s \in C] < \bar{s}_i(x; \varepsilon, \mu_l), \end{cases} \tag{4}$$

where $E_i[s \mid s \in C] = \int_C s f(s; \mu_i) ds$. With the complementary probability $1 - \pi_i(C; \lambda)$ such a voter i optimally votes according to

$$v_i^*(\emptyset \mid C, \lambda) = \begin{cases} A & \text{if } \mu_i \geq 0; \\ R & \text{if } \mu_i < 0. \end{cases} \tag{5}$$

Obviously, if voter i is an educated voter ($x_i = e$) who does learn the true state of the world by herself, then she optimally votes $v_i^* = A$ if and only if $\omega \geq 0$, regardless of the leader's information disclosure strategy.

For the case where voter i does not learn the state of the world through her education level, note first that, for any pair of subsets $C, C' \in \mathcal{C}_i$, we have that $C \subseteq C'$ with $C \neq C'$ implies $\pi_i(C'; \lambda) > \pi_i(C; \lambda)$. The probability $\pi_i(C; \lambda)$, according to which voter i uses the concealed signals together with her own opinions μ_i, is strictly increasing in the set inclusion order—with the restriction to the class of subsets of interest $\mathcal{C}_i$. To fix ideas, consider for instance that $\bar{s}_i(x; \varepsilon, \mu_l) < \bar{s}_l$. Suppose that the leader conceals all signals that belong to a certain subset $C \in \mathcal{C}_i$ that satisfies $C_i(\mu_l; x) \subseteq C$ with $C_i(\mu_l; x) \neq C$. It follows then that $\pi_i(C; \lambda) > \pi_i(C_i(\mu_l; x); \lambda)$. By enlarging the set of concealed signals, the leader raises the probability that the voter uses the leader's concealed signals combined with her own opinions μ_i. Also, since we are supposing that $\bar{s}_i(x; \varepsilon, \mu_l) < \bar{s}_l$, it follows that, for signals $s \in C_i$ under which the leader always prefers to remain in the status quo, voter i will optimally choose $v_i^*(\emptyset \mid C, \lambda) = A$. Crucially, the voter will vote in such a way for a subset of signals C that includes strictly the interval $C_i(\mu_l; x)$. In other words, enlarging the set of concealed signals beyond $C_i(\mu_l; x)$ raises the voter's "skepticism" and leads to a voting behavior clearly unfavorable to the leader. This idea is nicely captured under the term *prejudicial effect* by Che and Kartik (2009). Due to the role of such a prejudicial effect, we observe that the leader has no incentives to conceal signals $s \notin C_i(\mu_l; x)$ to the single voter i. In other words, *the real interval $C_i(\mu_l; x)$ gives us the largest set that a leader with opinion μ_l may want to conceal from voter i.*

Finally, notice also that the probability $\pi_i = \pi_i(C; \lambda)$ is strictly increasing and convex in the investigation effort λ. In particular, for a given subset $C \in \mathcal{C}$, we can compute

$$\frac{\partial \pi_i}{\partial \lambda} = \frac{P_i[s \in C]}{\left(1 - \lambda P_i[s \notin C]\right)^2} > 0 \text{ and } \frac{\partial^2 \pi_i}{\partial \lambda^2} = \frac{2P_i[s \in C]P_i[s \notin C]}{\left(1 - \lambda P_i[s \notin C]\right)^3} > 0. \tag{6}$$

Thus, in order to determine her optimal vote upon a subset of signals C that the leader conceals, the voter relies less on her only own opinion exclusively (and places more importance on the signals concealed by the leader) when she observes that the leader invests more in obtaining evidence.

For the case of interest in our model, where the leader needs to persuade *all* voters, we will consider the class $\mathcal{C} \equiv \cup_{i \in N} \mathcal{C}_i$ of possible largest *concealment sets* $C_k(\mu_l; x)$ *under a majority rule* k. Then, we will use the logic presented in this Section 3 to investigate how a leader with opinion $\mu_l \in \{0, \overline{\mu}, \underline{\mu}\}$ will be interested in concealing all signals $s \in C_k^*(\mu_l; x) \subseteq C_k(\mu_l; x)$, for some largest concealment set $C_k(\mu_l; x) \in \mathcal{C}$, and disclosing the rest of signals that he obtains. Notice that, in addition to the leader's opinion μ_l, the optimal concealment set $C_k^*(\mu_l; x)$ will naturally depend on the education profile x and on the existing majority rule k.

4 Equilibrium Analysis

Equilibrium requires that each player best responds to the choices of the rest of players. In particular, the leader must choose the probability $\lambda \in (0, \bar{\lambda}]$ of his investigation being successful and a subset of signals $C_k^*(\mu_l; x) \in \mathcal{C}$ that he conceals to *all* voters, whereas each voter i must choose her vote $v_i \in \{A, R\}$ according solely to her preferred alternative.

4.1 Leader's (Optimal) Obfuscation Strategy

We turn to study how the leader designs his optimal concealment set $C_k^*(\mu_l; x)$ by taking into account the role played by the voting mechanism. Though similar in spirit to the case where the leader wants to persuade a single voter (Section 3), the analysis of information disclosure to several voters (that may possess information of their own as well) requires additional considerations.

Fix a given majority rule $k \in N$. Recall that the voters' opinions are ordered in a way that entails $-\mu_1 < \cdots < -\mu_{n/2} < 0 < -\mu_{(n/2)+1} < \cdots < -\mu_n$. However, given that some voters may be educated and others uneducated, the relevant ordering is the one induced over the set of critical signal realizations $\bar{s}_i(x; \varepsilon, \mu_l)$ defined in Eq. (1). We simply reorder the critical signal realizations of all the voters by considering

$$\bar{s}_{i_1}(x; \varepsilon, \mu_l) < \cdots < \bar{s}_{i_j}(x; \varepsilon, \mu_l) < \bar{s}_{i_j+1}(x; \varepsilon, \mu_l) < \ldots \bar{s}_{i_n}(x; \varepsilon, \mu_l). \tag{7}$$

Notice that we may need to relabel indexes as expressed above (say, from i to i_j). Let us then use $\sigma(x) = (\bar{s}_{i_1}, \ldots, \bar{s}_{i_j}, \ldots, \bar{s}_{i_n})$ to refer to such an ordering of the critical signal realization under a given education profile x. As mentioned in fn. 9, notice that all players incorporate the different opinions of everyone and are able to assess in a common manner the probability that the outcome of the election be either acceptance or rejection.

Suppose first that $C = \emptyset$ so that the leader does not conceal any signal that he obtains. Then, since approval of the new initiative requires that at least k voters vote in favor of acceptance, it follows that the particular voter i_k, whose name is associated to the reordering $\sigma(x)$ described in Eq. (7), would be *pivotal* in the election process. Rejection would be the outcome of the election conditional on the voters observing any signal $s < \bar{s}_{i_k}$, whereas acceptance would be the outcome of the election conditional on the voters observing any signal $s \geq \bar{s}_{i_k}$. In short, using the derivation of a voter's optimal voting behavior in Eq. (2), the probability that the outcome of the election be acceptance, conditional on $C = \emptyset$ and on observing a signal $s \in \mathbb{R}$, is simply given by $\Pr[o(v^*(s \mid C, \lambda)) = A] = 0$ if $s < \bar{s}_{i_k}$ and $\Pr[o(v^*(s \mid C, \lambda)) = A] = 1$ if $s \geq \bar{s}_{i_k}$. This tells us how the voting process would lead to either outcome, conditional on the leader not concealing any signal and on the voters observing a particular signal realization.

Now we turn to study how voters would determine their preferred alternatives conditional on signals that they do not observe, given a concealment interval $C \in \mathcal{C}$. Consider a majority rule k and take a given voter $i \in N$. Then, conditional on a concealment set $C \in \mathcal{C}$ such that $C \neq \emptyset$, we can derive the probability that voter i prefers acceptance when she receives a signal $s = \emptyset$ as

$$\Pr[v_i^*(\emptyset \mid C, \lambda) = A] = \pi_i(C, \lambda)\mathcal{I}(i, C) + [1 - \pi_i(C, \lambda)]\mathcal{J}(i), \tag{8}$$

where $\mathcal{I}(i, C)$ and $\mathcal{J}(i)$ are indicator functions specified, respectively, by $\mathcal{I}(i, C) = 1$ if $E_i[s \mid s \in C] \geq \bar{s}_i$, and $\mathcal{I}(i, C) = 0$ otherwise, and by $\mathcal{J}(i) = 1$ if $\mu_i \geq 0$, and $\mathcal{J}(i) = 0$ otherwise. The expression given in Eq. (8) follows from our earlier analysis (in Section 3, where optimal voting behavior was described by Eq. (4) and Eq. (5)) for the case where the leader is interested in persuading a single voter.

On the one hand, voter i considers that the leader obtained and concealed a signal with probability $\pi_i(C, \lambda)$. Conditional on this event, it follows from Eq. (4) that voter i wants to vote for acceptance if and only if $E_i[s \mid s \in C] \geq \bar{s}_i$. This leads directly to acceptance being the outcome of the election with probability $\pi_i(C, \lambda)$ if $E_i[s \mid s \in C] \geq \bar{s}_i$, and with probability zero if $E_i[s \mid s \in C] < \bar{s}_i$. On the other hand, voter i considers that the leader obtained no signal with probability $1 - \pi_i(C, \lambda)$. From Eq. (5), we know that, conditional on this this event, voter i wants to vote for acceptance if and only if $\mu_i \geq 0$. Since the two described events (that the leader is successful in his efforts and conceals the signal, and that he is unsuccessful) are disjoint, the resulting probability can be derived in an additive manner by applying the total probability rule. All these considerations lead to the expression in Eq. (8).

Finally, we comment on how concealed signals lead to voting outcomes, given a subset $C \neq \emptyset$. For a majority rule k, an investigation effort λ, and a set of signals $C \in \mathcal{C}$ that the leader may conceal, let us use

$$\phi_k(C, \lambda) \equiv \Pr[o(v^*(\emptyset \mid C, \lambda)) = A] \tag{9}$$

to denote the probability that the outcome of the election be acceptance conditional on the voters receiving $s = \emptyset$. First, consider the pivotal voter i_k that resulted from the ordering $\sigma(x)$ induced by the existing profile of education levels x in the previously described situation where no obtained signal were concealed. Recall, however, that we are now considering that $C \neq \emptyset$ so that some obtained signals are concealed. Note that if the associated critical signal realization $\bar{s}_{i_k}$ satisfies $\bar{s}_{i_k} \notin C$, then we directly obtain:

(a) because signals $s \in C$ such that $s \geq \bar{s}_{i_k}$ make at least k voters prefer to vote for acceptance, it follows that $\bar{s}_{i_k} < \inf C \Rightarrow \phi_k(C, \lambda) = 1$;

(b) because signals $s \in C$ such that $s < \bar{s}_{i_k}$ make less than k voters prefer to vote for acceptance, it follows that $\bar{s}_{i_k} \geq \sup C \Rightarrow \phi_k(C, \lambda) = 0$.

Secondly, for the case where $\bar{s}_{i_k} \in C$, the players want to assess whether or not the concealed signals $s \in C$ could induce at least a number k of voters to vote for acceptance. To this end, we will use the expression derived in Eq. (8) above to obtain insights, in Lemma 1 below, about the probability of attaining the voting outcome $o(v) = A$ when $\bar{s}_{i_k} \in C$. It is useful to distinguish between the three different categories of the leader.

Lemma 1. Consider a given a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader designs a concealment set $C \in \mathcal{C}$ such that $\bar{s}_{i_k} \in C$. Further, suppose that voters observe signal $s = \emptyset$. Then, the following implications about probability $\phi_k(C, \lambda)$ hold.

1. Moderate leader ($\mu_l = 0$).

(a) Suppose that $k \leq n/2$ and consider the largest concealment set $C_k = [\bar{s}_{i_k}, 0)$. Then, $\phi_k(C, \lambda) = 1$ for any subset $C \subseteq C_k$.

(b) Suppose that $k > n/2$ and consider the largest concealment set $C_k = [0, \bar{s}_{i_k})$. Then, $\phi_k(C, \lambda) = 0$ for any subset $C \subseteq C_k$.

2. Radical leader in favor of the new initiative ($\mu_l \to +\infty$). Consider the largest concealment set $C_k = (-\infty, \bar{s}_{i_k})$. Then, for any subset $C \subseteq C_k$, (a) $\phi_k(C, \lambda) > 0$ for $k \leq n/2$ and (b) $\phi_k(C, \lambda) = 0$ for $k > n/2$.

3. Radical leader in favor of the status quo ($\mu_l \to -\infty$). Consider the largest concealment set $C_k = [\bar{s}_{i_k}, +\infty)$. Then, for any subset $C \subseteq C_k$, (a) $\phi_k(C, \lambda) = 1$ for $k \leq n/2$, and (b) $\phi_k(C, \lambda) < 1$ for $k > n/2$.

The result in 1. of Lemma 1 leads directly to the implication that the leader would be indifferent between disclosing all obtained signals or concealing all signals $s \in C$.

The result in 2. (a) of Lemma 1 follows because, upon observing $s = \emptyset$, the set of voters $\{i_1, \dots, i_k\}$ prefer to vote for the new initiative with positive probability. This is the case since each of those voters will place a positive probability on the event that the leader investigation efforts were unsuccessful. In consequence, assessing their preferred alternative will be based (to some extent) on their priors. This leads to that the outcome $o(v) = A$ happens with positive probability (i.e., $\phi_k(C, \lambda) > 0$) when all signals $s \in C = (-\infty, \bar{s}_{i_k})$ are concealed. Crucially, if voters received instead signals $s < \bar{s}_{i_k}$, then the voting outcome $o(v) = A$ would have probability zero, as only less than k voters would want to vote for the new initiative. The result in 3. (b) of Lemma 1 follows because, upon observing $s = \emptyset$, the set of voters $\{i_k, \dots, i_n\}$ prefer to vote for the new initiative with probability less than one. Similarly to the situation described in 2. (a), each of those voters will place a positive probability on the event that the leader investigation efforts were unsuccessful. In consequence, assessing their preferred alternative will be based (to some extent) on their priors. This leads to that the outcome $o(v) = A$ happens with probability less that one (i.e., $\phi_k(C, \lambda) < 1$) when all signals $s \in C = [\bar{s}_{i_k}, +\infty)$ are concealed. If the voters instead received signals $s \geq \bar{s}_{i_k}$, then the voting outcome $o(v) = A$ would happen with probability one, as no less than k voters would want to vote for the new initiative.

The main implications of this analysis are that, conditional on his investigation effort being successful, the leader will have (sometimes, only weak) incentives to design the optimal concealment set $C_k^*(\mu_l; x) \subseteq C_k(\mu_l; x)$ so as to conceal either (a) all signals $s \in [\bar{s}_l, \bar{s}_{i_k})$ if $\bar{s}_l < \bar{s}_{i_k}$, or (b) all signals $s \in [\bar{s}_{i_k}, \bar{s}_l)$ if $\bar{s}_l > \bar{s}_{i_k}$. Notice that, in case (a), conditional on the leader observing a signal $s \in [\bar{s}_l, \bar{s}_{i_k})$, he prefers acceptance whereas the voting outcome would be rejection. Similarly, in case (b), for those signals $s \in [\bar{s}_{i_k}, \bar{s}_l)$, the leader prefers rejection, yet the outcome of the election would be acceptance. The key point is that, should the leader not conceal such signals, he would then obtain the negative payoff -1 with probability one. On the other hand, concealment of such signals allows him to obtain the expected loss of either $[1 - \phi_k(C_k^*, \lambda)]$, in the sort of situations described by (a) above, or of $\phi_k(C_k^*, \lambda)$, in situations as the ones described by (b). This gives us the formal description of the idea of obfuscation suggested by our model. In addition, the earlier arguments lead to that the leader only wants to conceal those signals. This the case because disclosing signals $s \notin C_k^*(\mu_l; x)$ already makes the outcome of the election coincide with the alternative preferred by the leader, whereas concealing any of those signals raises the voters' skepticism, which could be harmful for the leader (as argued in Section 3).

Observation 1. In general, it is not obvious whether or not to regard the voter with label i_k (that stems from the induced ordering $\sigma(x)$) as decisive, or pivotal, to switch the outcome, conditional on signals being concealed ($C \neq \emptyset$) and on the voters receiving no signal ($s = \emptyset$). In particular, the possibility of concealed signals poses difficulties to the intuitive idea of a pivotal voter in cases where voters different from i_k could change their preferred alternative upon receiving $s = \emptyset$. In turn, these difficulties affect the determination of the probability $\phi_k(C, \lambda)$ according to which the outcome of voting is acceptance.

In our setup, voter i_k stands formally as *the voter who would be pivotal in a (hypothetical) situation where all signals were disclosed*. When all signals are disclosed, we can straightforwardly consider such a notion of pivotal voter in a quite natural manner. This is the case because the preferred alternative of each voter is assessed in a deterministic manner. However, a notion of pivotal voter for situations in which signals are concealed is less clear, and needs further considerations. Crucially, in these cases, the preferred alternative of some voters can only be assessed in stochastic terms—as such preferred alternatives are given by mixed strategies. Therefore, we need to propose a notion of pivotal voters in the presence of concealed signals. Our particular notion for such cases seeks to (i) rely, as much as possible, on deterministic optimal choices by the voters, and to (ii) select a single voter as being pivotal. The practical goal of our notion is to verify whether or not voters who would be clearly pivotal when all signals are disclosed continue to be pivotal (under such a notion) when signals are concealed and they receive no signal. Following the criteria described in (i) and (ii) above, and taking into account the order $\sigma(x)$ of the voters' critical signal realizations, we propose the following notion.

Definition 1. Consider the order $\sigma(x)$ of critical signal realizations induced by the profile of education levels x. We say that voter i_k continues to be *pivotal to the voting process in the presence of concealed signals*, if given a concealment set $C \neq \emptyset$, and conditional on the voters receiving signal $s = \emptyset$, then either (a) all voters $\{i_1, \ldots, i_{k-1}\}$ vote for acceptance of the new initiative with probability one, or (b) all voters $\{i_{k+1}, \ldots, i_n\}$ vote for rejection of the new initiative with probability one.

As a consequence, in case (a) above, if voter i_k votes for acceptance with probability one as well, then the outcome of the voting process would be $o(v) = A$ with probability one. Even further, if voter i_k votes for acceptance with probability $\phi_k(C, \lambda)$, then the outcome of the voting process would be $o(v) = A$ with such a probability $\phi_k(C, \lambda)$. Analogously, in case (b), if voter i_k votes for rejection with probability $1 - \phi_k(C, \lambda)$, then the outcome of the voting process would be $o(v) = R$ with such a probability $1 - \phi_k(C, \lambda)$.

However, the conditions (a) and (b) provided by our notion in Definition 1 do not cover all possible cases. To fix ideas about the difficulties that concealed signals pose to the problems of (a) proposing a notion of pivotal voter and of (b) identifying pivotal voters in particular situations, consider the following example. Suppose that the leader is a radical in favor of the new initiative (so that $\bar{s}_l \to -\infty$) and consider the set $C = (\bar{s}_l, \bar{s}_{i_k}]$ of concealed signals. In addition, suppose that the voting rule satisfies $k < n/2$ so that $i_k < j$ for some voter j with $\mu_j > 0$. Note then that, conditional on C and on the voters receiving signal $s = \emptyset$, we know from the specification in Eq. (8) that all voters $i_{(n/2)+1}, \ldots, i_n$ prefer rejection with probability one. Such a number $(n/2)$ of voters, though, would be not sufficient to reject the proposal in this example. In this case, our notion of pivotal voter in the presence of concealed signals (given by Definition 1) is not useful to conclude whether voter i_k is the pivotal voter, neither to identify a pivotal voter. The key point in this example is that (even when we invoke such a notion in Definition 1) whether voter i_k ends up being pivotal or not depends crucially on the preferred alternative (conditional on C and on the voters receiving signal $s = \emptyset$) of those voters j with $i_k < j \leq n/2$, so that $\mu_j > 0$. Obviously, further structure and assumptions would be necessary to obtain general messages about whether or not voter i_k continue to be pivotal (when moving from a situation where no signals are concealed to another with concealed signals) in this sort of particular situations. In particular, further key considerations on the differences between all the players opinions would be necessary. Given that a general analysis to explore all possible situations would be ill-suited to overcome such difficulties, in Section 5 we will introduce a reduced-form assumption to focus on interesting situations for our investigation of the well-beings of the leader (for each of his possible categories) and of the group of voters.

Following the previous arguments, and the results provided by Lemma 1, Proposition 1–Proposition 3 below characterize the optimal design of concealment sets by the leader.

Proposition 1. Consider a given a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Then, the moderate leader ($\mu_l = 0$) designs the concealment set $C^*_k(0; x)$ as follows:

(a) for $k \leq n/2$, the leader selects any subset $C^*_k(0; x) \subseteq C_k(0; x) = [\bar{s}_{i_k}(x; \varepsilon, 0), 0)$;

(b) for $k > n/2$, the leader selects any subset $C^*_k(0; x) \subseteq C_k(0; x) = [0, \bar{s}_{i_k}(x; \varepsilon, 0))$.

The moderate leader has *weak* incentives to conceal signals that would critically influence the voters whose opinions are closer to his own opinions. Nevertheless, a profound multiplicity of optimal concealment sets arises for the case of the moderate leader. In particular $C^*_k(0; x) = \emptyset$ is included in the description given by Proposition 1. Thus, *disclosing all signals obtained through his investigation efforts is also part of the optimal behavior of the moderate leader*. In some equilibria, the moderate leader chooses not to obfuscate the voters by concealing evidence. We make no formal claims regarding equilibrium selection. However, in environments where we could naturally consider that disclosing the obtained signals involves any sort of cost for the leader, our setup would deliver the message that concealing $C^*_k(0; x) = C_k(0; x)$ appears as a relatively more reasonable behavior.

In sharp contrast with the case of the moderate leader, *radical leaders have strict incentives to conceal all signals that would critically influence those voters whose opinions are closer to his own's*. For radical leaders, unique non-empty concealment sets may arise as part of their behavior in equilibrium. Furthermore, whether such optimal concealment sets end up being unique or multiple depends crucially on the majority rule. In other words, depending on the majority rule, there could be a unique equilibrium in which the radical leader obfuscates those voters that are closer in opinions by concealing evidence. In particular, the radical leader in favor of the new initiative *always* obfuscates for majority rules $k \leq n/2$. He also obfuscates more (according to the set inclusion order) as the majority rule becomes more unanimous. On the other hand, the radical leader in favor of the status quo *always* obfuscates for majority rules $k > n/2$, and he obfuscates more as the majority rule becomes more dictatorial.

Proposition 2. Consider a given a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Then, the radical leader biased in favor of the new initiative ($\mu_l = \overline{\mu} > \mu_1$) designs the concealment set $C_k^*(\overline{\mu}; x)$ as follows:

(a) for $k \leq n/2$, the leader selects the interval $C_k^*(\overline{\mu}; x) = C_k(\overline{\mu}; x) = [\bar{s}_l, \bar{s}_{i_k}(x; \varepsilon, \overline{\mu}))$ with $C_k^*(\overline{\mu}; x) = C_k(\overline{\mu}; x) = (-\infty, \bar{s}_{i_k}(x; \varepsilon, \overline{\mu}))$ for $\overline{\mu} \longrightarrow +\infty$;

(b) for $k > n/2$, the leader selects any subset $C_k^*(\overline{\mu}; x) \subseteq C_k(\overline{\mu}; x)$ with the form $C_k^*(\overline{\mu}; x) = [\bar{s}_l, \bar{s}_{i_{n/2}}) \cup B$, for any subset $B \subseteq [\bar{s}_{i_{n/2}}, \bar{s}_{i_k}(x; \varepsilon, \overline{\mu}))$. Moreover, $\bar{s}_l \longrightarrow -\infty$ for $\overline{\mu} \longrightarrow +\infty$.

Proposition 3. Consider a given a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Then, the radical leader biased in favor of the status quo ($\mu_l = \underline{\mu} < \mu_n$) designs the concealment set $C_k^*(\underline{\mu}; x)$ as follows:

(a) for $k \leq n/2$, the leader selects any subset $C_k^*(\underline{\mu}; x) \subseteq C_k(\underline{\mu}; x)$ with the form $B \cup C_k^*(\underline{\mu}; x) = (\bar{s}_{i_{(n/2)+1}}, \bar{s}_l]$, for any subset $B \subseteq (\bar{s}_{i_k}(x; \varepsilon, \underline{\mu}), \bar{s}_{i_{(n/2)+1}}]$. Moreover, $\bar{s}_l \longrightarrow +\infty$ for $\underline{\mu} \longrightarrow -\infty$

(b) for $k > n/2$, the leader selects the interval $C_k^*(\underline{\mu}; x) = C_k(\underline{\mu}; x) = [\bar{s}_{i_k}(x; \varepsilon, \underline{\mu}), \bar{s}_l)$ with $C_k^*(\underline{\mu}; x) = C_k(\underline{\mu}; x) = [\bar{s}_{i_k}(x; \varepsilon, \underline{\mu}), +\infty)$ for $\underline{\mu} \longrightarrow -\infty$.

The following example illustrates the construction of the (optimal) concealment sets $C_k^*(0; x)$ described by Proposition 1.

Example 1. Suppose that $n = 4$ and consider the moderate leader ($\mu_l = 0$). Suppose that there are exactly two voters located on each side of the leader over the opinion spectrum, that is, $-\mu_1 < -\mu_2 < 0 < -\mu_3 < -\mu_4$. Consider a situation where $x = (ne, ne, ne, ne)$ so that no voter can obtain information through an external source. In this case, it follows that the induced ordering $\sigma(x)$ of critical signal realizations is simply given by $\bar{s}_1 < \bar{s}_2 < \bar{s}_3 < \bar{s}_4$ so that $i_k = k$.

On one extreme of the plausible majority rules, consider first that $k = 1$ so that the vote of a single voter in favor of acceptance is sufficient to approve the new initiative. Then, conditional on observing a signal $s < 0$, the leader has weak incentives to conceal all signals $s \in [-\mu_i, 0)$ from each of the two voters i such that $\mu_i > 0$. Notice that, conditional on such negative signals, the leader strictly prefers rejection but observing them would make any of such two voters $i = 1, 2$ (with opinions $\mu_i > 0$) to vote for acceptance instead. Under the dictatorship majority rule $k = 1$, the leader clearly wants to avoid this. The optimal strategy of the leader is then to obfuscate these two voters $i = 1, 2$. The incentives are weak though. Given the obfuscation that the leader can induce by concealing evidence, these two voters would continue to vote for acceptance with probability one as well. The moderate leader is then indifferent between concealing signals from any subset $C \subseteq [-\mu_i, 0)$. On the other hand, if the leader observes a signal $s \geq 0$, then he does have incentives to conceal such a nonnegative signal. This is so because, upon observing such signals, already two voters vote for acceptance (those two voters $i = 1, 2$ with $\mu_i > 0$). In this case, new initiative is approved regardless of the votes cast by those two voters with $\mu_i < 0$. It follows that the optimal concealment set takes the form of any set $C_1(0; x)$ such that $C_1(0; x) \subseteq [\bar{s}_1, 0) = [-\mu_1, 0)$.

On the other extreme, consider now a majority rule $k = 4$ so that unanimity in favor of acceptance is required to approve the new initiative. Then, conditional on observing a signal $s < 0$, the leader does not care if the two voters $i = 1, 2$ with $\mu_i > 0$ vote for acceptance since the remaining two voters (i.e., those two voters $i = 3, 4$ with $\mu_i < 0$) will vote for rejection upon disclosing such negative signals. The leader would then disclose all negative signals because four votes are now required for the new initiative to be approved. On the other hand, conditional on observing a signal $s \geq 0$, the leader wishes that all voters vote for acceptance. In this case, he would be indifferent between concealing all signals $s \in C$ for any subset $C \subseteq [0, -\mu_4)$. Thus the optimal concealment set takes the form of any set $C_4(0; x)$ such that $C_4(0; x) \subseteq [0, \bar{s}_4) = [0, -\mu_4)$.

Finally, consider $k = 2$ so that simple majority is sufficient to approve the new initiative. Then, conditional on observing a signal $s < 0$, the leader has weak incentives to obfuscate only one voter with positive priors, in particular, the voter with the smallest $|\bar{s}_i|$ among those two voters $i = 1, 2$ with $\mu_i > 0$. If the leader did not obfuscate such a voter, then two voters would vote for acceptance for signals under which the leader strictly prefers rejection. Thus, the leader would be indifferent between concealing all signals $s \in C$ for any set $C \subseteq [-\mu_2, 0)$. On the other hand, conditional on observing a signal $s \geq 0$, the leader already can count (with probability one) on the vote for acceptance of the two voters with positive priors. Therefore, we obtain that the optimal concealment set takes the form of any $C_2(0; x)$ such that $C_2(0; x) \subseteq [\bar{s}_2, 0) = [-\mu_2, 0)$.

How do education levels influence the leader's optimal design of the concealment set? In situations where no voter had education ($x = (ne, \ldots, ne)$), we could simply resort to the ordering $-\mu_1 < \cdots < -\mu_{n/2} < 0 < -\mu_{(n/2)+1} < \cdots < -\mu_n$ to determine directly the critical signal realization $\bar{s}_k = -\mu_k$ associated to voter k. However, in situations where some voters are educated, we observe from the specification in Eq. (1) that such an ordering needs further qualification to determine the critical signal realization associated to the voter that would be pivotal in a situation where all signals were disclosed. In particular, for those situations, the relevant ordering is given by the condition expressed in Eq. (7). Leaving aside the required analytical expressions, such comparative implications convey a quite intuitive message. The basic idea is that varying degrees of access to external sources of information across voters make them learn differently. As a consequence, such discrepancies in access to external means of information affect the distribution of cutoff signals for the voters to prefer one alternative or the other.

Let us restrict attention to the (largest) selection $C^*_k(0; x) = C_k(0; x)$ of optimal concealment sets. Then, the general message in the presence of education is that optimal concealment sets shrink, compared to the situation where no voter possesses education. Furthermore, the new optimal concealment sets shrink relative more as the probability ε of the education efforts being fruitful increases. The message conveyed is that the leader is aware that his obfuscation strategy becomes less effective when voters are able to obtain larger amounts of information by themselves about ω.

The most interesting situations arise when some voters are educated, whereas others are not so that the information disclosed by the leader stands as their unique source of additional information about ω. The following example illustrates how the moderate leader would optimally design the concealment set $C^*_k(0; x)$ when some voters are educated. Example 2 highlights differences about the leader's optimal behavior, relative to the case where voters are not educated (as in Example 1).

Example 2. Suppose that $n = 4$ and consider the moderate leader ($\mu_l = 0$). There are exactly two voters located on each side of the leader over the opinion spectrum, that is, $-\mu_1 < -\mu_2 < 0 < -\mu_3 < -\mu_4$. Let us focus on the dictatorship majority rule $k = 1$ so that the vote of a single voter in favor of acceptance is sufficient to approve the new initiative. Recall that, for situations where all signals were disclosed, voter 1 would be the pivotal voter in the absence of education. As argued above, we will see that this might change in the presence of education.

Suppose that voters 3 and 4 are educated. Since the vote of a single voter in favor of the initiative is sufficient to achieve the outcome preferred by the leader when he receives signals $s \geq 0$, he does not care about the education levels of such voters 3 and 4. In particular, the leader finds optimal to disclose all nonnegative signals. He does care, however, about whether or not to conceal negative signals, depending on the education levels of voters 1 and 2. We turn now to consider plausible education levels for voters 1 and 2.

First, suppose that $x_1 = x_2 = e$. Then, we obtain the following induced ordering $\sigma(x)$ of the critical signal realizations $\bar{s}_i$.

$$-(1-\varepsilon)\mu_1 < -(1-\varepsilon)\mu_2 < 0 < -(1-\varepsilon)\mu_3 < -(1-\varepsilon)\mu_4.$$

Thus, conditional on observing a signal $s < 0$, the leader wants to conceal signals so as to obfuscate the two voters with positive priors. The optimal concealment strategy of the leader takes the form now of any subset $C_1(0; x) \subseteq [-(1-\varepsilon)\mu_1, 0)$. We observe that the largest set within the family of optimal concealment sets shrinks relative to the largest set within the family of subsets that we derived in Example 1 for the case in which voters were not educated.

Secondly, suppose that $x_1 = ne$ and $x_2 = e$. Then, we obtain the following induced ordering $\sigma(x)$ of the critical signal realizations $\bar{s}_i$.

$$-\mu_1 < -(1-\varepsilon)\mu_2 < 0 < -(1-\varepsilon)\mu_3 < -(1-\varepsilon)\mu_4.$$

Thus, conditional on observing a signal $s < 0$, the leader wants to conceal signals so as to obfuscate the two voters with positive priors. The optimal concealment strategy of the leader takes the form now of any subset $C_1(0; x) \subseteq [-\mu_1, 0)$. This family of subsets indeed coincides with the family of optimal concealment sets that we derived in Example 1 for the case in which voters were not educated.

Lastly, suppose that $x_1 = e$ and $x_2 = ne$. Then, we can obtain two different orderings of the critical signal realizations, depending on their particular opinions μ_1 and μ_2, and on the probability ε that education efforts are fruitful. In particular, if $\varepsilon < (\mu_1 - \mu_2)/\mu_1$, then it follows that

$$-(1-\varepsilon)\mu_1 < -\mu_2 < 0 < -(1-\varepsilon)\mu_3 < -(1-\varepsilon)\mu_4.$$

The optimal concealment set in this case is any subset $C_1(0; x) \subseteq [-(1-\varepsilon)\mu_1, 0)$. The largest possible optimal concealment set shrinks relative to the largest possible set within the family of subsets that was optimally chosen when voters were not educated. On the other hand, if $\varepsilon > (\mu_1 - \mu_2)/\mu_1$, we then obtain

$$-\mu_2 < -(1-\varepsilon)\mu_1 < 0 < -(1-\varepsilon)\mu_3 < -(1-\varepsilon)\mu_4.$$

In this case, the optimal concealment set takes the form of any subset $C_1(0; x) \subseteq [-\mu_2, 0)$. Again, the largest possible optimal concealment set shrinks relative to the largest set within the family of subsets that was optimally chosen when voters were not educated.

5 Leader's Utility in Equilibrium

We turn now to investigate the (ex ante) utility that the leader receives from his investigation effort and obfuscation behavior in equilibrium. Notably, even though there is a profound multiplicity of equilibria in the proposed model, in terms of optimal concealment sets, equilibrium payoffs are unique.

Following our previous comments on Observation 1, we now make the reduced form assumption of focusing on situations in which voter i_k continues to be the pivotal voter (according to Definition 1) when the leader moves from a hypothetical situation of not concealing signals to doing so (so that $C_k \neq \emptyset$), and voters receive no signal (i.e., they receive $s = \emptyset$). Notice that, without imposing further assumptions on the differences among the players opinions, focusing in such situations is crucial to obtain insights about all possible categories of the leader. Given our notion of pivotal voter in the presence of concealed signals in Definition 1, this assumption would hold always when the leader is moderate, when the leader is a radical in favor of the new initiative and the majority rule is not below simple majority, or when the leader is a radical in favor of the status quo and the majority rule is not above simple majority.

Suppose that the leader has an opinion $\mu_l \in \{0, \overline{\mu}, \underline{\mu}\}$ and that the profile of education levels is x. Conditional on the selection of the concealment set $C_k^*(\mu_l; x) \in \mathcal{C}$, let $U_{\mu_l}(\lambda; x, k)$ be the leader's ex ante expected utility for an investigation effort λ, given the profile of education levels x and the majority rule k. Then, Lemma 2–Lemma 4 derive useful expressions for the (ex ante) expected utility of the leader, provided that he designs optimally a concealment set $C_k^*(\mu_l; x)$ at the interim stage of the game.

Lemma 2. Consider a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader has a moderate opinion ($\mu_l = 0$). Then, the leader's (ex ante) expected utility for an investigation effort $\lambda \in (0, \bar{\lambda}]$, conditional on the optimal selection of a concealment set $C_k^*(0; x) \subseteq C_k(0; x)$, can be expressed as

$$U_0(\lambda; x, k) = -(1/2)\big[1 - \lambda P_l[s \notin C_k^*]\big] - c(\lambda),$$

where $C_k^* = C_k^*(0; x) \subseteq [\bar{s}_{i_k}, 0)$ for $k \leq n/2$ and $C_k^* = C_k^*(0; x) \subseteq [0, \bar{s}_{i_k})$ for $k > n/2$.

Lemma 3. Consider a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader has a radical opinion in favor of the new initiative ($\mu_l = \overline{\mu} \to +\infty$). Then, the leader's (ex ante) expected utility for an investigation effort $\lambda \in (0, \bar{\lambda}]$, conditional on the optimal selection of a concealment set $C_k^* = C_k^*(\overline{\mu}; x) \subseteq C_k(\overline{\mu}; x)$, can be expressed as

(a) for $k \leq n/2$, $U_{\overline{\mu}}(\lambda; x, k) = -\pi_k(C_k^*, \lambda)\big[1 - \lambda P_l[s \notin C_k^*]\big] - c(\lambda)$;

(b) for $k > n/2$, $U_{\overline{\mu}}(\lambda; x, k) = -\big[1 - \lambda P_l[s \notin C_k^*]\big] - c(\lambda)$.

Lemma 4. Consider a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader has a radical opinion in favor of the status quo ($\mu_l = \underline{\mu} \to -\infty$). Then, the leader's (ex ante) expected utility for an investigation effort $\lambda \in (0, \bar{\lambda}]$, conditional on the optimal selection of a concealment set $C_k^* = C_k^*(\underline{\mu}; x) \subseteq C_k(\underline{\mu}; x)$, can be expressed as

(a) for $k \leq n/2$, $U_{\underline{\mu}}(\lambda; x, k) = -\big[1 - \lambda P_l[s \notin C_k^*]\big] - c(\lambda)$;

(b) for $k > n/2$, $U_{\underline{\mu}}(\lambda; x, k) = -\pi_k(C_k^*, \lambda)\big[1 - \lambda P_l[s \notin C_k^*]\big] - c(\lambda)$.

The descriptions of the families of optimal concealment sets $C_k^*(\mu_l; x)$ that appear in Lemma 2–Lemma 4 correspond to those derived, respectively, in Proposition 1–Proposition 3.

With the tractable expressions for the leader's utility in equilibrium derived in Lemma 2–Lemma 4 at hand, we turn now to explore the optimal investigation effort of each category of the leader in equilibrium. Given an leader $\mu_l \in \{0, \overline{\mu}, \underline{\mu}\}$, a profile x of education levels, and a majority rule k, we will find useful to consider explicitly the function $\varphi_{\mu_l}(\lambda; x, k) \equiv \partial U_{\mu_l}(\lambda; x, k)/\partial\lambda$, which gives us the marginal change in the leader's (ex ante) utility due to changes in his investigation effort.

Proposition 4. Consider a given a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader is moderate ($\mu_l = 0$). Then, there is a unique equilibrium investment effort $\lambda^* \in (0, \overline{\lambda})$, which is characterized by the condition

$$P_l[s \notin C_k^*(0; x)] = 2c'(\lambda^*), \tag{10}$$

where we have $C_k^*(0; x) \subseteq [\bar{s}_{i_k}, 0)$ for $k \leq n/2$ and $C_k^*(0; x) \subseteq [0, \bar{s}_{i_k})$ for $k > n/2$.

Proposition 5. Consider a given a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader is a radical biased in favor of the new initiative ($\mu_l = \overline{\mu} \to +\infty$). Then,

(a) for $k \leq n/2$, there is a unique equilibrium investment effort $\lambda^* \in (0, \overline{\lambda})$, which is characterized by the condition

$$\pi_k P_l[s \notin C_k^*] - (\partial\pi_k/\partial\lambda)\big[1 - \lambda P_l[s \notin C_k^*]\big] = c'(\lambda^*), \tag{11}$$

where $C_k^* = C_k^*(\overline{\mu}; x)$ and $\pi_k = \pi_k(C_k^*, \lambda^*)$;

(b) for $k > n/2$, there is a unique equilibrium investment effort $\lambda^* \in (0, \overline{\lambda})$, which is characterized by the condition

$$P_l[s \notin C_k^*] = c'(\lambda^*), \tag{12}$$

where $C_k^* = C_k^*(\overline{\mu}; x)$.

Proposition 6. Consider a given a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader is a radical biased in favor of the status quo ($\mu_l = \underline{\mu} \to -\infty$). Then,

(a) for $k \leq n/2$, there is a unique equilibrium investment effort $\lambda^* \in (0, \overline{\lambda})$, which is characterized by the condition

$$P_l[s \notin C_k^*] = c'(\lambda^*), \tag{13}$$

where $C_k^* = C_k^*(\underline{\mu}; x)$.

(b) for $k > n/2$, there is a unique equilibrium investment effort $\lambda^* \in (0, \overline{\lambda})$, which is characterized by the condition

$$\pi_k P_l[s \notin C_k^*] - (\partial\pi_k/\partial\lambda)\big[1 - \lambda P_l[s \notin C_k^*]\big] = c'(\lambda^*), \tag{14}$$

where $C_k^* = C_k^*(\underline{\mu}; x)$ and $\pi_k = \pi_k(C_k^*, \lambda^*)$.

Two qualitatively different insights emerge from Proposition 4–Proposition 6. On the one hand, the characterizations of the leader's optimal effort provided by Eq. (10), Eq. (11), and Eq. (14) describe the neat requirement that the marginal benefit from the investigation effort must be equal to its marginal cost. In this case, the marginal benefit is directly given by the probability that the received signal be actually disclosed by the leader. Unlike this, the conditions provided by Eq. (12) and Eq. (13) include in the expressions of the marginal benefit both the probability that voter i_k uses the concealed signals (according to her own opinions) and the rate of change of such probability with respect to the investigation effort. Such discrepancies in the characterization of the optimal investigation effort are driven by the following forces.

On the one hand, for those situations in which (1) the leader is moderate, (2) the leader is a radical in favor of the new initiative and the majority rule is more unanimous, or (3) the leader is a radical in favor of the status quo and the majority rule is more dictatorial, it follows that, conditional on concealed signals $s \in C_k^*$, the voting outcome obtained is not affected by whether they use only their opinions or their opinions combined with the concealed signals. The idea here is that the whether or not taking into account concealed signals is relevant to change the outcome of the election.[21]

On the other hand, for the situations in which (4) the leader is a radical in favor of the new initiative and the majority rule is more dictatorial, or (5) the leader is a radical in favor of the status quo and the majority rule is more unanimous, it follows that, conditional on concealed signals $s \in C_k^*$, the outcome of the election process depends crucially on whether the voters use only their opinions or their opinions together with the concealed signals. In particular, for the case described in (4), it follows that the radical leader who prefers the new initiative benefits from the requirement that only a relatively small number of voters—which are relatively close to her own opinions—be sufficient to approve the new initiative. Conditional on concealed signals, such voters wish to choose the leader's preferred alternative when they use only their priors. This benefits the leader when the majority rule is more dictatorial. Similarly, for the case described in (5), we have that the radical leader who prefers to remain in the status quo benefits from the requirement that a high number of voters be required in order to change the status quo. Conditional on concealed signals, voter i_k—who is relatively close to the leader's own opinions—prefers rejection based solely on her own opinions. This again benefits the leader. Such mechanisms explain why, in the situations described in (4) and (5), the leader incorporates explicitly—as part of the marginal benefit of his efforts—the probability π_k that voter i_k uses the undisclosed signals, as well as the rate $\partial \pi_k / \partial \lambda$ according to which such a probability changes with his investigation efforts.

[21] For instance, suppose that the leader is strongly biased in favor of the new initiative and that approval of the new initiative requires a relatively large number of votes in favor of it. Then, the leader needs to persuade a relatively large number of voters. The priors of some of those voters will be necessarily far away from the opinion of the leader. Furthermore, upon concealed signals, such voters will give importance to their own priors (using either their own priors solely or their own priors combined with the leader's "technology"). As a result, they will vote against the new initiative, regardless of the leader's obfuscation efforts.

For the case of the moderate leader, we can use our previous insights to comment (in Observation 2 below) about (i) how would the investigation efforts of the leader change as a function of the majority rule? and (ii) which would be the preferred majority rules of the leader?

Observation 2. Consider the moderate leader ($\mu_l = 0$) and suppose that he faces a profile of education levels x and a majority rule $k \in N$. Consider the induced ordering $\sigma(x)$ of critical signal realizations described in Eq. (7) and the associated voter i_k. Now, suppose that there is a one-unit increase ($\Delta k = 1$) in the number of votes required to approve the new initiative so that we move from the initial rule k to the (slightly modified) rule $k + 1$. As to our first question, (i) above, note that the implicit value theorem (adapted to the discrete change in k) can then be used on the condition $\varphi_0(\lambda^*; x, k) = 0$—which guarantees that $\lambda^* \in (0, \overline{\lambda})$ is part of the leader's best response in equilibrium—to derive a reasonable approximation of the induced change $\Delta\lambda^*$ in the leader's optimal investigation effort. In particular, given $\Delta k = 1$, it follows that

$$\frac{\Delta\lambda^*}{\Delta k} \approx -\frac{\Delta\varphi_0/\Delta k}{\partial\varphi_0/\partial\lambda} = (-1/2)^r \Big(P_l[s \geq \bar{s}_{i_k}] - P_l[s \geq \bar{s}_{i_{k+1}}]\Big) \Big/ c''(\lambda^*), \tag{15}$$

where $r = 2$ for majority rules $k, k + 1 \leq n/2$ and $r = 1$ for rules $k, k + 1 > n/2$. Thus, since $c''(\lambda^*) > 0$, we observe that increases in the required number of votes in favor of acceptance incentivizes the moderate leader (a) to invest more in acquiring information (as a result, λ^* raises) when the majority rule is more dictatorial than simple majority, and (b) to invest less (as a result, λ^* lowers) when the majority rule is more unanimous than simple majority. By combining those insights, it follows that the moderate leader wants to invest more in acquiring information as the majority rule becomes closer to the simple majority. The key point here is that the leader invests more and, therefore, provides more accurate information, as the majority rule narrows the discrepancy between the opinion of voter i_k and the leader's opinion.

As to our second question of interest, (ii) above, observe that, by plugging the requirement given by Proposition 4 into the expression for the (ex ante) utility of the leader provided in Lemma 2, the expression

$$U_0(\lambda^*; x, k) = -(1/2) + \lambda^* c'(\lambda^*) - c(\lambda^*) \tag{16}$$

gives us the optimal (ex ante) utility of the leader in equilibrium. Now, for the considered one-unit increase in the number of votes required to approve the new initiative, we can use the approximation given in Eq. (15), together with the expression in Eq. (16), to derive the induced change

$$\begin{aligned}\Delta U_0(\lambda^*; x, k) &= \lambda^* c''(\lambda^*)\Delta\lambda^* \\ &\approx (-1/2)^r \lambda^* \Big(P_l[s \geq \bar{s}_{i_k}] - P_l[s \geq \bar{s}_{i_{k+1}}]\Big),\end{aligned}$$

where $r = 2$ for majority rules $k \leq n/2$ and $r = 1$ for majority rules $k > n/2$. In fact, the expression above for the change $\Delta U_0(\lambda^*; x, k)$ induced by a change $\Delta k = 1$ simply follows from (a discrete-perturbation adaptation of) the envelope theorem:

$$\Delta U_0(\lambda^*; x, k)/\Delta k = U_0(\lambda^*; x, k+1) - U_0(\lambda^*; x, k).$$

Since $\Delta U_0(\lambda^*; x, k) > 0$ as k increases for majority rules $k > n/2$ and $\Delta U_0(\lambda^*; x, k) < 0$ as k increases for majority rules $k > n/2$, it follows that the moderate leader prefers rules either $k = n/2$ or $k = (n/2) + 1$ over the rest of majority rules.[22]

Pushing further, we investigate where (each category of) the leader would wish to place the opinion of voter i_k, if he had the opportunity to do so.

Proposition 7. Consider a given majority rule k and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader had the possibility of choosing the location in the real line of the critical signal realization $\bar{s}_{i_k} = \bar{s}_{i_k}(x; \varepsilon, \mu_l)$—conditional on the model's restriction that $\bar{s}_{i_k} \in (-\infty, 0)$ for $k \leq n/2$ and $\bar{s}_{i_k} \in (0, +\infty)$ for $k > n/2$. Then,

(a) the moderate leader ($\mu_l = 0$), would choose $\bar{s}_{i_k} \to 0$ for the largest selection of optimal concealment subsets ($C^*_k(0; x) = C_k(0; x)$), whereas he would not care about $\bar{s}_{i_k}$ for the selection in which he discloses all signals ($C^*_k(0; x) = \emptyset$);

(b) the radical leader in favor of the new initiative ($\mu_l = \overline{\mu} \to +\infty$) would always prefer $\bar{s}_{i_k} \to -\infty$;

(c) the radical leader in favor of the status quo ($\mu_l = \underline{\mu} \to -\infty$) would always prefer $\bar{s}_{i_k} \to +\infty$.

The general message is that the leader would prefer that voter i_k be as much aligned as possible with his own opinions. For the equilibrium selection in which the leader conceals all signals within the disagreement set $C_k(0; x)$, the moderate leader prefers a rule as close as possible to the simple majority $k = n/2$. Importantly, for the equilibrium election in which he discloses all obtained signals, the leader is indifferent between majority rules. In sharp contrast with these insights, we observe that a radical leader in favor of the new initiative always prefers a dictatorial majority rule, $k = 1$, and a radical leader in favor of the status quo always prefers a unanimous rule, $k = n$.

The insights provided by Proposition 7 could be useful think about what levels of turnout would leaders prefer in some forms of the so-called popular consultations. Such voting environments have become common in some countries to gauge the sense of the

[22] From the expression derived in Eq. (16), we observe that determining which of the two rules, either $k = n/2$ or $k = (n/2) + 1$, is preferred by the leader depends largely on (1) the particular shape of the cost function c and (2) the magnitudes of the respective biases $\mu_{n/2}$ and $\mu_{(n/2)+1}$. In short, though, our model delivers the message that the moderate leader's (ex ante) utility in equilibrium is harmed by both very dictatorial and very unanimous majority rules.

public opinion about particular questions. Two important features of consultations are that participation is not enforced and that there is no clear commitment about implementing the outcome of the consultation.[23] Arguably, an enormous fraction of the voters who participate in popular consultations typically vote for accepting the initiative proposed. Both the absence of participation enforcement and of commitment to the outcome incentivize only those voters strongly in favor of the initiative proposed to show up. According to the available data, this has been the case in most prominent popular consultations. Thus, conditional on the new initiative being accepted with a large support, we would like to view turnout levels as an indicator of the practical "switch-point" majority rule that grants the acceptance outcome.

Consider then the following two illustrative examples. In December 2020, the opposition leaders in Venezuela pushed forward a consultation where the alternatives where either to remain in the political status quo, which was set by the previous legislative election, or to accept the initiative of holding a new election (with a broader international verification process). The Venezuelan government publicly positioned itself as a radical leader in favor of the status quo and, therefore, of rejecting the raised initiative. More than 90 percent of the votes resulted in favor of the new initiative. In August 2021, the Mexican government organized a consultation where the alternatives where either to remain in the legal status quo, of not probing former presidents, or to accept the initiative of investigating such ex-presidents. The Mexican government publicly positioned itself as a radical leader in favor of the new initiative. Approximately, 98 percent of the votes were in favor of the new initiative. The clear directions of the votes in both examples support our view that turnout can be a good indicator of the majority rule which in practice sets the "switch-point" for the collective choice being one or another in such voting environments. Of course, such "switch-point" majority rules are merely hypothetical as no decision was implemented from the outcomes of any of the two consultations.

Our model's insights on the leaders' well-beings in equilibrium would suggest that, conditional on having an acceptance outcome, if the leaders could choose turnout in their respective consultations, then the Venezuelan government would prefer a relatively high turnout while the Mexican government would prefer a relatively low turnout. As a matter of fact, that was roughly what happened in both consultations. According to the organizers, 6.3 million voters participated in the Venezuelan consultation.[24] According the the Venezuelan government, 5.2 million voters participated. Either way, turnout was

[23] Most consultations are basically "symbolic," with no practical effects in terms of policy implementation. In some cases, commitment is absent due to the circumstances under which the consultation takes place and, in other cases, it is even explicitly dropped by the rules of the consultation itself. In consequence, beyond a presumed goal of opinion gathering by policy-makers, consultations are mainly used as a mean of making public the opinions that lead to the outcome of the consultation.

[24] A qualification is in order here. Knowing that the international community would be more likely to back their political position if the consultation displayed majority rules relatively more unanimous than dictatorial, opposition leaders in Venezuela made huge counter-efforts to increase the turnout for their consultation. In particular, vote online, as well as vote by voters living abroad the country, was strongly promoted by the opposition leaders.

very high for this sort of unofficial popular consultations, accounting for approximately 30-33 percent of the eligible electorate. According to official data, only 5 percent to the eligible electorate participated in the Mexican consultation.

5.1 *The Role of Education*

We turn now to investigate (i) how the information obtained by voters from external sources interacts with the leader's obfuscation strategy and (ii) which profiles of education levels are preferred by each category of the leader, depending on the existing majority rule.

For a profile of education levels x, Let $N_e(x) \equiv \{i \in N \mid x_i = e\}$ be the set of educated voters and $n_e(x) \equiv |N_e(x)|$ the associated number of educated voters. The most interesting situations arise when there are certain (exogenous) restrictions such that a number less than the required majority k (to accept the new initiative through the voting process) can be educated. In such situations, a certain number of votes required to leave the status quo will then necessarily correspond to voters that do not have access to external sources of information. By focusing on this class of situations we want to avoid (less interesting) situations where all the voters who support the new initiative could in principle obtain information from an external source, and not from the leader's disclosure of information. Therefore, as in Example 3 and Example 4 below, in the sequel we will restrict attention to situations where $0 < n_e(x) < k$ for the existing majority rule $k \in N$. As a consequence, in order to explore the role of education in a meaningful way, from here onwards, we need to restrict attention to majority rules that are not fully dictatorial, i.e., $1 < k \leq n$.[25]

Example 3. Consider a leader in favor of the new initiative ($\mu_l = \overline{\mu} \to \infty$). Suppose that there are 8 voters and that the existing majority rule is simple majority, $k = 4$. Consider a situation where exactly 3 voters receive education. Let us then consider two alternative profiles of education levels to illustrate plausible answers to the questions asked in (i) and (ii) above.

First, consider an education profile x such that voters 1, 2, and 3 are educated, $x_1 = x_2 = x_3 = e$. Then, it follows that $\bar{s}_i(x; \varepsilon, \overline{\mu}) = -[(1-\varepsilon)\mu_i + \varepsilon\overline{\mu}] \to -\infty$ for the three educated voters $i = 1, 2, 3$. The induced ordering $\sigma(x)$ is given by

$$\bar{s}_1(x; \varepsilon, \overline{\mu}) \leq \bar{s}_2(x; \varepsilon, \overline{\mu}) \leq \bar{s}_3(x; \varepsilon, \overline{\mu}) < \bar{s}_4(x; \varepsilon, \overline{\mu}) < \cdots < \bar{s}_8(x; \varepsilon, \overline{\mu}),$$

so that the pivotal voter i_4 (for a situation where all signals were disclosed) is voter 4, exactly as it would be the case in a hypothetical situation where no voter receives education.

Secondly, consider another education profile x' such that voters 6, 7, and 8 are educated, $x'_6 = x'_7 = x'_8 = e$. Then, it follows that $\bar{s}_i(x'; \varepsilon, \overline{\mu}) = -[(1-\varepsilon)\mu_i + \varepsilon\overline{\mu}] \to -\infty$ for the

[25] Example 2 was not restricted by these considerations as the sort of questions illustrated there were substantially different, and not affected by the restrictions that we consider from this point onwards.

three educated voters $i = 6, 7, 8$. The induced ordering $\sigma(x)$ is given by

$$\bar{s}_6(x'; \varepsilon, \overline{\mu}) \leq \bar{s}_7(x'; \varepsilon, \overline{\mu}) \leq \bar{s}_8(x'; \varepsilon, \overline{\mu}) < \bar{s}_1(x'; \varepsilon, \overline{\mu}) < \cdots < \bar{s}_5(x'; \varepsilon, \overline{\mu}),$$

so that voter i_4 is now voter 1.[26]

Notably, we learned from Proposition 2 that the respective optimal concealment sets are uniquely given by $C_4^*(\overline{\mu}; x) = (-\infty, -\mu_4)$ and by $C_4^*(\overline{\mu}; x') = (-\infty, -\mu_1)$. The optimal concealment set chosen by the leader diminishes (in the set inclusion order) when we move from the profile x to the profile x'. In addition, from the insights in Proposition 7, we observe that, in equilibrium, the leader prefers the education profile x' over the education profile x. In particular, under the restriction that exactly three voters get education, the preferred concealment set by the leader is necessarily $(-\infty, -\mu_1)$.

Finally, it can be checked that other education profiles x'' can be similarly proposed in a way such that they lead to $C_4^*(\overline{\mu}; x'') = (-\infty, -\mu_1)$ as well. For example, any profile of education levels such that a subset of exactly three voters $j \in \{2, \ldots, 8\}$ get educated will make their critical signal realizations $\bar{s}_{i_j} \to -\infty$. As a consequence, under the majority rule $k = 4$, we would have $i_4 = 1$. In short, to maximize his (ex ante) utility in equilibrium, all that this radical leader wants is that voter 1 be not one of the three educated voters. This message that the radical leader prefers that those voters that are closer to his own opinion do not get education is a general one (as we will see in Proposition 8).

Example 4. Consider the moderate leader ($\mu_l = 0$). Suppose that there are 10 voters and that the existing majority rule is now $k = 3$. Consider a situation where exactly 2 voters receive education. We observe that the size of the optimal concealment set $C_3^*(0; x)$ according to the (largest) selection $C_3^*(0; x) = [\bar{s}_{i_3}(x; \varepsilon, 0), 0)$ is minimized when the corresponding critical signal realization $\bar{s}_{i_3}(x; \varepsilon, 0)$ is as close as possible to zero. Since only two voters can receive education, this goal is achieved if voters 3 and 4 are chosen to get education. In this case, a voter $i_3 \in \{3, 4, 5\}$ will ultimately be the pivotal voter (in a situation were all signals were disclosed) and $\bar{s}_{i_3}(x; \varepsilon, 0) = \min\{-\mu_5, -(1 - \varepsilon)\mu_3\}$. This gives us a plausible optimal concealment set with minimal size that can be induced by x in the described situation. Of course, note that other optimal concealment sets $C_3^*(0; x) \subseteq [\bar{s}_{i_3}(x; \varepsilon, 0), 0)$ arise as part of equilibrium, regardless of the existing profile of education levels.

Some qualitative observations can be derived from Example 3 above, in which the leader is a radical in favor of the new initiative. First, (i) the incentives of the leader to obfuscate voters lower when the voters that become educated are *not* those with opinions closer to the leader's opinion. In addition, (ii) the leader prefers such situations where the

[26] Furthermore, from the order of the voters' opinions μ_i, we know that the following relation holds

$$\bar{s}_1(x; \varepsilon, \overline{\mu}) \leq \bar{s}_2(x; \varepsilon, \overline{\mu}) \leq \bar{s}_3(x; \varepsilon, \overline{\mu}) \leq \bar{s}_6(x'; \varepsilon, \overline{\mu}) \leq \bar{s}_7(x'; \varepsilon, \overline{\mu}) \leq \bar{s}_8(x'; \varepsilon, \overline{\mu})$$

when comparing some critical signal realizations associated to the two profiles x and x'.

educated voters are not those whose opinions are more similar to his own's. The message conveyed by this Example 3, though, is very particular to any of the two radical leaders. A quite interesting mechanism lies behind such implications. In particular, the radical leader assesses what educated voters can learn about the underlying state *by using his own radical view about the state*. Thus, given his own "extreme" perspective, the radical leader's anticipation of the reorder of the voters' critical signal realizations upon education makes him regard as of no value the education of voters whose opinions are more similar to his own's.

That message, however, does not follow for the case of the moderate leader, as illustrated in Example 4. The moderate leader's anticipation of the change in the order to the voters' critical signal realizations (when moving from a hypothetical situation of absence of education to another where some voters may get education) is less "extreme," compared to the case of a radical leader. In particular, under the equilibrium selection $C^*_k(0; x) = C_k(0; x)$, the moderate leader only wants to make sure that the voter that would be pivotal (in a hypothetical situation where all signals were disclosed and no education were available) indeed gets education if she were allowed to. Given his own moderate perspective about the state of the world, this would reduce his incentives to obfuscate voters and, in turn, maximize his (ex ante) utility in equilibrium. Furthermore, in other equilibrium selections the moderate leader would care even less about who gets educated. In the extreme case given by the equilibrium selection $C^*_k(0; x) = \emptyset$, in which the leader discloses all obtained signals, he is indifferent in equilibrium between any profiles of education levels.

The insights provided by Proposition 8 bellow for radical leaders follow closely the arguments laid out in Example 3. As in the example, the proposition benefits from the results obtained earlier in Proposition 2 and Proposition 3, together with Proposition 7.

Proposition 8. Consider a given majority rule $1 < k \leq n$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that a number $0 < n_e(x) < k$ of voters are educated under profile x. Suppose that the leader had the possibility of choosing which voters are educated and which voters are not (i.e., the possibility of choosing the composition of the set $N_e(x)$) under the restriction that exactly $n_e = n_e(x)$ voters are educated. Then,

(a) the radical leader in favor of the new initiative ($\mu_l = \overline{\mu} \to +\infty$) would choose $N_e(x)$ in any way such that $N_e(x) \subset N \setminus \{1, \ldots, k - n_e\}$;

(b) the radical leader in favor of the status quo ($\mu_l = \underline{\mu} \to -\infty$) would choose $N_e(x)$ in any way such that $N_e(x) \subset N \setminus \{k - n_e, \ldots, n\}$.

In natural situations where there are constraints that restrict the amount of education that can be provided to the set of voters, radical leaders would prefer that the external means of information not be provided to voters that have opinions relatively close to their own's.

6 Voters' Welfare in Equilibrium

In this section, we investigate certain features, both of (i) majority rules and of (ii) profiles of education levels, that are preferred by the group of voters. We continue to invoke the reduced-form assumption considered in Section 5 to focus on situations in which voter k continues to be the pivotal voter when the leader conceals signals and voters receive $s = \emptyset$.

Suppose that the leader has an opinion $\mu_l \in \{0, \overline{\mu}, \underline{\mu}\}$ and that the profile of education levels is x. Conditional on the optimal selection of the concealment set $C_k^*(\mu_l; x)$, let $V_j^{\mu_l}(\lambda; x, k)$ be voter j's ex ante expected utility for an investigation effort λ, given the profile of education levels x and the majority rule k. Consider the critical signal realization $\bar{s}_j = \bar{s}_j(x; \varepsilon, \mu_l)$ for voter j.

Suppose first that the leader does obtain a signal (an event which happens with probability λ). Then, we can express the ex ante expected utility of voter j as

$$\begin{aligned} V_j^{\mu_l, s \neq \emptyset} = -\big\{ & P[o(v) = A \mid s < \bar{s}_j] P_j[s < \bar{s}_j] \\ & + \Pr[o(v) = R \mid s \geq \bar{s}_j] P_j[s \geq \bar{s}_j] \big\}. \end{aligned} \tag{17}$$

Secondly, suppose that the leader does not receive a signal (an event which happens with probability $1 - \lambda$). In this case, we can express the ex-ante expected utility of voter j as

$$\begin{aligned} V_j^{\mu_l, s = \emptyset} = -\big\{ & \Pr[o(v) = A \mid v_j = R] P_j[v_j = R] \\ & + \Pr[o(v) = R \mid v_j = A] P_j[v_j = A] \big\}. \end{aligned} \tag{18}$$

Thus, note that voter j suffers a loss whenever the outcome of the voting process is different from what she prefers. The ex ante expected utility of voter j is then

$$V_j^{\mu_l} = \lambda V_j^{\mu_l, s \neq \emptyset} + (1 - \lambda) V_j^{\mu_l, s = \emptyset}. \tag{19}$$

We adopt an utilitarian view and specify the relevant welfare function $W^{\mu_l}(\lambda; x, k)$ for the group of voters as the sum of their ex ante expected utilities, $W^{\mu_l}(\lambda; x, k) = \sum_{j \in N} V_j^{\mu_l}(\lambda; x, k)$. By embedding the derivations in Eq. (17), Eq. (18), and Eq. (19) above into this definition of voters' welfare, we will be able to investigate how the welfare of the voters depends on the distribution of their opinions, for each category of the leader. Nonetheless, for expositional reasons, we relegate such detailed derivations that describe the welfare function of the group of voters to Appendix B (Lemma 5-Lemma 7). Using such derivations, we are able to establish that, when the leader is moderate, each voter j prefers that her own critical signal realization $\bar{s}_j$ be as close as possible to the critical signal realization $\bar{s}_k$ of voter k. This makes it more likely that the preferred alternative of each voter coincides with the outcome of the voting process. This particular insight is provided by Lemma 5.

As to the question (i) which are the majority rules preferred by the group of voters prefer, we focus on the case of a moderate leader. Recall that a moderate leader prefers either majority rules $k = n/2$ or $k = (n/2) + 1$ (Proposition 7). However, this needs not be always the case for the group of voters. In particular, if we allow k to vary, then our insights (Lemma 5) lead to that some voters may benefit when the critical signal realization of the relevant voter k becomes closer to their own while, at the same time, other voters may be harmed. Assessing the overall impact on the entire groups of voters becomes quite specific, depending on the particular discrepancies between the players' opinions. In Observation 3 below, we study a particular situation (for which the distributions of the initial opinions and of the critical signal realizations are suitably chosen) such that the group of voters either prefer majority rules $k = n/2$ or $k = (n/2) + 1$ (i.e., majority rules close to simple majority).

Observation 3. Suppose that the leader is a moderate ($\mu_l = 0$) and consider a majority rule $k \leq n/2$. Suppose that the voters' initial opinions are arranged such that $\mu_1 = |\mu_n|$, $\mu_2 = |\mu_{n-1}|, \ldots, \mu_{n/2} = |\mu_{(n/2)+1}|$. In addition, suppose that the induced ordering $\sigma(x)$ of critical signal realizations is such that, for each voter j with $\bar{s}_j < 0$, there exists a voter $m(j)$ with $\bar{s}_{m(j)} > 0$ such that $\bar{s}_{m(j)} \geq |\bar{s}_j|$.

Consider now a one-unit increase, $\Delta k = 1$, in the number of votes k required to approve the new initiative so that we move from the initial majority rule k to the (slightly modified) rule $k + 1$. From the proposed distribution of opinions and of critical signal realizations, we obtain the following implication. For each voter j such that $\bar{s}_j < \bar{s}_k < 0$, there exists another voter $m(j)$ such that $\bar{s}_{m(j)} > 0$. Then note that, as k increases, each such voter j suffers a loss, whereas each such voter $m(j)$ benefits. Moreover, it follows that $P_{m(j)}[s < \bar{s}_{m(j)}] \geq P_j[s \geq \bar{s}_j]$. This implication follows simply from the assumption that, from the perspective of a voter i, signals are normally distributed with mean μ_i. Thus, the group of voters prefers $k = n/2$, because aggregate gains overcome aggregate loses, given the suggested one-unit increase $\Delta k = 1$.

More in detail, given $\Delta k = 1$, it follows that the voters who have negative critical signal realizations that lie above the one of voter i_{k+1} gain. In addition, the voter who passes to position i_{k+1} does not lose either because she becomes pivotal now. We can then restrict attention to study the change in welfare that stems from those voters with positive critical signal realizations and those voters with negative critical signal realizations that lie below the one of voter i_{k+1}. This specific change in welfare can be expressed as

$$
\begin{aligned}
-\lambda\Big\{ & \overbrace{P_l[s \in [\bar{s}_k, \bar{s}_k)]}^{(a)}]P_j[s \geq \bar{s}_k] \\
& + \overbrace{\big[P_l[s \in [\bar{s}_j, \bar{s}_{k+1})] - P_l[s \in [\bar{s}_j, \bar{s}_k)]\big]}^{(b)} \sum_{j=1}^{k-1} P_j[s \geq \bar{s}_j] \\
& + \overbrace{\big[P_l[s \in [\bar{s}_{k+1}, \bar{s}_j)] - P_l[s \in [\bar{s}_k, \bar{s}_j)]\big]}^{(c)} \sum_{j=n/2+1}^{n} P_j[s < \bar{s}_j]\Big\}.
\end{aligned}
\tag{20}
$$

The expression of the welfare of voters that allow us to derive the change in Eq. (20) is formally established in Lemma 5–(a). A key point here is that all players anticipate how the leader will optimally conceal and disclose signals. Therefore, all voters consider in a common manner how the leader's obfuscation strategy will affect the probability that the outcome of election be either acceptance or rejection. This is why the probabilities that appear in the expression in Eq. (20), according to which some signals are concealed and others are disclosed, are considered from the perspective of the leader. In the expression in Eq. (20) above, term (a) captures the decrease in the utility of voter k (under the modified majority rule $k+1$), term (b) captures the decrease in the utilities of the voters whose critical signal realizations are below the one of voter k and term (c) captures the increase in the utilities of the voters with positive critical signal realizations. Notice that the magnitudes of the terms in (a), (b) and (c) are the same (in absolute value, (a) and (b) have positive sign, whereas (c) has negative sign). Then, since for each voter j with negative critical signal realization, there exists a voter $m(j)$ with positive critical signal realization such that $P_{m(j)}[s < \bar{s}_{m(j)}] \geq P_j[s \geq \bar{s}_j]$, it follows that a unit increase in k raises the welfare of voters.

A totally analogous analysis would follow for majority rules $k > n/2$. Therefore, under the particular description of opinions and critical signal realizations suggested here, the group of voters prefer that majority rules be either $k = n/2$ or $k = (n/2) + 1$.

As to the question (ii) which distribution of external sources of information would the voters prefer, we offer insights for both the cases of a moderate leader (Proposition 9) and of radical leaders (Proposition 10). We should emphasize that our welfare analysis here restricts attention to situations in which, following any rearrangement of the profile x of education levels, voter k remains in the k-th position (within the spectrum of opinions), conditional on n_e voters being educated. In this way, we can ensure that voter k continues to be pivotal after we modify the profile of education levels. To ease the exposition, it will be useful to set

$\alpha \equiv \sum_{j=k+1}^{n} P_j[s < \bar{s}_j] - \sum_{j=1}^{k-1} P_j[s \geq \bar{s}_j]$ for $k \leq n/2$ and

$\beta \equiv \sum_{j=1}^{k-1} P_j[s \geq \bar{s}_j] - \sum_{j=k+1}^{n} P_j[s < \bar{s}_j]$ for $k > n/2$.

Such terms α and β will be useful to measure whether the group of voters either benefits or are harmed when voter k becomes educated.

Proposition 9. Consider the moderate leader ($\mu_l = 0$). Consider a given majority rule $1 < k \leq n$ and suppose that a number $0 < n_e < k$ of voters are educated under profile x. Then, voters' welfare $W^{\mu_l}(\lambda; x, k)$ is maximized when the set of educated voters takes the form: for $k \leq n/2$, we have $N_e(x) \subseteq N \setminus \{k+1, \ldots, n/2\}$ and for $k > n/2$, we have $N_e(x) \subseteq N \setminus \{(n/2)+1, \ldots, k-1\}$, provided that $3n/2 - k \geq n_e$. Moreover,

(a) for $k \leq n/2$, there exists a bound $\overline{\alpha}$ on the term α such that if (i) $\alpha < \overline{\alpha} < 0$, then the n_e voters $j \neq k$ whose critical signal realizations (in the absence of education) are the closest ones to zero must be educated, and (ii) if $\alpha > 0$ is sufficiently high, then the $n_e - 1$ voters $j \neq k$ whose critical signal realizations (in the absence of education) are the closest ones to zero, together with voter k, must be educated;

(b) for $k > n/2$, there exists a bound $\overline{\beta}$ on the term β such that (i) if $\beta < \overline{\beta} < 0$, then the n_e voters $j \neq k$ whose critical signal realizations in the absence of education are the closest to zero, and (ii) if $\beta > 0$ is sufficiently high, then the $n_e - 1$ voters $j \neq k$ whose critical signal realizations (in the absence of education) are the closest ones to zero, together with voter k, must be educated.

For $k \leq n/2$, the group of voters prefer that voters with negative critical signal realizations that lie above the one of voter k (i.e., those voters $\{k+1, \ldots, n/2\}$) not be educated. Recall that for more dictatorial majority rules the outcome is acceptance with probability one upon signals above the critical signal realization s_k of voter k. The likelihood that voters in the aforementioned set $\{k+1, \ldots, n/2\}$ prefer rejection increases when they become educated. In particular, voters' welfare decreases when such voters become educated. In addition, voters' welfare increases when other voters $j \notin \{k, k+1, \ldots, n/2\}$ become educated. Note that such voters $j \notin \{k, k+1, \ldots, n/2\}$ are precisely voters $\{1, \ldots, k-1\} \cup \{n/2, \ldots, n\}$. Using totally analogous arguments, it follows that, for $k > n/2$, the group of voters prefer that voters whose positive critical signal realizations lie below the one of voter k (i.e., those voters $\{(n/2)+1, \ldots, k-1\}$) not be educated.

Consider a majority rule $k \leq n/2$. Then, notice that if voter k moves from being uneducated to being educated (so that her critical signal realization increases), then the $k-1$ voters whose critical signal realizations lie below $\bar{s}_k$ suffer a lost, whereas the $n-k$ voters voters whose critical signal realizations lie above $\bar{s}_k$ benefit. As mentioned earlier, the term α measures whether the group of voters gains or loses when voter k becomes educated. Then, a relatively low value of α indicates that an educated voter k inflicts an aggregate utility loss to the group of voters. Then, in order to maximize voters' welfare, we would like that the n_e voters $j \neq k$ whose critical signal realizations (in the absence of education) are the closest ones to zero be educated. On the other hand, a relatively high value of α indicates that an educated voter k benefits the group of voters in aggregate. Then, voters would prefer that voter k and the $n_e - 1$ voters $j \neq k$ whose critical signal realizations (in the absence of education) are the closest ones to zero be educated. For majority rules $k > n/2$, totally analogous interpretations follow.

We previously derived that the moderate leader prefers that voter k were educated (as suggested in Example 4). Unlike this, we observe that is not necessarily what the group of voters prefer.

We close this section by providing a necessary condition for voters' welfare to be maximized when leaders are radical.

Proposition 10. Suppose that a number $0 < n_e(x) < k$ of voters are educated under profile x. Suppose that voters had the possibility of choosing which voters are educated in order to maximize their welfare $W^{\mu_l}(\lambda; x, k)$. Then,

(a) consider a radical leader in favor of the new initiative ($\mu_l = \overline{\mu} \to +\infty$) and consider a majority rule $k > n/2$. Then, in order to maximize voters' welfare, the set of educated voters must have the following form: $N_e(x) \subseteq N \setminus \{(n/2) + 1, \ldots, k - 1\}$, provided that $3n/2 - k \geq n_e$;

(b) consider a radical leader in favor of the status quo ($\mu_l = \underline{\mu} \to -\infty$) and consider a majority rule $k \leq n/2$. Then, in order to maximize voters' welfare, the set of educated voters must have the following form: $N_e(x) \subseteq N \setminus \{k + 1, \ldots, n/2\}$.

Similarly to our insights in Proposition 9, we obtain that (a) for a radical leader in favor of the new initiative and a more unanimous voting rule, welfare unambiguously decreases when the voters whose positive critical signal realizations lie below the one of voter k (i.e., those voters $\{(n/2)+1, \ldots, k-1\}$) become educated. Recall that in this case the outcome is rejection with probability one when the leader conceals signals below the critical signal realization of voter k. If such voters get educated, it becomes then more likely that they prefer acceptance, which raises their likelihood of suffering a welfare lost. Case (b), relative to the leader in favor of the status quo, offers a totally analogous interpretation.

7 Final Comments

We conclude by commenting on additional empirical evidence to illustrate some of the paper's insights. Let us first go back to some evidence available about the 2016 UK Brexit referendum discussed in the Introduction. According to research by the Centre for the Study of Media, Communication and Power (Moore and Ramsay, 2017) that surveyed 20 UK media outlets during the 10-week campaign, 65 percent of the front-page prints were backing the Leave option. Of these, the *Telegraph*, the *Express*, and the *Daily Mail* lead the field. Among the outlets that backed the Remain option, the newspapers with more front-page prints were the *Guardian*, the *Observer*, *The Times*, and *The Financial*

Times. The outlets backing the Leave option focused on immigration issues (79 percent of the front-pages on immigration were printed by the outlets arguing in favor of the Leave option). Front pages dealing with economic issues were split more evenly. However, the media group backing the Leave option focused on pensions and on the impact of migration on the NHS, whereas Remain supporters payed more attention to the effects of productivity (as the above mentioned statement by the CBI), on mortgages and housing prices, and on workers' rights. The survey gives details of studies and other pieces of evidence that both campaigns disclosed. We observe that each of the two media groups disclosed pieces of evidence (which were subsequently contested and, most of the times, shown to be incomplete) largely addressed to voters with opinions already biased towards the option backed by the corresponding media group.

Other empirical evidence to illustrate this paper's insights can be obtained from corporate governance environments. Some recent findings support our model's implication (argued in Observation 2) that moderate leaders are incentivized both to investing more in information and to obfuscating less when voting rules move away from dictatorial and get closer to simple majority. In particular, Mukhopadhyay and Shivakumar (2021) explore the information disclosure implications of regulators requiring firms to approve their proposals through shareholder voting. In 2006, the US Security and Exchange Commission (SEC) introduced direct disclosure regulations to companies that made mandatory the disclosure of compensation-relevant metrics. However, similar in spirit to the mechanism proposed in this paper, in practice, board leaders could still disclose null pieces of evidence to shareholders. Omitting details, or presenting them in "obscure" ways,[27] were commonly reported ways of concealing evidence after the 2006 SEC ruling. Subsequently, in 2011, the SEC introduced a "Say on Pay" voting requirement by shareholders of companies. No further ruling on disclosure was issued by the SEC at that time.

Mukhopadhyay and Shivakumar (2021) take advantage of those two separate regulations to propose an empirical strategy to isolate the role of introducing the simple majority rule for accepting new proposals. Specifically, the authors construct a measure of the key performance indicators disclosures of the companies listed as subject to regulation (between 2007 and 2017). Using such a measure, their analysis shows that the introduction of the simple majority as voting rule accounted for an increase (of roughly 20 percent) in the amount of evidence disclosed by board leaders. This finding can be compared to what our model delivers for the case of the moderate leader. In Observation 2, we derive the implication that, as the voting majority moves away from very dictatorial (which seems a reasonable proxy of situations where, in practice, there is no voting requirement) and approaches simple majority, the leader is incentivized to investing more in information and to obfuscating less (precisely those voters in favor of the proposal). Notably, Mukhopadhyay and Shivakumar (2021) use their empirical strategy to argue that, in such a 2011-2017 period, simple-majority voting incentivized board leaders to disclose more

[27] The corporate governance literature uses *fog indexes* to empirically account for difficulties in interpreting and digesting pieces of reported information.

information. Their particular interpretation is that concealing information would enhance the skepticism of the shareholders, which may rise the probability of the raised proposal being rejected. A totally analogous driving force of skepticism (which is fostered by concealed pieces of evidence) is captured by the model investigated in this paper.

Appendix A

Take a given majority rule $k \in N$ and suppose that the leader has an opinion $\mu_l \in \{0, \overline{\mu}, \underline{\mu}\}$. Then, upon observing a signal $s \in \mathbb{R}$, let $U_{\mu_l}(s \mid s; x, k)$ be the leader's *interim* expected utility conditional on disclosing such a signal. Analogously, let $U_{\mu_l}(\emptyset \mid s; x, k)$ be the leader's *interim* expected utility when he chooses to conceal such a signal. Recall that $\bar{s}_{i_k} = \bar{s}_{i_k}(x; \varepsilon, 0)$ gives us the signal realization that makes voter i_k's optimal decision to switch between the two alternatives.

Proof of Lemma 1. To ease notation in the following arguments, let us simply write $v_i^* = v_i^*(\emptyset \mid C, \lambda)$.

1. Moderate leader ($\mu_l = 0$).

(a) Suppose that $k \leq n/2$. Consider the concealment set $C_k = [\bar{s}_{i_k}, 0)$. Suppose that the leader's investigation efforts allow him to obtain a signal $s < 0$. For such a signal, the leader prefers to remain in the status quo. It follows that the new initiative is approved through the election process if voters $\{i_1, \dots, i_k\}$ vote for acceptance. Note first that only less than k voters (in particular, voters $\{i_1, \dots, i_{k-1}\}$) would prefer to accept when signals $s < \bar{s}_{i_k}$ are disclosed. In addition, using the expression in Eq. (8) of the probability that a single voter prefers alternative A when she observes signal $s = \emptyset$, we observe that voter i_k prefers to vote for the alternative A when she observes $s = \emptyset$. Therefore, $\phi_k(C, \lambda) = 1$ for any subset $C \subseteq C_k$.

On the other hand, if the leader obtains a signal $s \geq 0$, then he prefers to disclose such a signal. Conditional on $s \geq 0$, the leader prefers the new initiative and the voting process would lead to acceptance since at least $n/2$ voters would prefer acceptance (upon observing $s \geq 0$) and we are considering $k \leq n/2$.

(b) Suppose that $k > n/2$. Consider the concealment set $C_k = [0, \bar{s}_{i_k})$. Suppose that the leader investigation efforts allow him to obtain a signal $s \geq 0$. Conditional on such a signal, the leader prefers the new initiative. Notice that the new initiative is not approved through the election process if voters $\{i_{(n/2)+1}, \dots, i_k\}$ do not vote for acceptance. Note first that less than k voters (in particular, voters $\{i_{k+1}, \dots, i_n\}$) would prefer to reject when signals $s \geq \bar{s}_{i_k}$ are disclosed. In addition, using the expression in Eq. (8) of the probability that a single voter prefers alternative A when she observes signal $s = \emptyset$, we observe that voter i_k prefers to vote for the alternative R when she observes $s = \emptyset$. Therefore, $\phi_k(C, \lambda) = 0$ for any subset $C \subseteq C_k$.

On the other hand, if the leader obtains a signal $s < 0$, then he prefers to disclose such a signal. Conditional on $s < 0$, the leader prefers the status quo and the voting process would lead to rejection of the new initiative since less than $n/2$ voters would prefer acceptance (upon observing $s < 0$) and we are considering $k > n/2$.

2. Radical leader in favor of the new initiative ($\mu_l \to +\infty$). Consider the concealment set $C_k = (-\infty, \bar{s}_{i_k})$. Notice that the new initiative is approved through the election process if voters $\{i_1, \ldots, i_k\}$ vote for acceptance.

Suppose (a) $k \leq n/2$. Then, using the expression in Eq. (8) of the probability that a single voter prefers alternative A when she observes signal $s = \emptyset$, note that

$$\begin{aligned}\phi_k(C, \lambda) &\geq \Pr[v_{i_1}^* = A, \ldots, v_{i_k}^* = A] \\ &= \Pr[v_{i_1}^* = A]\Pr[v_{i_2}^* = A \mid v_{i_1}^* = A] \times \cdots \times \Pr[v_{i_k}^* = A \mid v_{i_1}^* = A, \ldots, v_{i_{k-1}}^* = A] \\ &\geq \Pi_{j=1}^{k}[1 - \pi_{i_j}(C, \lambda)] > 0.\end{aligned}$$

Suppose (b) $k > n/2$. Then, it follows directly that $\phi_k(C, \lambda) = 0$ for any subset $C \subseteq C_k$ since less than k voters want to vote for alternative A.

3. Radical leader in favor of the status quo ($\mu_l \to -\infty$). We obtain an analogous insight to the one derived in 2. above for the case of the radical leader in favor of the new initiative. In particular, consider now the concealment set $C_k = [\bar{s}_{i_k}, +\infty)$. Notice that the new initiative is rejected through the election process if voters $\{i_k, \ldots, i_n\}$ do not vote for acceptance.

Suppose (a) $k \leq n/2$. Then, it follows directly that $\phi_k(C, \lambda) = 1$ for any subset $C \subseteq C_k$ since at least k voters want to vote for alternative A.

Suppose (b) $k > n/2$. Then, using the expression in Eq. (8) of the probability that a single voter prefers alternative A when she observes signal $s = \emptyset$, note that

$$\begin{aligned}\phi_k(C, \lambda) &\leq \Pr[v_{i_k}^* = A, \ldots, v_{i_n}^* = A] \\ &= \Pr[v_{i_n}^* = A]\Pr[v_{i_{n-1}}^* = A \mid v_{i_n}^* = A] \times \cdots \times \Pr[v_{i_k}^* = A \mid v_{i_{k+1}}^* = A, \ldots, v_{i_n}^* = A] \\ &\leq \Pi_{j=k}^{n}[1 - \pi_{i_j}(C, \lambda)] < 1.\end{aligned}$$

This concludes the required arguments. ∎

Proof of Proposition 1. The required arguments make use of the derivations in Lemma 1. Suppose that the leader is moderate ($\mu_l = 0$).

(a) Consider a majority rule $k \leq n/2$. Note then that $\mu_{i_k} > 0$ and, therefore, $\bar{s}_{i_k}(x; \varepsilon, 0) < 0$ for voter i_k, given the considered arrangement of the voters' opinions.

Given a signal $s < 0$ observed by the leader, it follows that if any signal $s < \bar{s}_{i_k}$ is publicly observed, then a number less than k voters will vote $v_i = A$. Even though the leader prefers rejection for such signals, those voters are not sufficient to attain the acceptance outcome. By disclosing only those signals $s < \bar{s}_{i_k}$, the interim expected utility of the leader is $U_{\mu_l}(s \mid s; x, k) = 0$. Now, if for a signal realization $s < 0$, we have that $s \geq \bar{s}_{i_k}$, then a number of voters no less than k will vote $v_i = A$ with probability one. For those signals $s \in [\bar{s}_{i_k}, 0)$ the leader prefers rejection and, therefore, his interim utility is either $U_{\mu_l}(s \mid s; x, k) = -1$ or $U_{\mu_l}(\emptyset \mid s; x, k) = -\phi_k(C_k^*, \lambda)L$, for anysubset $C_k^* \subseteq [\bar{s}_{i_k}, 0)$. Now, since $\mu_k > 0$ and $E_{i_k}[s \mid s \in C_k^*] = \int_{C_k^*} s f(s; \mu_{i_k}) ds \geq \bar{s}_{i_k}$ for $C_k^* \subseteq [\bar{s}_{i_k}, 0)$, we observe from Eq. (8) that $\mathcal{J}(k) = 1$ and $\mathcal{I}(k, C_k^*) = 1$. Therefore, $\phi_k(C_k^*, \lambda) = 1$. It follows that the leader is indifferent between concealing any subset of signals $C_k^* \subseteq [\bar{s}_{i_k}, 0)$.

Given a signal $s \geq 0$ observed by the leader, if he chooses to disclose it, then a number no less than $n/2$ voters will vote $v_i = A$. Since the majority rule k satisfies $k \leq n/2$, it follows that the outcome of the election will be acceptance with probability one. For such signals $s \geq 0$ the leader strictly prefers acceptance so that, by disclosing them, his interim utility is $U_{\mu_l}(s \mid s; x, k) = 0$. Therefore, the leader optimally chooses any subset $C_k^*(0; x) \subseteq [\bar{s}_{i_k}, 0)$.

(b) Consider a majority rule $k > n/2$. Note then that $\mu_{i_k} < 0$ and $\bar{s}_{i_k} > 0$ for voter i_k, given the considered arrangement of the voters' opinions.

Given a signal $s < 0$ observed by the leader, it follows that no more than $n/2$ voters will now vote $v_i = A$ if the leader decides to disclose such signals. Since the majority rule k satisfies $k > n/2$, we know that the outcome of the election will be rejection with probability one if the leader discloses only such negative signals. For those signals $s < 0$ the leader prefers rejection and, therefore, his interim utility is $U_{\mu_l}(s \mid s; x, k) = 0$. It follows that the leader finds strictly beneficial to disclose all negative of signals.

Given a signal $s \geq 0$ observed by the leader, then at least $n/2$ voters will vote for acceptance upon observing such nonnegative signals (i.e., those voters i with opinions $\mu_i > 0$). For $0 \leq s < \bar{s}_{i_k}$, only $k - 1$ voters will vote for acceptance with probability one so that the outcome of the election will be rejection with probability one. In this case, $U_{\mu_l}(s \mid s; x, k) = -1$. On the other hand, if the leader chooses to conceal such signals $s \in C_k^* \subseteq [0, \bar{s}_{i_k})$, his interim utility is $U_{\mu_l}(\emptyset \mid s; x, k) = -[1 - \phi_k(C_k^*, \lambda)]L$. Now, since $\mu_{i_k} < 0$ and $E_{i_k}[s \mid s \in C_k^*] = \int_{C_k^*} s f(s; \mu_{i_k}) ds < \bar{s}_{i_k}$ for $C_k^* \subseteq [0, \bar{s}_{i_k})$, we observe from Eq. (8) that $\mathcal{J}(k) = 0$ and $\mathcal{I}(k, C_k^*) = 0$. Therefore, $\phi_k(C_k^*, \lambda) = 0$. Therefore, the leader is indifferent between concealing any subset of signals $C_k^* \subseteq [0, \bar{s}_{i_k})$.

If the leader observes and discloses a signal $s \geq \bar{s}_{i_k}$, then at least k voters will vote for acceptance with probability one. For those signals $s \in [\bar{s}_{i_k}, +\infty)$ the leader's interim utility when he discloses the signals is $U_{\mu_l}(s \mid s; x, k) = 0$. As a consequence, he will optimally disclose such signals $s \geq \bar{s}_{i_k}$. Therefore, the leader optimally chooses any subset $C_k^*(0; x) \subseteq [0, \bar{s}_{i_k})$. ∎

Proof of Proposition 2. The required arguments are similar to those provided in the proof of Proposition 1. Suppose that the leader is a radical in favor of the new initiative

($\mu_l = \overline{\mu} \to +\infty$).

(a) Consider a majority rule $k \le n/2$. Note then that $\mu_{i_k} > 0$ and, therefore, $\bar{s}_{i_k}(x;\varepsilon,0) < 0$ for voter i_k, given the considered arrangement of the voters' opinions. Given a signal $s \in \mathbb{R}$ observed by the leader, it follows that if any signal $s < \bar{s}_{i_k}$ is publicly observed, then a number less than k voters will vote $v_i = A$. This radical leader prefers acceptance for any signal that he obtains, and those voters are not sufficient to attain the acceptance outcome. By disclosing those signals $s < \bar{s}_{i_k}$, the interim expected utility of the leader is $U_{\mu_l}(s \mid s; x, k) = -1$. On the other hand, by concealing such signals, his interim utility is $U_{\mu_l}(\emptyset \mid s; x, k) = -[1 - \phi_k(C_k^*, \lambda)]L$, for anysubset $C_k^* \subseteq [\bar{s}_l, \bar{s}_{i_k})$. Now, since $\mu_k > 0$ and $E_{i_k}[s \mid s \in C_k^*] = \int_{C_k^*} s f(s;\mu_{i_k}) ds < \bar{s}_{i_k}$ for $C_k^* \subseteq [\bar{s}_l, \bar{s}_{i_k})$, we observe from Eq. (8) that $\mathcal{J}(k) = 1$ and $\mathcal{I}(k, C_k^*) = 0$. Therefore, $\phi_k(C_k^*, \lambda) = 1 - \pi_k(C_k^*, \lambda) \in (0,1)$. It follows that the leader is strictly better off by concealing any subset of signals $C_k^* \subseteq C_k(\overline{\mu}; x) = [\bar{s}_l, \bar{s}_{i_k})$. Notice that, in this case, $\bar{s}_l \to -\infty$ when $\overline{\mu} \to +\infty$.

(b) Consider a majority rule $k > n/2$. Note then that $\mu_{i_k} < 0$ and $\bar{s}_{i_k} > 0$ for voter i_k, given the considered arrangement of the voters' opinions. Given a signal $s < \bar{s}_{i_{n/2}}$ observed by the leader, it follows that $s < \bar{s}_{i_k}$. From the arguments given in (a) above, it follows then that the leader has strict incentives to conceal all negative signals $s \in [\bar{s}_l, \bar{s}_{i_k})$ for each $k \le n/2$. Therefore, the leader is strictly better off by concealing all signals $s \in [\bar{s}_l, \bar{s}_{i_{n/2}})$. On the other hand, given a signal $s \ge \bar{s}_{i_{n/2}}$ observed by the leader, it follows that concealment of signals $s \in B$ for any subset $B \subseteq [\bar{s}_{i_{n/2}}, \bar{s}_{i_k})$ implies $E_{i_k}[s \mid s \in B] = \int_B s f(s;\mu_{i_k}) ds < \bar{s}_{i_k}$. We then observe from Eq. (8) that $\mathcal{J}(k) = 0$ and $\mathcal{I}(k, B) = 0$. Therefore, $\phi_k(B, \lambda) = 0$. As a consequence, for signals $s \ge \bar{s}_{i_{n/2}}$ observed by the leader, he is indifferent between concealing signals in any subset $B \subseteq [\bar{s}_{i_{n/2}}, \bar{s}_{i_k})$. By putting together the optimal concealment sets that the leader designs for the cases of signals above and bellow the critical realization $\bar{s}_{i_{n/2}}$, it follows that he wishes to conceal all signals that belong to any set $C_k^*(\overline{\mu}; x)$ with the form $C_k^*(\overline{\mu}; x) = [\bar{s}_l, \bar{s}_{i_{n/2}}) \cup B$ for any subset $B \subseteq [\bar{s}_{i_{n/2}}, \bar{s}_{i_k})$. Recall that $\bar{s}_l \to -\infty$ when $\overline{\mu} \to +\infty$. ∎

Proof of Proposition 3. The required arguments are similar to those provided in the proof of Proposition 1. Suppose that the leader is a radical in favor of the status quo ($\mu_l = \underline{\mu} \to -\infty$). The exposition benefits from presenting the arguments for case (b) first.

(b) Consider a majority rule $k > n/2$. Note then that $\mu_{i_k} < 0$ and, therefore, $\bar{s}_{i_k}(x;\varepsilon,0) > 0$ for voter i_k, given the considered arrangement of the voters' opinions. Given a signal $s \in \mathbb{R}$ observed by the leader, it follows that if any signal $s \ge \bar{s}_{i_k}$ is publicly observed, then a number no less than k voters will vote $v_i = A$. This radical leader prefers rejection for any signal that he obtains, and those voters are sufficient to attain the acceptance outcome. By disclosing those signals $s \ge \bar{s}_{i_k}$, the interim expected utility of the leader is $U_{\mu_l}(s \mid s; x, k) = -1$. On the other hand, by concealing such signals, his interim utility is $U_{\mu_l}(\emptyset \mid s; x, k) = -\phi_k(C_k^*, \lambda)L$, for anysubset $C_k^* \subseteq [\bar{s}_{i_k}, \bar{s}_l)$. Now, since $\mu_k < 0$ and $E_{i_k}[s \mid s \in C_k^*] = \int_{C_k^*} s f(s;\mu_{i_k}) ds \ge \bar{s}_{i_k}$ for $C_k^* \subseteq [\bar{s}_{i_k}, \bar{s}_l)$, we observe from Eq. (8) that

$\mathcal{J}(k) = 0$ and $\mathcal{I}(k, C_k^*) = 1$. Therefore, $\phi_k(C_k^*, \lambda) = \pi_k(C_k^*, \lambda) \in (0, 1)$. It follows that the leader is strictly better off by concealing any subset of signals $C_k^* \subseteq C_k(\underline{\mu}; x) = [\bar{s}_{i_k}, \bar{s}_l)$. Notice that, in this case, $\bar{s}_l \to +\infty$ when $\underline{\mu} \to -\infty$.

(a) Consider a majority rule $k \leq n/2$. Note then that $\mu_{i_k} > 0$ and $\bar{s}_{i_k} < 0$ for voter i_k, given the considered arrangement of the voters' opinions. Given a signal $s \leq \bar{s}_{i_{(n/2)+1}}$ observed by the leader, it follows that $s \leq \bar{s}_{i_k}$. From the arguments given in (b) above, it follows then that the leader has strict incentives to conceal all negative signals $s \in [\bar{s}_{i_k}, \bar{s}_l)$ for each $k \geq (n/2) + 1$. Therefore, the leader is strictly better off by concealing all signals $s \in [\bar{s}_{i_{(n/2)+1}}, \bar{s}_l)$. On the other hand, given a signal $s < \bar{s}_{i_{(n/2)+1}}$ observed by the leader, it follows that concealment of signals $s \in B$ for any subset $B \subseteq [\bar{s}_{i_k}, \bar{s}_{i_{n/2}})$ implies $E_{i_k}[s \mid s \in B] = \int_B s f(s; \mu_{i_k}) ds \geq \bar{s}_{i_k}$. We then observe from Eq. (8) that $\mathcal{J}(k) = 1$ and $\mathcal{I}(k, B) = 1$. Therefore, $\phi_k(B, \lambda) = 1$. As a consequence, for signals $s \leq \bar{s}_{i_{(n/2)+1}}$ observed by the leader, he is indifferent between concealing signals in any subset $B \subseteq [\bar{s}_{i_k}, \bar{s}_{i_{n/2}})$. By putting together the optimal concealment sets that the leader designs for the cases of signals above and bellow the critical realization $\bar{s}_{i_{(n/2)+1}}$, it follows that he wishes to conceal all signals that belong to any set $C_k^*(\underline{\mu}; x)$ with the form $C_k^*(\underline{\mu}; x) = B \cup [\bar{s}_{i_{(n/2)}+1}, \bar{s}_l)$ for any subset $B \subseteq [\bar{s}_{i_k}, \bar{s}_{i_{n/2}})$. Recall that $\bar{s}_l \to +\infty$ when $\underline{\mu} \to -\infty$. ∎

Proof of Lemma 2. Suppose that the leader is moderate ($\mu_l = 0$).

(a) Consider a majority rule $k \leq n/2$. Suppose that the leader chooses a research effort $\lambda \in (0, \bar{\lambda}]$. Then, with such a probability λ the leader receives a signal s, and with probability $1 - \lambda$ obtains no signal ($s = \emptyset$). First, since $\mu_l = 0$, it follows that, conditional on obtaining a signal, the leader (strictly) prefers the election outcome of rejection with probability $\int_{-\infty}^{0} f(s; 0) ds = 1/2$. Proposition 1–(a) showed that, at the interim stage, the leader optimally chooses to conceal a subset of signals $C_k^* = C_k^*(0; x) \subseteq [\bar{s}_{i_k}, 0)$. In this case, the proof of Proposition 1–(a) showed that the leader is able to induce an outcome of acceptance with probability $\phi_k(C_k^*, \lambda)$, so that his expected payoff is $-\phi_k(C_k^*, \lambda) \int_{C_k^*} f(s; 0) ds$. As mentioned, this outcome is attained (from an ex ante perspective) with probability $(1/2)\lambda$. Similarly, if the leader obtains no signal, then he has no choice to make with respect to the concealment set. In this case, the leader (strictly) prefers the election outcome of rejection with probability $\int_{-\infty}^{0} f(\omega; 0) d\omega = 1/2$. Since voter i_k is voting according to the probability $\phi_k(C_k^*, \lambda)$, the leader will obtain an expected payoff $-\phi_k(C_k^*, \lambda)$, which is attained (from an ex ante perspective) with probability $(1/2)(1 - \lambda)$. Furthermore, the leader (strictly) prefers the election outcome of acceptance with probability $\int_{0}^{+\infty} f(s; 0) ds = 1/2$, conditional on obtaining a signal, and, similarly, with probability $\int_{0}^{+\infty} f(\omega; 0) d\omega = 1/2$, conditional on obtaining no signal. However, provided that $k \leq n/2$, the proof of Proposition 1–(a) showed that in such cases the leader chooses optimally to disclose all obtained signals and, at the same time, the election outcome is acceptance with probability one. Thus, the leader obtains a zero payoff in all those cases.

By combining all the arguments above, it follows that the (ex ante) expected utility of the moderate leader takes the form

$$\begin{aligned} &U_0(\lambda; x, k) = \\ &(1/2)\lambda\Big[-\phi_k(C_k^*, \lambda)\int_{C_k^*} f(s;0)ds\Big] + (1/2)(1-\lambda)\Big[-\phi_k(C_k^*, \lambda)\Big] - c(\lambda) \\ &\qquad = -(1/2)\phi_k(C_k^*, \lambda)\big[1 - \lambda P_l[s \notin C_k^*]\big] - c(\lambda). \end{aligned} \tag{21}$$

(b) Consider a majority rule $k > n/2$. Suppose that the leader chooses a research effort $\lambda \in (0, \bar{\lambda}]$. Then, with such a probability λ the leader receives a signal s, and with probability $1-\lambda$ obtains no signal ($s = \emptyset$). First, since $\mu_l = 0$, it follows that, conditional on obtaining a signal, the leader (strictly) prefers the election outcome of acceptance with probability $\int_0^{+\infty} f(s;0)ds = 1/2$. Proposition 1–(b) showed that, at the interim stage, the leader optimally chooses to conceal a subset of signals $C_k^* = C_k^*(0; x) \subseteq [0, \bar{s}_{i_k})$. In this case, the proof of Proposition 1–(b) showed that the leader is able to induce rejection with probability $1 - \phi_k(C_k^*, \lambda)$, so that his expected payoff is $-\big[1 - \phi_k(C_k^*, \lambda)\big]\int_{C_k^*} f(s;0)ds$. As mentioned, this outcome is attained from an ex ante perspective with probability $(1/2)\lambda$. Similarly, if the leader obtains no signal, then he has no choice to make with respect to the concealment set. In this case, the leader (strictly) prefers the election outcome of acceptance with probability $\int_0^{+\infty} f(\omega;0)d\omega = 1/2$. Since voter $i = k$ is voting according to the probability $\phi_k(C_k^*, \lambda)$, the leader will obtain an expected payoff $-\big[1-\phi_k(C_k^*, \lambda)\big]$, which is attained from an ex ante perspective with probability $(1/2)(1-\lambda)$. Secondly, the leader (strictly) prefers the election outcome of rejection with probability $\int_{-\infty}^0 f(s;0)ds = 1/2$, conditional on obtaining a signal, and, similarly, with probability $\int_{-\infty}^0 f(\omega;0)d\omega = 1/2$, conditional on obtaining no signal. However, provided that $k > n/2$, the proof of Proposition 1–(b) showed that in such cases the leader chooses optimally to disclose all obtained signals and, at the same time, the election outcome is rejection with probability one. Thus, the leader obtains a zero payoff in all those cases.

By combining all those arguments, it follows that the (ex ante) expected utility of the moderate leader takes the form

$$\begin{aligned} &U_0(\lambda; x, k) = \\ &(1/2)\lambda\Big[-\big[1-\phi_k(C_k^*, \lambda)\big]\int_{C_k^*} f(s;0)ds\Big] + (1/2)(1-\lambda)\Big[-\big[1-\phi_k(C_k^*, \lambda)\big]\Big] - c(\lambda) \\ &\qquad = -(1/2)\phi_k\big[1 - \phi_k(C_k^*, \lambda)\big]\big[1 - \lambda P_l[s \notin C_k^*]\big] - c(\lambda). \end{aligned} \tag{22}$$

We can now proceed as follows.

(a) Consider a majority rule $k \leq n/2$. Then, since $\mu_k > 0$ and $E_{i_k}[s \mid s \in C_k^*] = \int_{C_k^*} s f(s; \mu_{i_k})ds \geq \bar{s}_{i_k}$ for $C_k^* = [\bar{s}_{i_k}, 0)$, we observe from Eq. (8) that $\mathcal{J}(k) = 1$ and

$\mathcal{I}(k, C_k^*) = 1$. Therefore, $\phi_k(C_k^*, \lambda) = 1$. From Eq. (21), we obtain then

$$U_0(\lambda; x, k) = -(1/2)\big[1 - \lambda P_l[s \notin C_k^*]\big] - c(\lambda). \tag{23}$$

(b) Consider a majority rule $k > n/2$. Then, since $\mu_{i_k} < 0$ and $E_{i_k}[s \mid s \in C_k^*] = \int_{C_k^*} s f(s; \mu_{i_k}) ds < \bar{s}_{i_k}$ for $C_k^* = [0, \bar{s}_{i_k})$, we observe from Eq. (8) that $\mathcal{J}(k) = 0$ and $\mathcal{I}(k, C_k^*) = 0$. Therefore, $\phi_k(C_k^*, \lambda) = 0$. From Eq. (22), we obtain then that $U_0(\lambda; x, k)$ takes the same expression as in Eq. (23) above. ∎

Proof of Lemma 3. We provided complete arguments for the the proof of Lemma 2, i.e., for the case in which the leader is moderate ($\mu_l = 0$). Most of the arguments required for the case in which the leader is a radical leader in favor of the new initiative ($\mu_l = \overline{\mu}$) are completely analogous. Therefore, we build upon such arguments for the case $\mu_l = 0$ developed in the proof of Lemma 2. Suppose that the leader is a radical in favor of approving the new initiative ($\mu_l = \overline{\mu} \to +\infty$). Arguments totally analogous to the ones used for the case in which the leader is moderate yield

$$U_{\overline{\mu}}(\lambda; x, k) = -\big[1 - \phi_k(C_k^*, \lambda)\big]\big[1 - \lambda P_l[s \notin C_k^*]\big] - c(\lambda), \tag{24}$$

for $C_k^* = C_k^*(\overline{\mu}; x)$ and for any $k \in N$. We can now proceed as follows.

(a) Consider a majority rule $k \leq n/2$. Then, since $\mu_{i_k} > 0$ and $E_{i_k}[s \mid s \in C_k^*] = \int_{C_k^*} s f(s; \mu_{i_k}) ds < \bar{s}_{i_k}$ for $C_k^* = (-\infty, \bar{s}_{i_k})$, it follows that $\mathcal{J}(k) = 1$ and $\mathcal{I}(k, C_k^*) = 0$. We then observe from Eq. (8) that $\phi_k(C_k^*, \lambda) = 1 - \pi_k(C_k^*, \lambda)$. From Eq. (24), we obtain then that

$$U_{\overline{\mu}}(\lambda; x, k) = -\pi_k(C_k^*, \lambda)\Big[1 - \lambda P_l[s \notin C_k^*]\Big] - c(\lambda). \tag{25}$$

(b) Consider a majority rule $k > n/2$. Then, since $\mu_{i_k} < 0$ and $E_{i_k}[s \mid s \in C_k^*] = \int_{C_k^*} s f(s; \mu_{i_k}) ds < \bar{s}_{i_k}$ for C_k^*, it follows that $\mathcal{J}(k) = 0$ and $\mathcal{I}(k, C_k^*) = 0$. We then observe from Eq. (8) that $\phi_k(C_k^*, \lambda) = 0$. From Eq. (24), we obtain

$$U_{\overline{\mu}}(\lambda; x, k) = -\Big[1 - \lambda P_l[s \notin C_k^*(\overline{\mu})]\Big] - c(\lambda). \tag{26}$$

This concludes the required arguments. ∎

Proof of Lemma 4. We provided complete arguments for the the proof of Lemma 2, i.e., for the case in which the leader is moderate ($\mu_l = 0$). Most of the arguments required for the case in which the leader is a radical leader in favor of the status quo ($\mu_l = \underline{\mu}$) are completely analogous. Therefore, we build upon such arguments for the case $\mu_l = 0$ developed in the proof of Lemma 2. Suppose that the leader is a radical in favor of approving the status quo ($\mu_l = \underline{\mu} \to -\infty$). Arguments totally analogous to the ones used for the case in which the leader is moderate yield for any $k \in \{1, \dots, n\}$, we have

$$U_{\underline{\mu}}(\lambda; x, k) = -\phi_k(C_k^*, \lambda)\big[1 - \lambda P_l[s \notin C_k^*]\big] - c(\lambda), \tag{27}$$

for $C_k^* = C_k^*(\underline{\mu}; x)$ and for any $k \in N$.

(a) Consider a majority rule $k \leq n/2$. Then, since $\mu_{i_k} > 0$ and $E_{i_k}[s \mid s \in C_k^*] = \int_{C_k^*} s f(s; \mu_{i_k}) ds \geq \bar{s}_{i_k}$ for $C_k^* = [\bar{s}_{i_k}, +\infty)$, it follows that $\mathcal{J}(k) = 1$ and $\mathcal{I}(k, C_k^*) = 1$. Therefore, $\phi_k(C_k^*, \lambda) = 1$. From Eq. (28), we obtain

$$U_{\underline{\mu}}(\lambda; x, k) = -\big[1 - \lambda P_l[s \notin C_k^*]\big] - c(\lambda). \tag{28}$$

(b) Consider a majority rule $k > n/2$. Then, since $\mu_{i_k} < 0$ and $E_{i_k}[s \mid s \in C_k^*] = \int_{C_k^*} s f(s; \mu_{i_k}) ds \geq \bar{s}_{i_k}$ for C_k^*, we have $\mathcal{J}(k) = 0$ and $\mathcal{I}(k, C_k^*) = 1$. It follows from Eq. (8) that $\phi_k(C_k^*, \lambda) = \pi_k(C_k^*, \lambda)$.From Eq. (28), we obtain then

$$U_{\underline{\mu}}(\lambda; x, k) = -\pi_k(C_k^*, \lambda)\big[1 - \lambda P_l[s \notin C_k^*]\big] - c(\lambda). \tag{29}$$

This concludes the required arguments. ∎

Proof of PROPOSITION 4. The proof of the proposition makes use of the expression for the ex ante utility of the leader derived in Lemma 2.

Suppose that the leader is moderate ($\mu_l = 0$). Partial derivation of the expression for $U_0(\lambda; x, k)$ in Eq. (23) with respect to λ yields

$$\frac{\partial U_0(\lambda; x, k)}{\partial \lambda} = (1/2) P_l[s \notin C_k^*] - c'(\lambda). \tag{30}$$

We can resort to explore the required first order conditions in order to maximize the expression for $U_0(\lambda; k)$ in Eq. (23). First, note that $\lambda^* \to 0$ is consistent with a (corner) optimal investigation behavior only if $\lim_{\lambda \to 0} \varphi_0(0; x, k) = \lim_{\lambda \to 0} \partial U_0(0; x, k)/\partial \lambda \leq 0$. Given that $P_l[s \notin C_k^*] > 0$, such a plausible optimal behavior is ruled out by the Inada condition $\lim_{\lambda \to 0} c'(\lambda) = 0$ because it directly leads to $\lim_{\lambda \to 0} \varphi_0(\lambda; x, k) = P_l[s \notin C_k^*] > 0$. Secondly, $\lambda^* = \overline{\lambda}$ can be a (corner) solution to the associated problem only if $\varphi_0(\overline{\lambda}; k) = \partial U_0(\overline{\lambda}; x, k)/\partial \lambda \geq 0$. Given that $P_l[s \notin C_k^*] \in (0, 1)$, this possible solution is ruled out by the Inada condition $\lim_{\lambda \to \overline{\lambda}} c'(\lambda) = +\infty$ because it implies that $\lim_{\lambda \to \overline{\lambda}} \varphi_0(\lambda; x, k) < 0$. We are then left only with (well-behaved) interior solutions as possible candidates to maximize the expression for $U_0(\lambda; x, k)$ in Eq. (23).

Then, the first order condition $\varphi_0(\lambda^*; x, k) = \partial U_0(\lambda^*; x, k)/\partial \lambda = 0$ yields

$$P_l[s \notin C_k^*] = 2c'(\lambda^*),$$

where $C_k^* = C_k^*(0; x) \subseteq [\bar{s}_{i_k}, 0)$ for $k \leq n/2$ and $C_k^* = C_k^*(0; \bar{s}_{i_k}) \subseteq [0, \bar{s}_{i_k})$ for $k > n/2$.

Inspection of the partial derivative derived in Eq. (30) leads us to conclude that the function $U_0(\lambda; x, k)$ is strictly concave in λ because the cost $c(\lambda)$ is assumed to be strictly convex in λ. Finally, note that our assumptions on the cost function c directly imply that $\varphi_0(\lambda; x, k)$ is a continuous function in the interval $(0, \overline{\lambda})$. Given that the Inada on conditions on c guarantee that $\lim_{\lambda\to 0} \varphi_0(\lambda; k) > 0$ and $\lim_{\lambda\to\overline{\lambda}} \varphi_0(\lambda; x, k) < 0$, it follows from the intermediate value theorem that we can ensure the existence of a value $\lambda^* \in (0, \overline{\lambda})$ such that $\varphi_0(\lambda^*; x, k) = 0$. Furthermore, since the function c is strictly increasing, it follows that such a value $\lambda^* \in (0, \overline{\lambda})$ is unique. ∎

Proof of Proposition 5. The proof of the proposition makes use of the expression for the ex ante utility of the leader derived in Lemma 3. Suppose that the leader is a radical in favor of approving the new initiative ($\mu_l = \overline{\mu} \to +\infty$).

(a) Consider a majority rule $k \le n/2$. Partial derivation of the expression for $U_{\overline{\mu}}(\lambda; x, k)$ given by Eq. (25) with respect to λ yields

$$\frac{\partial U_{\overline{\mu}}(\lambda; x, k)}{\partial \lambda} = (L)\Big\{\pi_k P_l[s \notin C_k^*] - (\partial \pi_k/\partial \lambda)\big[1 - \lambda P_l[s \notin C_k^*]\big]\Big\} - c'(\lambda). \tag{31}$$

From the expression given by Eq. (31), notice first that (in a manner totally analogous as argued earlier in the proof of Proposition 4) the plausible corner behaviors $\lambda \to 0$ and $\lambda = \overline{\lambda}$ that can be derived from the problem that faces the leader when he chooses λ are ruled out by the respective Inada conditions $\lim_{\lambda\to 0} c'(\lambda) = 0$ and $\lim_{\lambda\to\overline{\lambda}} c'(\lambda) = +\infty$. We are then left only with (well-behaved) interior solutions (bounded away from $\lambda = 0$) as possible candidates to maximize the expression for $U_{\overline{\mu}}(\lambda; k)$ in Eq. (25). The first order condition $\varphi_{\overline{\mu}}(\lambda^*; x, k) = \partial U_{\overline{\mu}}(\lambda^*; x, k)/\partial \lambda = 0$ yields

$$\pi_k P_l[s \notin C_k^*] - (\partial \pi_k/\partial \lambda)\big[1 - \lambda P_l[s \notin C_k^*]\big] = c'(\lambda^*),$$

where $C_k^* = C_k^*(\overline{\mu}; x)$.

To guarantee that the condition above is also sufficient for an interior choice λ^* to maximize the leader's ex ante utility still need to check for the concavity of the expression for the ex ante utility $U_{\overline{\mu}}(\lambda; x, k)$ given by Eq. (25) (with respect to λ). By using the expressions for $\partial \pi_k(C_k^*, \lambda)/\partial \lambda$ and $\partial^2 \pi_k(C_k^*, \lambda)/\partial \lambda^2$ derived in Eq. (6) (for $C = C_k^*$), some further algebra over the expression in Eq. (31) yields

$$\begin{aligned}\frac{\partial^2 U_{\overline{\mu}}(\lambda; x, k)}{\partial \lambda^2} &= (L)\Big\{2\frac{\partial \pi_k}{\partial \lambda} P_l[s \notin C_k^*] - \frac{\partial^2 \pi_k}{\partial \lambda^2}\big[1 - \lambda P_l[s \notin C_k^*]\big]\Big\} - c''(\lambda)\\ &= \frac{2LP_k[s \in C_k^*]}{\big(1 - \lambda P_k[s \notin C_k^*]\big)^3}\Big(P_k[s \in C_k^*] - P_l[s \in C_k^*]\Big) - c''(\lambda),\end{aligned}$$

for $C_k^* = C_k^*(\overline{\mu}; x) \subseteq (-\infty, \bar{s}_{i_k})$. Then, recall that $P_k[s \in C_k^*] = \int_{-\infty}^{\bar{s}_{i_k}} f(s; \mu_{i_k}) ds$ and $P_l[s \in C_k^*] = \int_{-\infty}^{\bar{s}_{i_k}} f(s; \overline{\mu}) ds$. Therefore, for $\bar{s}_{i_k} < 0 < \mu_{i_k} < \overline{\mu} \to +\infty$, we have that

$P_k[s \in C_k^*] < P_l[s \in C_k^*]$.[28] It follows that $\partial^2 U_{\overline{\mu}}(\lambda; x, k)/\partial\lambda^2 < 0$ and, therefore, that the ex ante utility of $U_{\overline{\mu}}(\lambda; x, k)$ is (strictly) concave for each $\lambda \in (0, \overline{\lambda})$.

Finally, using the expressions of π_k and $\partial\pi_k/\partial\lambda$ given, respectively, in Eq. (3) and Eq. (6), we observe that $\varphi_{\overline{\mu}}(\lambda; x, k)$ is continuous in $\lambda \in (0, \overline{\lambda})$. In a manner totally analogous as in the proof of Proposition 4, we can invoke the intermediate value theorem to conclude that there exists a unique $\lambda^* \in (0, \overline{\lambda})$ such that $\varphi_{\overline{\mu}}(\lambda^*; x, k) = 0$.

(b) Consider a majority rule $k > n/2$. Partial derivation of the expression for $U_{\overline{\mu}}(\lambda; x, k)$ given by Eq. (26) with respect to λ yields

$$\frac{\partial U_{\overline{\mu}}(\lambda; k)}{\partial\lambda} = (L)P_l[s \notin C_k^*(\overline{\mu})] - c'(\lambda). \tag{32}$$

From the expression given by Eq. (32), notice first that (as argued earlier in the proof of Proposition 4) the possible corner behaviors $\lambda \to 0$ and $\lambda = \overline{\lambda}$ that might result from the problem that faces the leader when he chooses λ are ruled out by the respective Inada conditions $\lim_{\lambda\to 0} c'(\lambda) = 0$ and $\lim_{\lambda\to\overline{\lambda}} c'(\lambda) = +\infty$. We are then left only with interior solutions (bounded away from $\lambda = 0$) as possible candidates to maximize the expression for $U_{\overline{\mu}}(\lambda; x, k)$ in Eq. (26). The first order condition $\varphi_{\overline{\mu}}(\lambda^*; x, k) = \partial U_{\overline{\mu}}(\lambda^*; x, k)/\partial\lambda = 0$ yields

$$P_l[s \notin C_k^*(\overline{\mu})] = c'(\lambda^*),$$

where $C_k^*(\overline{\mu}; x) \subseteq (-\infty, \bar{s}_{i_k})$.

Finally, inspection of the derivative obtained in Eq. (32) leads to that, contingent on majority rules $k > n/2$, $U_{\overline{\mu}}(\lambda; x, k)$ is strictly concave in λ because the cost $c(\lambda)$ is assumed to be strictly convex in λ. In addition, under our assumptions on the cost c, we can again apply again the intermediate value theorem to the function $\varphi_{\overline{\mu}}(\lambda; x, k)$ to conclude that there exists a unique $\lambda^* \in (0, \overline{\lambda})$ such that $\varphi_{\overline{\mu}}(\lambda^*; x, k) = 0$. ∎

Proof of Proposition 6. The proof of the proposition makes use of the expression for the ex ante utility of the leader derived in Lemma 4. Suppose that the leader is a radical in favor of remaining in the status quo ($\mu_l = \underline{\mu} \to -\infty$).

(a) Consider a majority rule $k \leq n/2$. Partial derivation of the expression for $U_{\underline{\mu}}(\lambda; x, k)$ given by Eq. (28) with respect to λ yields

$$\frac{\partial U_{\underline{\mu}}(\lambda; x, k)}{\partial\lambda} = (L)P_l[s \notin C_k^*(\underline{\mu})] - c'(\lambda). \tag{33}$$

From the expression given by Eq. (33), notice first that (as argued earlier in the proof of Proposition 4) the plausible corner behaviors $\lambda \to 0$ and $\lambda = \overline{\lambda}$ that might result from the problem that faces the leader when he chooses λ are ruled out by the respective Inada

[28] Leaving aside the technical requirement $\overline{\mu} \to +\infty$, the condition that in fact guarantees the stated argument is that the difference between opinions μ_{i_k} and $\overline{\mu}$ be sufficiently large.

conditions $\lim_{\lambda\to 0} c'(\lambda) = 0$ and $\lim_{\lambda\to\overline{\lambda}} c'(\lambda) = +\infty$. We are then left only with interior solutions as possible candidates to maximize the expression for $U_{\underline{\mu}}(\lambda; x, k)$ in Eq. (28). The first order condition $\varphi_{\underline{\mu}}(\lambda^*; x, k) = \partial U_{\underline{\mu}}(\lambda^*; x, k)/\partial\lambda = 0$ yields

$$P_l[s \notin C_k^*(\underline{\mu})] = c'(\lambda^*),$$

where $C_k^*(\underline{\mu}; x) \subseteq [\bar{s}_{i_k}, +\infty)$.

Finally, inspection of the derivative obtained in Eq. (33) leads to that, contingent on majority rules $k \leq n/2$, $U_{\underline{\mu}}(\lambda; x, k)$ is strictly concave in λ because the cost $c(\lambda)$ is assumed to be strictly convex in λ. In addition, under our assumptions on the cost c, we can again apply again the intermediate value theorem to the function $\varphi_{\underline{\mu}}(\lambda; x, k)$ to conclude that there exists a unique $\lambda^* \in (0, \overline{\lambda})$ such that $\varphi_{\underline{\mu}}(\lambda^*; x, k) = 0$.

(b) Consider a majority rule $k > n/2$. Partial derivation of the expression for $U_{\overline{\mu}}(\lambda; x, k)$ given by Eq. (29) with respect to λ yields

$$\frac{\partial U_{\underline{\mu}}(\lambda; x, k)}{\partial\lambda} = (L)\Big\{\pi_k P_l[s \notin C_k^*] - (\partial\pi_k/\partial\lambda)\big[1 - \lambda P_l[s \notin C_k^*]\big]\Big\} - c'(\lambda). \tag{34}$$

From the expression given by Eq. (34), notice first that (as argued earlier in the proof of Proposition 4) the possible corner behaviors $\lambda \to 0$ and $\lambda = \overline{\lambda}$ that might result from the problem that faces the leader when he chooses λ are ruled out by the respective Inada conditions $\lim_{\lambda\to 0} c'(\lambda) = 0$ and $\lim_{\lambda\to\overline{\lambda}} c'(\lambda) = +\infty$. We are then left only with interior solutions (bounded away from $\lambda = 0$) as possible candidates to maximize the expression for $U_{\underline{\mu}}(\lambda; x, k)$ in Eq. (29). The first order condition $\partial U_{\underline{\mu}}(\lambda^*; x, k)/\partial\lambda = 0$ yields

$$\pi_k P_l[s \notin C_k^*] - (\partial\pi_k/\partial\lambda)\big[1 - \lambda P_l[s \notin C_k^*]\big] = c'(\lambda^*),$$

where $C_k^* = C_k^*(\underline{\mu}; x) \subseteq [\bar{s}_{i_k}, +\infty)$.

To guarantee that the condition above is also sufficient for an interior choice λ^* to maximize the leader's ex ante utility still need to check for the concavity of the expression for the ex ante utility $U_{\underline{\mu}}(\lambda; x, k)$ given by Eq. (29) (with respect to λ). By proceeding exactly as in 2.–(a), we derive

$$\frac{\partial^2 U_{\underline{\mu}}(\lambda; x, k)}{\partial\lambda^2} = \frac{2LP_k[s \in C_k^*]}{\left(1 - \lambda P_k[s \notin C_k^*]\right)^3}\Big(P_k[s \in C_k^*] - P_l[s \in C_k^*]\Big) - c''(\lambda),$$

for $C_k^* = C_k^*(\underline{\mu}; x) \subseteq [\bar{s}_{i_k}, +\infty)$. Then, recall that $P_k[s \in C_k^*] = \int_{\bar{s}_{i_k}}^{\infty} f(s; \mu_{i_k})ds$ and $P_l[s \in C_k^*] = \int_{\bar{s}_{i_k}}^{\infty} f(s; \underline{\mu})ds$. Therefore, for $\underline{\mu} < \mu_{i_k} < 0 < \bar{s}_{i_k}$, with $\underline{\mu} \to -\infty$, we have that $P_k[s \in C_k^*] < P_l[s \in C_k^*]$.[29] It follows that $\partial^2 U_{\overline{\mu}}(\lambda; x, k)/\partial\lambda^2 < 0$ and, therefore, that the

[29] Leaving aside the technical requirement $\underline{\mu} \to -\infty$, the condition that in fact guarantees the stated

ex ante utility of $U_{\overline{\mu}}(\lambda; x, k)$ is (strictly) concave for each $\lambda \in (0, \overline{\lambda})$.

Finally, using the expressions of π_k and $\partial \pi_k / \partial \lambda$ given, respectively, in Eq. (3) and Eq. (6), we observe that $\varphi_{\underline{\mu}}(\lambda; x, k)$ is continuous in $\lambda \in (0, \overline{\lambda})$. In a manner totally analogous as in the proof of Proposition 4, we can invoke the intermediate value theorem to conclude that there exists a unique $\lambda^* \in (0, \overline{\lambda})$ such that $\varphi_{\underline{\mu}}(\lambda^*; x, k) = 0$. ∎

Proof of Proposition 7. The proof of the proposition makes use of the expressions for the (ex ante) expected utility of the leader which were derived in Lemma 2–Lemma 4.

(a) Consider the moderate leader ($\mu_l = 0$). Consider any majority rule $k \in N$. Then, note that $P_l[s \in C_k^*(0)] = \int_{\bar{s}_{i_k}}^{0} f(s; 0) ds = (1/2) - F(\bar{s}_{i_k}; 0)$ for $k \leq n/2$, whereas $P_l[s \in C_k^*(0)] = \int_{0}^{\bar{s}_{i_k}} f(s; 0) ds = F(\bar{s}_{i_k}; 0) - (1/2)$ for $k > n/2$. Consider a given optimal investigation effort $\lambda^* \in (0, \overline{\lambda})$ for this leader. Let us use the short-hand notation $U_0 = U_0(\lambda^*; x, k)$ for simplicity. Then, by the envelope theorem, it follows from the expression derived in Lemma 2 that $\partial U_0 / \partial \bar{s}_{i_k} = (1/2) \lambda^* f(\bar{s}_{i_k}; 0) > 0$ for $k \leq n/2$, whereas $\partial U_0 / \partial \bar{s}_{i_k} = -(1/2) \lambda^* f(\bar{s}_{i_k}; 0) < 0$ for $k > n/2$. Therefore, if the moderate leader had the ability to choose $\bar{s}_{i_k}$, he would prefer $\bar{s}_{i_k} \to 0$.

(b) Consider a radical leader in favor of the new initiative ($\mu_l = \overline{\mu} \to +\infty$). Consider any majority rule $k \in N$. Then, note that $P_i[s \in C_k^*(\overline{\mu}; x)] = F(\bar{s}_{i_k}; \mu_i)$ so that we can compute

$$\pi_k = \pi_k(C_k^*(\overline{\mu}; \lambda); \mu_{i_k}) = \frac{\lambda F(\bar{s}_{i_k}; \mu_{i_k})}{\lambda F(\bar{s}_{i_k}; \mu_{i_k}) + (1 - \lambda)},$$

and, therefore,

$$\frac{\partial \pi_k}{\partial \bar{s}_{i_k}} = \frac{\lambda (1 - \lambda) f(\bar{s}_{i_k}; \mu_{i_k})}{\left[\lambda F(\bar{s}_{i_k}; \mu_{i_k}) + (1 - \lambda)\right]^2} > 0.$$

Consider a given optimal investigation effort $\lambda^* \in (0, \overline{\lambda})$ for this leader. Let us use the short-hand notation $U_{\overline{\mu}} = U_{\overline{\mu}}(\lambda^*; x, k)$ for simplicity. Then, by the envelope theorem, it follows from the expression derived in (a) of Lemma 3 for the case where $k \leq n/2$, that

$$\frac{\partial U_{\overline{\mu}}}{\partial \bar{s}_{i_k}} = -L \left[\frac{\partial \pi_k}{\partial \bar{s}_{i_k}} \left[(1 - \lambda^*) + \lambda^* F(\bar{s}_{i_k}; \overline{\mu}) \right] + \pi_k \lambda^* f(\bar{s}_{i_k}; \overline{\mu}) \right] < 0.$$

In addition, it follows from the expression derived in (b) of Lemma 3 for the case where $k > n/2$, that

$$\frac{\partial U_{\overline{\mu}}}{\partial \bar{s}_{i_k}} = -L \lambda^* f(\bar{s}_{i_k}; \overline{\mu}) < 0.$$

Therefore, if the radical leader had the ability to choose $\bar{s}_{i_k}$, he would prefer $\bar{s}_{i_k} \to -\infty$.

(c) Consider a radical leader in favor of the status quo ($\mu_l = \underline{\mu} \to -\infty$). Consider any majority rule $k \in N$. Then, note that $P_i[s \in C_k^*(\underline{\mu}; x)] = F(\overline{\bar{s}_{i_k}}; \mu_{i_k})$ so that we can

argument is that the difference between opinions μ_{i_k} and $\underline{\mu}$ be sufficiently large.

compute

$$\pi_k = \pi_k(C_k^*(\underline{\mu}; x); \mu_{i_k}) = \frac{\lambda - \lambda F(\bar{s}_{i_k}; \mu_{i_k})}{1 - \lambda F(\bar{s}_{i_k}; \mu_{i_k})},$$

and, therefore,

$$\frac{\partial \pi_k}{\partial \bar{s}_{i_k}} = \frac{-\lambda(1-\lambda) f(\bar{s}_{i_k}; \mu_{i_k})}{\left[1 - \lambda F(\bar{s}_{i_k}; \mu_{i_k})\right]^2} < 0.$$

Consider a given optimal investigation effort $\lambda^* \in (0, \overline{\lambda})$ for this leader. Let us use the short-hand notation $U_{\underline{\mu}} = U_{\underline{\mu}}(\lambda^*; x, k)$ for simplicity. Then, by the envelope theorem, it follows from the expression derived in (a) of Lemma 4 for the case where $k \leq n/2$, that

$$\frac{\partial U_{\underline{\mu}}}{\partial \bar{s}_{i_k}} = L\lambda^* f(\bar{s}_{i_k}; \underline{\mu}) > 0.$$

In addition, it follows from the expression derived in (b) of Lemma 3 for the case where $k > n/2$, that

$$\frac{\partial U_{\underline{\mu}}}{\partial \bar{s}_{i_k}} = -L\left[\frac{\partial \pi_k}{\partial \bar{s}_{i_k}}\left[1 - \lambda^* F(\bar{s}_{i_k}; \underline{\mu})\right] - \pi_k \lambda^* f(\bar{s}_{i_k}; \underline{\mu})\right] > 0.$$

Therefore, if the radical leader had the ability to choose $\bar{s}_{i_k}$, he would prefer $\bar{s}_{i_k} \to +\infty$. ∎

Proof of Proposition 8. Consider a majority rule $1 < k \leq n$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that a number $0 < n_e(x) < k$ of voters are educated under profile x. Suppose that the leader has the possibility of choosing the composition of the set $N_e(x)$.

(a) Consider a radical leader in favor of the new initiative ($\mu_l = \overline{\mu} \to +\infty$). From the results of Proposition 2, we know that the optimal concealment set has the form $C_k^*(\overline{\mu}; x) = (-\infty, \bar{s}_{i_k}(x; \varepsilon, \overline{\mu}))$ for any majority rule $k \in N$. Then, notice that the size of the optimal concealment set $C_k^*(\overline{\mu}; x)$ is minimized if the profile of education levels x induces an ordering $\sigma(x)$ such that the corresponding signal realization $\bar{s}_{i_k}(x; \varepsilon, \overline{\mu})$ is as low as possible. Since $\bar{s}_i(x; \varepsilon, \overline{\mu}) \to -\infty$ for each $i \in N_e(x)$ the resulting $\bar{s}_{i_k}(x; \varepsilon, \overline{\mu})$ results as low as possible if we choose that voters $i = 1, 2, \ldots, k - n_e$ do not receive education and that n_e voters from the remaining set $N \setminus \{1, \ldots, k - n_e\}$ receive education. This strategy makes voter $i_k = k - n_e$ to be the pivotal voter. The corresponding optimal concealment is therefore given by $C_k^*(\overline{\mu}; x) = (-\infty, -\mu_{k-n_e})$. This gives the set of minimal size that can be attained for the optimal concealment set for any ordering of critical signal realizations $\sigma(x)$, under the restriction that $n_e = n_e(x)$. In turn, from the results of Proposition 7, we observe that such an induced minimal size optimal concealment set maximizes the leader's (ex ante) utility in equilibrium when $\mu_l = \overline{\mu} \to +\infty$.

(b) Consider a radical leader in favor of the status quo ($\mu_l = \underline{\mu} \to -\infty$). From the results of Proposition 3, we know that the optimal concealment set has the form

$C_k^*(\overline{\mu}; x) = (\bar{s}_{i_k}(x; \varepsilon, \underline{\mu}), +\infty)$ for any majority rule $k \in N$. Then, notice that the size of the optimal concealment set $C_k^*(\underline{\mu}; x)$ is minimized if the profile of education levels x induces an ordering $\sigma(x)$ such that the corresponding signal realization $\bar{s}_{i_k}(x; \varepsilon, \underline{\mu})$ is as high as possible. Since $\bar{s}_i(x; \varepsilon, \underline{\mu}) \to +\infty$ for each $i \in N_e(x)$ the resulting $\bar{s}_{i_k}(x; \varepsilon, \underline{\mu})$ results as high as possible if we choose that voters $i = k - n_e, k - n_e + 1, \ldots, n$ do not receive education and that n_e voters from the remaining set $N \setminus \{k - n_e, \ldots, n\}$ receive education. This strategy makes voter $i_k = k - n_e$ to be the pivotal voter. The corresponding optimal concealment is therefore given by $C_k^*(\underline{\mu}; x) = (-\mu_{k-n_e}, +\infty)$. This gives the set of minimal size that can be attained for the optimal concealment set for any ordering of critical signal realizations $\sigma(x)$, under the restriction that $n_e = n_e(x)$. In turn, from the results of Proposition 7, we observe that such an induced minimal size optimal concealment set maximizes the leader's (ex ante) utility in equilibrium when $\mu_l = \underline{\mu} \to -\infty$. ∎

Proof of Proposition 9. Let us use Δ_W^j to capture the change in the welfare of voters when an arbitrary voter j moves from being uneducated to being educated. Then, let $\Delta_W^j > (\leq)\ 0$ indicate the presence of an increase (respectively, no change, and decrease) in voters' welfare.

Consider a moderate leader, $\mu_l = 0$, and suppose that $k \leq n/2$. Then, voters' welfare is given by Lemma 5-(a). Let us begin from a situation where no voter is educated. To propose distributions of education levels across voters in order to maximize their welfare, notice first that those $(n/2) - k$ voters whose negative critical signal realizations lie above the one of voter k (e.g., those voters $\{k + 1, \ldots, n/2\}$) must not be educated. The reason for this is that we are interested in minimizing the distance between the critical signal realizations of such voters and the one of voter k. In this way, we are able to decrease the the probability according to which, conditional on the leader not obtaining any signal, such voters prefer rejection. Recall, in this case the outcome is acceptance with probability one, that is, $\phi(C_k^*; \lambda) = 1$. Therefore, voters' welfare unambiguously decreases when such voters are educated, whereas voters' welfare unambiguously increases when a voter $j \neq k$ outside of this set, is educated. In consequence, $N_e(x) \subseteq N \setminus \{k + 1, \ldots, n/2\}$.

Consider now the set of voters $\{1, \ldots, k-1\} \cup \{n/2, \ldots, n\}$. Then, in order to maximize voters' welfare, we wish that the $n_e(x) - 1$ voters whose critical signal realizations (in the absence of education) are the closest one to zero be educated. The reason for this lies in that, from the perspective of any voter j, the signals received by the leader are normally distributed with mean μ_j. Thus, the voters j whose critical signal realizations in the absence of education are closest to zero, experience the highest possible reduction in the probability that the signal obtained by the leader lis below their own critical signal realizations. In other words, $P_j[s < \bar{s}_j]$, when such voters j become educated. Also, when such critical signal realizations of such voters tend to zero, we obtain the highest possible reduction in probability $P_l[s \in [\bar{s}_k, \bar{s}_j)]$. The reason for this lies in that, from the point of view of the moderate leader, signals are normally distributed with zero mean. More specifically, consider two voters, j and m, such that $0 < \bar{s}_j < \bar{s}_m$. If voter j becomes educated so that $0 < \bar{s}'_j < \bar{s}_j$, it then follows that

$$\Delta_W^j \equiv \overbrace{P_l[s \in [\bar{s}_k, \bar{s}_j)]}^{a} \overbrace{P_j[s < \bar{s}_j]}^{b} - \overbrace{P_l[s \in [\bar{s}_k, \bar{s}'_j)]}^{a'} \overbrace{P_j[s < \bar{s}'_j]}^{b'} > 0.$$

If voter m becomes educated so that $0 < \bar{s}'_m < \bar{s}_m$, we have that:

$$\Delta_W^m \equiv \overbrace{P_l[s \in [\bar{s}_k, \bar{s}_m)]}^{c} \overbrace{P_m[s < \bar{s}_m]}^{d} - \overbrace{P_l[s \in [\bar{s}_k, \bar{s}'_m)]}^{c'} \overbrace{P_m[s < \bar{s}'_m]}^{d'} > 0.$$

The claim is that $\Delta_W^j > \Delta_W^m$. Given that form the point of view of voters j and m signals are normally distributed with means $0 > \mu_j > \mu_m$, we have that:

$$\overbrace{P_l[s \in [\bar{s}_k, \bar{s}_j)]}^{a} - \overbrace{P_l[s \in [\bar{s}_k, \bar{s}'_j)]}^{a'} > \overbrace{P_l[s \in [\bar{s}_k, \bar{s}_m)]}^{c} - \overbrace{P_l[s \in [\bar{s}_k, \bar{s}'_m)]}^{c'}$$

and

$$\overbrace{P_j[s < \bar{s}_j]}^{b} - \overbrace{P_j[s < \bar{s}'_j]}^{b'} > \overbrace{P_j[s < \bar{s}_m]}^{d} - \overbrace{P_j[s < \bar{s}'_m]}^{d'},$$

that is: $a - a' > c - c' > 0$ and $b - b' > d - d' > 0$.

The inequality $a - a' > c - c' > 0$ and the fact that $b - b' > 0$, imply that $(a - c)b > (a' - c')b' > 0$, or equivalently $ab - a'b' > cb - c'b'$. The inequality $b - b' > d - d' > 0$ and the fact that $c - c' > 0$, imply that $(b - d)c > (b' - d')c'$, or equivalently $cb - c'b' > cd - c'd'$. Hence, $ab - a'b' > cd - c'd'$, that is, $\Delta_W^j > \Delta_W^m$. The case in which a pair of individuals have critical signal realizations of negative sign or of different sign, is analogous.

At his point, notice that if the number of voters j such that $\bar{s}_j < |\bar{s}_k|$ is smaller than n_e, then all these voters must be educated in order to maximize the welfare of the voters. Then, there remain those voters j for whom $|\bar{s}_j| > |\bar{s}_k|$, as well as voter k, that might still become educated.

The education of voter k benefits the $n - k$ voters whose critical signal realizations are above the one of this voter because and hurts the $k - 1$ voters whose critical signal realizations are below. Specifically,[30]

$$\Delta_W^k \equiv \left\{P_l[s \in [\bar{s}_k, \bar{s}_j)] - P_l[s \in [\bar{s}'_k, \bar{s}_j)]\right\}\alpha + \gamma.$$

[30] Notice that $P_l[s \in [\bar{s}_k, \bar{s}_j] - P_l[s \in [\bar{s}'_k, \bar{s}_j]] > 0$ takes the same value for each voter j whose critical signal realization is above the one of voter k. That is also the case for $P_l[s \in [\bar{s}_j, \bar{s}_k] - P_l[s \in [\bar{s}_j, \bar{s}'_k]] < 0$ and each voter each voter j whose critical signal realization is below the one of voter k. Moreover, both expressions have the same value and opposite sign.

In the expression above, we have $\bar{s}'_k > \bar{s}_k$,

$\alpha \equiv \sum_{j=k+1}^{n} P_j[s < \bar{s}_j] - \sum_{j=1}^{k-1} P_j[s \geq \bar{s}_j]$, and

$\gamma = \sum_{k+1}^{n/2} \pi_j \mathbf{1}_{E_{i_j}[s|s \in C_k^*] < \bar{s}_j} - \sum_{k+1}^{n/2} \pi'_j \mathbf{1}_{E_{i_j}[s|s \in C_k^{*\prime}] < \bar{s}_j} \geq 0$, with $\pi_j > \pi'_j$ and $C_k^{*\prime} \subseteq C_k^*$.

Suppose that the voter j who has the smallest critical signal realization such that $\bar{s}_j > |\bar{s}_k|$ is educated. Then, it follows that

$$\Delta_W^j \equiv P_l[s \in [\bar{s}_k, \bar{s}_j)] P_j[s < \bar{s}_j] - P_l[s \in [\bar{s}_k, \bar{s}'_j)] P_j[s < \bar{s}'_j] > 0,$$

where $\bar{s}'_j < \bar{s}_j$. The reasoning here is analogous to the situation in which the voter m who has the smallest signal realization $\bar{s}_m < 0$ such that $|\bar{s}_m| > |\bar{s}_k|$, is educated. In this case, it follows $\Delta_W^m > 0$ as well. Therefore, if

$$\alpha \leq \overline{\alpha} \equiv -\gamma / \{[P_l[s \in [\bar{s}_k, \bar{s}_j)] - P_l[s \in [\bar{s}'_k, \bar{s}_j)]\} < 0$$

so that $\Delta_W^k \leq 0$, then voters' welfare is maximized when the n_e voters $j \neq k$ whose critical signal realizations are the closest ones to zero be educated.

Consider now that $\alpha > \overline{\alpha}$. Given that signals are normally distributed with zero mean from the point of view of the leader, for the aforementioned voter j such that $\bar{s}_j > |\bar{s}_k|$ we have that $P_l[s \in [\bar{s}_k, \bar{s}_j)] - P_l[s \in [\bar{s}'_k, \bar{s}_j)] > P_l[s \in [\bar{s}_k, \bar{s}_j)] - P_l[s \in [\bar{s}_k, \bar{s}'_j)]$. Thus, $\overline{\Delta}_W^j \equiv P_l[s \in [\bar{s}_k, \bar{s}_j)] P_j[s < \bar{s}_j] - P_l[s \in [\bar{s}'_k, \bar{s}_j)] P_j[s < \bar{s}'_j]$ is an upper bound for Δ_W^j. Further, $\Delta_W^k \geq \overline{\Delta}_W^j$ if and only if:

$$P_l[s \in [\bar{s}_k, \bar{s}_j)][\alpha - P_j[s < \bar{s}_j]] + \beta \geq P_l[s \in [\bar{s}'_k, \bar{s}_j)][\alpha - P_j[s < \bar{s}'_j]]. \tag{35}$$

Given that signals are normally distributed with means zero and $\mu_j < 0$, respectively, from the point of view of the leader and the aforementioned voter j, it follows that

$$P_l[s \in [\bar{s}_k, \bar{s}_j)] - P_l[s \in [\bar{s}'_k, \bar{s}_j)] > P_j[s < \bar{s}_j] - P_j[s < \bar{s}'_j],$$

or equivalently,

$$P_j[s < \bar{s}'_j] > P_j[s < \bar{s}_j] - \left[P_l[s \in [\bar{s}_k, \bar{s}_j)] - P_l[s \in [\bar{s}'_k, \bar{s}_j)]\right].$$

Thus, since $P_j[s < \bar{s}'_j] = P_j[s < \bar{s}_j] - [P_l[s \in [\bar{s}_k, \bar{s}_j)] - P_l[s \in [\bar{s}'_k, \bar{s}_j)]$ ensures that the difference in right hand side of Eq. (35) attains its highest possible value, we are able to obtain the following sufficient condition in order to maximize voters' welfare: $\alpha \geq \alpha_j \equiv P_j[s < \bar{s}_j] + P_l[s \in [\bar{s}'_k, \bar{s}_j)]$. As a consequence, in order to maximize the welfare of the voters, voter k must be educated. Finally, for a voter m with $\bar{s}_m < 0$ such that $|\bar{s}_m| > |\bar{s}_k|$, the analysis is completely analogous and, therefore, $\alpha \geq \alpha_m \equiv P_m[s \geq \bar{s}_m] + P_l[s \in [\bar{s}_l, \bar{s}'_k)]$ is a sufficient condition. Again, in order to maximize the welfare of the voters, voter k must be educated.

In summary, if $\alpha \leq \overline{\alpha}$, then voters' welfare is maximized when the n_e voters $j \neq k$ whose critical signal realizations (in the absence of education) are the closest ones to zero be educated. On the other hand, if $\alpha > 0$ is sufficiently high, then voters' welfare is maximized when voter k and the $n_e - 1$ voters $j \neq k$ whose critical signal realizations in the absence of education are the closest ones to zero be educated.

The case in which the number of voters j such that $\bar{s}_j < |\bar{s}_k|$ is at least n_e, is analogous. Once the $n_e - 1$ voters $j \neq k$ whose critical signal realizations are the closest ones to zero become educated, there still remains one voter that might be educated. This voter would be either voter k or a certain voter $h \neq j, k$ who satisfies that $0 < \bar{s}_h < |\bar{s}_k|$ and, at the same time, that her critical signal realization $\bar{s}_h$ is the closest one to zero within the opinion spectrum. The change in voters' welfare when voters k and h become educated, are defined in case (a). If $\alpha \leq \overline{\alpha}$, voters' welfare is maximized when the n_e, voters $j \neq k$ whose critical signal realizations are the closest one to zero be educated. Otherwise, $\alpha \geq \alpha_h = P_h[s < \bar{s}_h] + P_l[s \in [\bar{s}'_k, \bar{s}_h)]$ is a sufficient condition for voters' welfare to be maximized when voter k and the $n_e - 1$ voters $j \neq k$ whose critical signal realizations are the closest ones to zero be educated.

The reasoning for $k > n/2$ is analogous and hence omitted. The expression for voters' welfare is provided in Lemma 5-(b). In this case voters in the set $\{(n/2)+1, \ldots, k-1\}$ must not be educated in order to maximize voters' welfare. That can be achieved if $3n/2-k \geq n_e$, that is, if the number of voters $j \neq k$ outside of the set $\{(n/2) + 1, \ldots, k - 1\}$ is at least as high as the required number of educated voters, n_e. With regard to the notation in Proposition 9, let

$$\overline{\beta} \equiv -\gamma' / \left\{[P_l[s \in [\bar{s}_k, \bar{s}_j] - P_l[s \in [\bar{s}'_k, \bar{s}_j)]\right\} < 0.$$

Analogously to case (a), $\gamma' \geq 0$ captures the change in voters' welfare when voter k becomes educated. This change is non-negative because: (i) the concealment set may shrink and hence the expected value of the concealed signals may decrease and (ii) for any voter j whose positive critical signal realization is below the one of voter k, π_j decreases if the concealment set shrinks. ∎

Proof of Proposition 10. Suppose that leader is a radical in favor of the new initiative, $\mu_l = \overline{\mu} \to +\infty$, and that the majority rule satisfies $k > n/2$. Then, the expression for the welfare of the voters is given by Lemma 6. Let us begin from a situation in which no voter is educated. By analogous reasons to the ones provided in the proof of Proposition 9, we know that, in order to maximize voters' welfare, the $k - 1 - n/2$ voters whose positive critical signal realizations lie below the one of voter k, must not be educated. These voters are the ones in the set $\{(n/2) + 1, \ldots, k - 1\}$. This would allow to minimize the distance between the critical signal realizations of these voters and the one of voter k. In turn, this would minimize the probability that these voters prefer acceptance, as recall, in this case the outcome is rejection with probability one. The feature that such voters become educated unambiguously reduces voters' welfare. Additionally, if a voter from the set $N \setminus \{(n/2)+1, \ldots, k-1\}$ becomes educated, then voters' welfare unambiguously increases. Therefore, provided that $3n/2 - k \geq n_e$, the set of educated voters must satisfy $N_e(x) \subseteq N \setminus \{(n/2)+1, \ldots, k-1\}$.

The analysis is fully analogous when we consider a radical leader in favor of the status quo, $\mu_l = \underline{\mu} \to -\infty$, and consider that the majority rule satisfies $k \leq n/2$. The expression for the welfare of the voters is given by Lemma 7. Thus, in order to maximize voters' welfare, it must be the case that the set of educated voters satisfies $N_e(x) \subseteq N \setminus \{k+1, \ldots, n/2\}$. This is always possible because there are at least $n/2$ voters such that, if they become educated, then voters' welfare increases whenever $n_e < k \leq n/2$. ∎

Appendix B

The following Lemma 5- Lemma 7 provide the expressions for voters' welfare depending on the leader's type. For simplicity, as in Section 5, we focus on situations in which voter i_k continues to be pivotal when the leader moves from a hypothetical situation of not concealing signals to doing so (so that $C_k \neq \emptyset$), and voters receive no signal (i.e., they receive $s = \emptyset$).

In order to understand the expression for voters' welfare, it is important to emphasize that all players anticipate how the leader will optimally conceal and disclose signals. Therefore, all voters consider in a common manner how the leader's obfuscation strategy will affect the probability that the outcome of election be either acceptance or rejection. This is why the probabilities that appear in the subsequent expressions in Lemma 5 - Lemma 7, according to which some signals are concealed and others are disclosed, are considered from the perspective of the leader.

Lemma 5. Consider a majority rule $k \in N$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader has a (centrist) moderate opinion $\mu_l = 0$. Then, conditional on the optimal selection of the concealment set $C_k^*(0; x)$, and for an investigation effort λ, voters' welfare is expressed as follows:

(a) For $k \leq n/2$

$$W^0(\lambda; x, k) = -\Big\{ \sum_{j=1}^{k-1} \lambda P_l[s \in [\bar{s}_j, \bar{s}_{i_k})] P_j[s \geq \bar{s}_j] + \sum_{j=k+1}^{n} \lambda P_l[s \in [\bar{s}_{i_k}, \bar{s}_j)] P_j[s < \bar{s}_j] + (1-\lambda)[n/2 + \sum_{j=k+1}^{n/2} \pi_j \mathbf{1}_{E_j[s|s \in C_k] < \bar{s}_j}] \Big\}.$$

(b) For $k > n/2$

$$W^0(\lambda; x, k) = -\Big\{ \sum_{j=1}^{k-1} \lambda P_l[s \in [\bar{s}_j, \bar{s}_{i_k})] P_j[s \geq \bar{s}_j] + \sum_{j=k+1}^{n} \lambda P_l[s \in [\bar{s}_{i_k}, \bar{s}_j)] P_j[s < \bar{s}_j]$$
$$+ (1-\lambda)[n/2 + \sum_{j=n/2+1}^{k-1} \pi_j \mathbf{1}_{E_j[s|s \in C_k] \geq \bar{s}_j}]\Big\}.$$

Proof of Lemma 5. The expression for voters' welfare when $k \leq n/2$ directly comes from observing that given the induced ordering $\sigma(x)$ of the voters' critical signal realizations, it follows that the leader's optimal concealment set has the form $C_k^* \subseteq [\bar{s}_{i_k}, 0)$. It is important to advance that although the expressions for voters' welfare are stated by asuming that for $k \leq n/2$, $C_k^* = [\bar{s}_{i_k}, 0)$, they are basically the same, and we arrive to the same conclusions, when $C_k^* \subset [\bar{s}_{i_k}, 0)$ and non-empty as $\phi_k(C_k^*; \lambda) = 1$.[31] The same observation holds for $k > n/2$ where $C_k^* \subseteq [0, \bar{s}_{i_k})$ and we focus on $C_k^* = [0, \bar{s}_{i_k})$, as $\phi_k(C_k^*; \lambda) = 0$. See Lemma 1.

Consider a given voter $j \in N$. When the leader receives a signal (an event which happens with probability λ), we have that:

$\Pr[o(v) = A \mid s < \bar{s}_j] = 0$ if $\bar{s}_j < \bar{s}_{i_k}$,

$\Pr[o(v) = A \mid s < \bar{s}_j] = \phi_k(C_k^*, \lambda) P_l[s \in [\bar{s}_{i_k}, \bar{s}_j)]$ if $\bar{s}_{i_k} \leq \bar{s}_j < 0$ and

$\Pr[o(v) = A \mid s < \bar{s}_j] = P_l[s \in [0, \bar{s}_j)] + \phi_k(C_k^*, \lambda) P_l[s \in C_k^*]$ if $\bar{s}_j \geq 0$.

Similarly:

$\Pr[o(v) = R \mid s \geq \bar{s}_j] = P_l[s \in [\bar{s}_j, \bar{s}_{i_k})] + [1 - \phi_k(C_k^*, \lambda)] P_l[s \in C_k^*]$ if $\bar{s}_j < \bar{s}_{i_k}$,

$\Pr[o(v) = R \mid s \geq \bar{s}_j] = [1 - \phi_k(C_k^*, \lambda)] P_l[s \in [\bar{s}_j, 0)]$ if $\bar{s}_{i_k} \leq \bar{s}_j < 0$, and

$\Pr[o(v) = R \mid s \geq \bar{s}_j] = 0$ if $\bar{s}_j \geq 0$.

In this case, by the proof of Lemma 1, $\phi_k(C_k^*, \lambda) = 1$.

When the leader does not receive a signal (an event which happens with probability $1 - \lambda$), voters in the set $\{i_1, \ldots, i_k\}$ vote for acceptance. Thus, as stated, $\phi_k(C_k^*, \lambda) = 1$. Then, the voters who lose are those with positive critical signal realizations, as they vote for rejection. Also, voters j such that $\bar{s}_{i_k} \leq \bar{s}_j < 0$ whenever they vote for rejection. That happens for each of these latter voters with probability π_j if $E_j[s|s \in C_k^*] < \bar{s}_j$. See

[31] The only minor difference when $C_k^* = \emptyset$ is that when the leader does not receive a signal, the voters whose negative (respectively positive) critical signal realizations are above (respectively below) the one of voter i_k for $k \leq n/2$ (respectively $k > n/2$) accept (respectively reject) with probability one. Recall that the outcome is indeed acceptance (respectively rejection). All the stated results go through in this case.

Eq. (8). We therefore express voters' welfare as:

$$W^0(\lambda;x,k) = -\Big\{\sum_{j=1}^{k-1}\lambda P_l[s\in[\bar{s}_j,\bar{s}_{i_k})]P_j[s\geq\bar{s}_j] + \sum_{j=k+1}^{n}\lambda P_l[s\in[\bar{s}_{i_k},\bar{s}_j)]P_j[s<\bar{s}_j]$$
$$+ (1-\lambda)[n/2 + \sum_{j=k+1}^{n/2}\pi_j\mathbf{1}_{E_j[s|s\in C_k^*]<\bar{s}_j}]\Big\}.$$

Consider now that $k > n/2$. Consider a given voter $j \in N$ and that. When the leader receives a signal (an event which happens with probability λ), we have that:

$\Pr[o(v) = A \mid s < \bar{s}_j] = P_l[s \in [\bar{s}_{i_k}, \bar{s}_j)] + \phi_k(C_k^*, \lambda)P_l[s \in C_k^*]$ if $\bar{s}_{i_k} \leq \bar{s}_j$,

$\Pr[o(v) = A \mid s < \bar{s}_j] = \phi_k(C_k^*, \lambda)P_l[s \in [0, \bar{s}_j)]$ if $0 < \bar{s}_j < \bar{s}_{i_k}$ and

$\Pr[o(v) = A \mid s < \bar{s}_j] = 0$ if $\bar{s}_j \leq 0$.

Similarly:

$\Pr[o(v) = R \mid s \geq \bar{s}_j] = 0$ if $\bar{s}_{i_k} \leq \bar{s}_j$,

$\Pr[o(v) = R \mid s \geq \bar{s}_j] = [1 - \phi_k(C_k^*, \lambda)]P_l[s \in [\bar{s}_j, \bar{s}_{i_k})]$ if $0 < \bar{s}_j < \bar{s}_{i_k}$, and

$\Pr[o(v) = R \mid s \geq \bar{s}_j] = P_l[s \in [\bar{s}_j, 0)] + (1 - \phi_k(C_k^*, \lambda))P_l[s \in C_k^*]$ if $\bar{s}_j \leq 0$.

In this case, by the proof of Lemma 1, $\phi_k(C_k^*, \lambda) = 0$.

When the leader does not receive a signal (an event which happens with probability $1-\lambda$), voters in the set $\{i_k, \ldots, i_n\}$ vote for rejection. Thus, as stated, $\phi_k(C_k^*, \lambda) = 0$ as less than k voters vote for acceptance. The voters who lose are those with negative critical signal realizations, as they vote for acceptance. Also, voters j such that $0 < \bar{s}_j < \bar{s}_{i_k}$ lose whenever they vote for acceptance. For each of these latter voters that happens with probability π_j if $E_j[s|s \in C_k^*] \geq \bar{s}_j$. See Eq. (8). We therefore express voters' welfare as:

$$W^0(\lambda;x,k) = -\Big\{\sum_{j=1}^{k-1}\lambda P_l[s\in[\bar{s}_j,\bar{s}_{i_k})]P_j[s\geq\bar{s}_j] + \sum_{j=k+1}^{n}\lambda P_l[s\in[\bar{s}_{i_k},\bar{s}_j)]P_j[s<\bar{s}_j]$$
$$+ (1-\lambda)[n/2 + \sum_{j=n/2+1}^{k-1}\pi_j\mathbf{1}_{E_j[s|s\in C_k^*]\geq\bar{s}_j}]\Big\}.$$

∎

Lemma 6. Consider a majority rule $k > n/2$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader is a radical in favor of the new initiative, $\mu_l = \overline{\mu} \to +\infty$. Then, conditional on the optimal selection of the concealment set $C_k^*(\overline{\mu}; x)$ and for an investigation effort λ voters' welfare is expressed as follows:

$$W^{\overline{\mu}}(\lambda; x, k) = -\Big\{ \sum_{j=1}^{k-1} \lambda P_l[s \in [\bar{s}_j, \bar{s}_{i_k})] P_j[s \geq \bar{s}_j] + \sum_{j=k+1}^{n} \lambda P_l[s \in [\bar{s}_{i_k}, \bar{s}_j)] P_j[s < \bar{s}_j]$$
$$+ (1-\lambda)[\sum_{j=1}^{n/2} \mathbf{1}_{E_j[s|s\in C_k^*]\geq \bar{s}_j} + (1 - \mathbf{1}_{E_{i_j}[s|s\in C_k^*]\geq \bar{s}_j})(1-\pi_j) + \sum_{j=n/2+1}^{k-1} \pi_j \mathbf{1}_{E_j[s|s\in C_k^*]\geq \bar{s}_j} \Big\}.$$

Proof of Lemma 6. The expression for voters' welfare when $k > n/2$ directly comes from observing that given the induced ordering $\sigma(x)$ of the voters' critical signal realizations it follows that $C_k^* = [-\infty, \bar{s}_{i_{n/2}}) \cup B$, for $B \subseteq [\bar{s}_{i_{n/2}}, \bar{s}_{i_k}(x; \varepsilon, \overline{\mu}))$ and $\overline{\mu} \to +\infty$.[32] Consider a given voter $j \in N$. When the the leader receives a signal (an event that happens with probability λ), we have that:

$\Pr[o(v) = A \mid s < \bar{s}_j] = \phi_k(C_k^*, \lambda) P_l[s \in (-\infty, \bar{s}_j)]$ if $\bar{s}_j < \bar{s}_{i_k}$, and

$\Pr[o(v) = A \mid s < \bar{s}_j] = P_l[s \in [\bar{s}_{i_k}, \bar{s}_j)] + \phi_k(C_k^*, \lambda) P_l[s \in C_k^*]$ if $\bar{s}_j > \bar{s}_{i_k}$.

Similarly:

$\Pr[o(v) = R \mid s \geq \bar{s}_j] = (1 - \phi_k(C_k^*, \lambda)) P_l[s \in [\bar{s}_j, \bar{s}_{i_k})]$, if $\bar{s}_j < \bar{s}_{i_k}$ and

$\Pr[o(v) = R \mid s \geq \bar{s}_j] = 0$ if $\bar{s}_j > \bar{s}_{i_k}$.

By the proof of Lemma 3, in this case $\phi_k(C_k^*, \lambda) = 0$.

When the leader does not receive a signal (an event which happens with probability $1-\lambda$), voters in the set $\{i_k, \dots, i_n\}$ vote for rejection. Thus, as stated, $\phi_k(C_k^*, \lambda) = 0$, as less than k voters vote for acceptance. Then, voters j with $\bar{s}_j < 0$ lose when they voter for acceptance. That happens for each of them with probability one if $E_j[s|s \in C_k^*] \geq \bar{s}_j$ or with probability $1-\pi_j$ otherwise. Also, the voters j such that $0 < \bar{s}_j < \bar{s}_{i_k}$ lose when they vote for acceptance. That happens for each of tem with probability π_j if $E_j[s|s \in C_k^*] \geq \bar{s}_j$. See Eq. (8). We therefore express voters' welfare as:

$$W^{\overline{\mu}}(\lambda; x, k) = -\Big\{ \sum_{j=1}^{k-1} \lambda P_l[s \in [\bar{s}_j, \bar{s}_{i_k})] P_j[s \geq \bar{s}_j] + \sum_{j=k+1}^{n} \lambda P_l[s \in [\bar{s}_{i_k}, \bar{s}_j)] P_j[s < \bar{s}_j]$$
$$+ (1-\lambda)[\sum_{j=1}^{n/2} \mathbf{1}_{E_j[s|s\in C_k^*]\geq \bar{s}_j} + (1 - \mathbf{1}_{E_j[s|s\in C_k^*]\geq \bar{s}_j})(1-\pi_j) + \sum_{j=n/2+1}^{k-1} \pi_j \mathbf{1}_{E_j[s|s\in C_k^*]\geq \bar{s}_j}] \Big\}.$$

[32] As in the previous Lemma 5, the expressions for voters' welfare are stated by assuming that for $k > n/2$, C_k^* is the largest concealment set. The analysis holds when the optimal concealment set shrinks, in the set inclusion order, as in this case by the proof of Lemma 1, $\phi(C_k^*; \lambda) = 0$. The only minor difference when C_k^* shrinks is that, when the leader does not receive a signal, some voters whose positive critical signal realizations are below the one of voter i_k may reject with probability one. All the stated results go through in this case.

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Lemma 7. Consider a majority rule $k \leq n/2$ and a profile of education levels x that induces an ordering $\sigma(x)$ of the voters' critical signal realizations. Suppose that the leader is a radical in favor of the status quo initiative, $\mu_l = \underline{\mu} \to -\infty$. Then, conditional on the optimal selection of the concealment set $C_k^*(\underline{\mu}; x)$ and for an investigation effort λ voters' welfare is expressed as follows:

$$W^{\underline{\mu}}(\lambda; x, k) = -\Big\{ \sum_{j=1}^{k-1} \lambda P_l[s \in [\bar{s}_j, \bar{s}_{i_k})] P_j[s \geq \bar{s}_j] + \sum_{j=k+1}^{n} \lambda P_l[s \in [\bar{s}_{i_k}, \bar{s}_j)] P_j[s < \bar{s}_j]$$

$$+ (1-\lambda)[\sum_{j=k+1}^{n/2} \pi_j \mathbf{1}_{E_j[s|s \in C_k^*] < \bar{s}_j} + \sum_{j=n/2+1}^{n} \mathbf{1}_{E_j[s|s \in C_k^*] < \bar{s}_j} + (1 - \mathbf{1}_{E_j[s|s \in C_k^*] < \bar{s}_j})(1 - \pi_j)]\Big\}.$$

Proof of Lemma 7. The expression for voters' welfare when $k \leq n/2$ directly comes from observing that given the induced ordering $\sigma(x)$ of the voters' critical signal realizations it follows that that $C_k^* = (\bar{s}_{i_{(n/2)+1}}, +\infty] \cup B$, for $B \subseteq (\bar{s}_{i_k}(x; \varepsilon, \underline{\mu}), \bar{s}_{i_{(n/2)+1}}]$ and $\underline{\mu} \to -\infty$.[33] Consider a given voter $j \in N$. When the leader receives a signal (an event which happens with probability λ), we have that:

$\Pr[o(v) = A \mid s < \bar{s}_j] = 0$ if $\bar{s}_j < \bar{s}_{i_k}$ and

$\Pr[o(v) = A \mid s < \bar{s}_j] = \phi_k(C_k^*, \lambda) P_l[s \in [\bar{s}_{i_k}, \bar{s}_j)]$ if $\bar{s}_{i_k} \leq \bar{s}_j$.

Similarly:

$\Pr[o(v) = R \mid s \geq \bar{s}_j] = P_l[s \in [\bar{s}_j, \bar{s}_{i_k})] + (1 - \phi_k(C_k^*, \lambda)) P_l[s \in C_k]$ if $\bar{s}_j < \bar{s}_{i_k}$ and

$\Pr[o(v) = R \mid s \geq \bar{s}_j] = (1 - \phi_k(C_k^*, \lambda)) P_l[s \in (s_j, +\infty)]$ if $\bar{s}_{i_k} \leq \bar{s}_j$.

By the proof of Lemma 1, in this case $\phi_k(C_k^*, \lambda) = 1$.

When the leader does not receive a signal (an event which happens with probability $1-\lambda$), voters in the set $\{i_1, \ldots, i_k\}$ vote for acceptance. Thus, as stated, $\phi_k(C_k^*, \lambda) = 1$. Then, voters j such that $\bar{s}_{i_k} < \bar{s}_j < 0$ lose whenever they vote for rejection. That happens for each of them with probability π_j if $E_j[s|s \in C_k^*] < \bar{s}_j$. Voters j such that $\bar{s}_{i_k} \geq 0$ also lose whenever they vote for rejection. That happens for each of them with probability one if $E_j[s|s \in C_k^*] < \bar{s}_j$ or with probability $1 - \pi_j$ otherwise. See Eq. (8). We therefore express

[33] As in the previous Lemma 5- Lemma 6, the expressions for voters' welfare are stated by assuming that for $k \leq n/2$, C_k^* is the largest concealment set. The analysis holds when the optimal concealment set shrinks, in the set inclusion order, as in this case by the proof of Lemma 1, $\phi(C_k^*; \lambda) = 1$. The only minor difference when C_k^* shrinks is that when the leader does not receive a signal, some voters whose negative critical signal realizations are above the one of voter i_k, may accept with probability one. All the stated results go through in this case.

voters' welfare as:

$$W^{\underline{\mu}}(\lambda; x, k) = -\Big\{ \sum_{j=1}^{k-1} \lambda P_l[s \in [\bar{s}_j, \bar{s}_{i_k})] P_j[s \geq \bar{s}_j] + \sum_{j=k+1}^{n} \lambda P_l[s \in [\bar{s}_{i_k}, \bar{s}_j)] P_j[s < \bar{s}_j]$$

$$+ (1-\lambda)[\sum_{j=k+1}^{n/2} \pi_j \mathbf{1}_{E_j[s|s \in C_k^*] < \bar{s}_j} + \sum_{j=n/2+1}^{n} \mathbf{1}_{E_j[s|s \in C_k^*] < \bar{s}_j} + (1 - \mathbf{1}_{E_j[s|s \in C_k^*] < \bar{s}_j})(1 - \pi_j)] \Big\}.$$

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