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ANTONIO JIMÉNEZ-MARTÍNEZ AND OSCAR GONZÁLEZ-GUERRA

Discrimination through "Versioning" with Advertising in Random Networks

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Oficina de Coordinación Editorial
editorial@cide.edu
Tel. 5727 9800

Abstract

This paper proposes a framework of second-degree discrimination with two different versions of a service that are served in random networks with positive externalities. In the model, consumers must choose between purchasing a premium version of the service or a free version that comes with advertising about a certain good (unrelated to the service). The ads attached to the free version influence the free version adopters' opinions and, given the induced effects on the good sales, they affect the optimal pricing of the premium version. We relate the optimal pricing strategy to the underlying hazard rate and degree distribution of the random network. Under increasing hazard rates, hazard rate dominance always implies higher prices for the service. In some applications of the model, decreasing hazard rates are often associated to extreme situations where only the free version of the service is provided. The model provides foundations for empirical analysis since key features of social networks can be related to their underlying hazard rate functions and degree distributions.

Keywords: Social networks, second-degree discrimination, advertising, degree distributions, hazard rate

JEL Classification: D83, D85, L1, M3

Resumen

Este artículo propone un marco de discriminación de segundo grado con dos versiones diferentes de un servicio que se proveen a través de redes aleatorias con externalidades positivas. En el modelo, los consumidores eligen entre comprar una versión premium del servicio o una versión gratuita que incluye publicidad sobre un cierto bien (no relacionado con el servicio). La publicidad adjunta a la versión gratuita influencia las opiniones de quienes adoptan esa versión y, a causa de los efectos en la venta del bien, afecta la estrategia óptima de precios. Relacionamos la polática óptima de precios con la hazard rate y con la distribución de grado de la red aleatoria. Con hazard rates crecientes, dominancia en hazard rate siempre implica mayores precios del servicio. En algunas aplicaciones del modelo, hazard rates decrecientes se asocian con frecuencia a situaciones extremas donde sólo la versión gratuita es provista. El modelo proporciona fundamentos teóricos para el estudio empírico dado que importantes propiedades de las redes sociales están relacionadas con sus hazard rates y distribuciones de grado

Palabras clave: Redes sociales, discriminación de segundo grado, publicidad, distribuciones de grado, tasa de riesgo

Clasificación JEL: D83, D85, L1, M3

Discrimination through “Versioning” with Avertising in Random Networks*

Oscar González-Guerra
CIDE, Division of Economics

and

Antonio Jiménez-Martínez†
CIDE, Division of Economics

August 2016

Abstract

This paper proposes a framework of second-degree discrimination with two different versions of a service that are served in random networks with positive externalities. In the model, consumers must choose between purchasing a premium version of the service or a free version that comes with advertising about a certain good (unrelated to the service). The ads attached to the free version influence the free version adopters' opinions and, given the induced effects on the good sales, they affect the optimal pricing of the premium version. We relate the optimal pricing strategy to the underlying hazard rate and degree distribution of the random network. Under increasing hazard rates, *hazard rate dominance* always implies higher prices for the service. In some applications of the model, decreasing hazard rates are often associated to extreme situations where only the free version of the service is provided. The model provides foundations for empirical analysis since key features of social networks can be related to their underlying hazard rate functions and degree distributions.

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† *Corresponding Author*

1 Introduction

We develop a model for exploring second-degree discrimination through advertising that is widely applicable to the provision of services over random social networks in the presence of positive externalities. The foundation of our model is the observation that most providers of online services through social networks are able to offer two different versions of their services: a *premium version* where consumers purchase it at a price, and a *free version* where consumers pay no price but, in exchange, must receive advertisements of some other products (which are usually unrelated to the service). For those consumers who choose the free version, the attached ads have two effects. First, they incur a cost from their exposition to ads, which establishes the relevant trade-off in order to choose one version or the other. Secondly, advertising influences the consumers' opinions about the advertised good, which affects the revenue from the sales of such a good. As a consequence, the revenue to the service provider from the free version of the service and, hence, the optimal pricing of the premium version are also affected. Since consumers's opinions are influenced, the effects of this type of discrimination are more complex than those of the classical second-degree price discrimination.

Over the last decade, the expansion of the Internet, mobile devices, and other communication technologies has triggered a dramatic increase of online services that are provided through social netwoks.[1] The social networks targeted by these service providers tend to feature positive externalities: the usage of the service is more or less beneficial to a consumer depending on whether or not his neighbors (e.g., friends or co-workers) are using it as well. Given these network externalities, the service provider would benefit from implementing some type of price discrimination. One plausible strategy is to offer individualized prices as a function of the consumer's position in the social network. On this important issue of perfect price discrimination, the theoretical literature on networks has recently yielded a number of key insights which relate the firm's pricing strategy to the consumers' centrality (Candogan et al. (2012) and Bloch & Quérou (2013)). Perfect price discrimination, however, can become difficult to implement in many real-world large social networks.[2]

Alternatively, since the platform technologies used for providing online services has made it possible for other firms (from a variety of industries, in principle, unrelated to the service provider) to use it as a channel to advertise their own products, the service provider may opt for implementing a second-degree discrimination strategy with advertising.[3] An additional

[1]Providing services through social networks is the business model of companies such as Google (general communication and information), Facebook and Twitter (social interaction), Whatsapp, Skype, and Line (communication) AirBnB (accommodation search), Waze (traffic and route forecasting), The Weather Channel (weather forecasting), Yelp and Foursquare (review and rating), YouTube, Vimeo, and Spotify (entertaining and information), Strava (exercise and health tracking), Box (second-hand selling), or Tinder (dating).

[2]First, it requires the firm to have full information about the entire social network, which may be unfeasible in complex (and, sometimes, rapid evolving) networks. Secondly, setting a particular price to each consumer depending on his position in the network may result too costly (or simply unpractical) for the firm even if it is able to identify completely the neighbors of each consumer.

[3]The most widely established pattern in practice is one where a service provider offers its product through

feature of these business models is that the premium version usually allows the consumers to enjoy the network externality to a larger extent than the free version. This type of second-degree discrimination, which is becoming extensively used in many real-world social networks, is commonly referred to in the business literature as "versioning." While the incentives for perfect price discrimination in social networks are now fairly well understood by economists, the process by which second-degree discrimination through "versioning" (with advertising) takes place has received little formal study. Understanding the simple economics of this type of second-degree discrimination is the goal of this paper.

To overcome the difficulties of dealing with complex networks, the service providers can rely on the empirical regularities that exhibit social networks.[4] Considering random networks allows for a tractable framework to investigate optimal discrimination over complex social networks. Therefore, based on random networks, we provide some general results about the optimal pricing of the premium version of the service (and, thus, the optimal discrimination strategy) as a function of fundamentals such as production and advertising costs, and of some key indicators of the degree distribution of the random network. Our model is able to capture some empirical regularities of second-degree discrimination through "versioning" in social networks.

Given that our main goal is the relationship between the service provider and the consumers, this paper abstracts from the plausible relations that may exist between the service provider and a different company whose product is being advertised through the online service provision. Using a reduced form, we model both firms as being perfectly integrated and acting as a single monopolist that offers both products.[5] The analysis assumes that the cost to the consumers from their exposition to ads is arbitrarily small, which directly implies that each consumer purchases either the premium or the free version of the service. This makes the analysis of the second-degree discrimination problem tractable.[6]

Our notion of advertising builds upon the *informative advertising* approach followed by Lewis & Sappington (1994) and Johnson & Myatt (2006). We consider that advertising informs consumers about the good's characteristics and let them improve their knowledge about their true underlying preferences for the good.

Our results show that the role of the random network in the monopolist's optimal discrim-

some social network and another firm makes use of the service diffusion platform to advertise its own product. Then, the firm that provides the service receives a compensation from the firm that advertises its product, which is usually based on the amount of ads served or on the profits from the advertising activity.

[4] See, e.g., Jackson (2008) where empirical regularities of social networks such as *small worlds*, *clustering*, or *assortativity* are discussed at length.

[5] Our main insights would follow qualitatively if we considered instead a model with two different firms, where the service provider receives a compensation proportional to the profits generated from the advertising activity.

[6] Under the premise that the architecture of the underlying social network is not affected when some consumers purchase no version of the service, considering the case where consumers may decide not to purchase any version of the service would only imply differences in the composition of the demand for the service. Yet, there would be no qualitative changes in the optimal discrimination strategy followed by the monopolist.

ination policy can be pinpointed by the (cummulative) *degree distribution* that generates the network and by its corresponding *hazard rate*. On the one hand, the shape of the degree distribution directly determines the demands of the premium and free versions of the service in our model. Other things equal, the aforementioned differences in externalities imply that consumers with relatively large neighbourhoods prefer the premium version rather than the free one. Then, the demand of the premium version, for each given price of the service, is positively related with the probability that the network generates relatively large neighbourhoods. On the other hand, the shape of the hazard rate of the degree distribution, among other implications, is useful to determine whether or not increasing the service price raises the profits from the service sales (to premium version adopters).

Including advertising into the monopolist's discrimination strategy leads to a more subtle analysis than that involving mere second-degree price discrimination. For instance, when the cost of advertising is relatively low, the firm is able to induce a high valuation for the good to the consumers, which naturally raises the revenue from the good sales. Thus, through this channel, it becomes profitable for the firm to raise the price of the service in order to increase the number of its free version adopters as these are the only consumers who are receiving the ads. If this effect is sufficiently high, then raising the service price can be profitable even for those random networks where this lowers the profits from the premium version sales. Taking into account the effects on total profits, both from the good and the service sales, Proposition 3 shows that, when the advertising cost lowers, the firm wishes to raise the price of the service if and only if the current proportion of free version adopters exceeds a certain bound, which depends only on fundamentals (production and advertising costs). In short, for the existing price for the service, the random network must have already allowed for relatively high proportions of free version adopters.

On the key issue of the role of the social network in the monopolist's optimal pricing strategy, the crucial criterion relies on a ranking between distributions which is stronger than the classical *first order stochastic dominance*. We show in Proposition 4 that, other things equal, *hazard rate dominance* implies higher optimal prices for the service when the compared social networks feature increasing hazard rates. For the case of decreasing hazard rate functions, the relation between the optimal prices is more ambiguous. Hazard rate functions are usually best interpreted from a dynamic perspective. High values of the hazard rate function indicate that, at the current price of the service, it is less likely that a randomly chosen consumer forms new links. Then, suppose that we compare two random networks, both with increasing hazard rates, and with degree distributions F_A and F_B such that F_A *Hazard Rate Dominates* F_B. By definition, this means that, for each fixed price of the service, there is a lower value for the hazard rate function under F_A than under F_B. Then, for each given price, the probability that a consumer increases the number of his current links is higher under F_A than under F_B so that premium version adopters are, in average, willing to accept higher prices for the service in the network generated by F_A.

Decreasing hazard rates are often associated in our model to situations where the monopolist prefers to serve the service only through its free version. Intuitively, decreasing hazard rates indicate that, as the price of the service increases, it becomes more likely for consumers to form new links in the future. Then, when the price of the service rises, this makes premium version adopters somehow more reluctant to switch to the free version because they expect a future increase in the number of their links and, thus, higher benefits from their relative externality gains.[7] As a consequence, the firm could find it beneficial to raise even further the price of the service and this might lead to situations where profits are always increasing in the service price. In these cases, the firm ends up seting an optimal price so high that all consumers opt for the free version and only the the premium version is provided. Corollary 1 provides conditions under which profits always increase with the price of the service.

Interestingly, the empirical literature on social networks suggests that decreasing hazard rates are often present in relatively large, but sparsely connected, social networks. In particular, most large and sparse social networks seem to adjust to the pattern of *scale-free networks* which are determined by a *power law* degree distribution. The power law and other associated distributions, such as the *Pareto* distribution, feature decreasing hazard rates.[8] On the other hand, recent empirical evidence on general graphs suggests that more densely connected graphs tend to diverge from the scale-free/power law pattern. In these cases, most graphs seem to adjust to the *exponential* degree distribution pattern,[9] which features constant hazard rate functions.

Of course, it remains quite debatable what forces are behind the actual strategies of real-world providers of services through social networks. There are many examples where providers that have access to relatively large and sparse social networks, such as Twitter, Waze, Yelp, Google, or YouTube are increasingly opting for providing only a free version (with advertising) of their services. On the other hand, there is evidence as well that firms which serve to relatively smaller, but denser, social networks, such as The Weather Channel, Spotify, and

[7]Of course, the price raise does lead to a transfer of service adopters from the premium to the free version. But this transfer is relatively smaller than the one induced under increasing hazard rates because, in this case, premium version adopters expect to have lower neighborhood sizes in the future.

[8]In their seminal study of the World Wide Network, Barabási & Albert (1999) concluded that the degree distribution of nodes on the Internet adjusts to a power law distribution. More recently, Clauset et al. (2009) found empirical evidence that both the nodes on the Internet (at the level of autonomous systems) and the number of links in websites adjust to a power law distribution. Also, Stephen & Toubia (2009) argue that the emerging social commerce network through the Internet follows a typical power law distribution. In perhaps the largest structural analysis conducted up to date, Ugander et al. (2011) conclude that the Facebook social network features decreasing hazard rates as well, though it does not fit the pattern of a power law distribution. Also, decreasing hazard rates are always generated by two ubiquitous models of the theoretical literature on random networks: the *preferencial attachment model* proposed by Barabási & Albert (1999) and the *network-based search model* suggested by Jackson & Rogers (2007).

[9]See, e.g., Rosas-Calals et al. (2007) or Ghoshal & Barabási (2011). At a theoretical level, increasing hazard rates arise in the classical models of Poisson random networks proposed by Erdös & Renyi (1959) and *Small Worlds* networks suggested by Watts & Strogatz (1998). Following a dynamic approach, Shin (2016) has recently provided a very interesting micro-founded model where increasing (or constant) hazard rates arise if an only if nodes are less likely (or equally likely) to engage in new links as their degrees increase.

most exercise tracking services and travel guides,[10] maintain both the premium and free version of their services. Our results provide empirical predictions of how the optimal pricing with "versioning" of service providers depends on the shape of the degree distribution and on its associated hazard rate function.

1.1 Related Literature

To the best of our knowledge, this is the first paper that explores second-degree discrimination in social networks by means of two versions of a product, one of which offers advertising. Nonetheless, our work is related to several fields of research in economics. The motivation of this article is influenced by the large literature, originated by Farrell & Saloner (1985) and Katz & Shapiro (1985), that explore the effects of network externalities on economic decisions. The study of the incentives to provide consumers with private information goes back to the seminal contribution of Lewis & Sappington (1991) and Lewis & Sappington (1994), and the work of Prat & Ottaviani (2001). Our notion of informative advertising builds upon the insights on advertising of Johnson & Myatt (2006). In particular, our model exploits their result that, in a wide variety of circumstances, the monopolist's profits are convex in the amount of information that is transmitted through advertising.[11] The idea that advertising may generate costs to the consumers from their exposition to ads goes back to the seminal paper of Becker & Murphy (1993). Including this type of costs allows us to model the critical trade-off for the consumers in order to choose one version or the other. The analysis of price discrimination where a monopolist offers a menu of different qualities of some product originated with the works of Mussa & Rosen (1978) and Maskin & Riley (1984).

Our analysis relies on the large literature on random networks. The study of random networks initiated with the influential work of Erdös & Renyi (1959), which served the basis for a plethora of models wherein economic agents only known that the actual social network is randomly drawn from a set of possible networks. Our dynamic interpretation of random networks builds on the canonical *configuration model* which was proposed by Bender & Canfield (1978) and used thereafter by a number of important papers in the social networks area (e.g., Bollobás (2001), Newman et al. (2001), Jackson & Yariv (2007), Galeotti & Goyal (2009), and Fainmesser & Galeotti (2016)).

Our paper complements a recent and prolific literature that deals with how firms can use the diffusion of information through social networks to increase their profits. The literature on optimal advertising in the presence of adoption externalities initiated with the insights of Butters (1977) and Grossman & Shapiro (1994). Optimal targeting and advertising strategies through *word of mouth communication* with local interactions have been investigated by a

[10] In contrast with most typical social networks, these are networks where each node is directly linked with almost all other nodes.

[11] As in their paper, we obtain that, through its advertising activity, the firm optimally wishes either to reveal no information whatsoever or to be completely informative.

number of papers since the seminal contributions of Ellison & Fudenberg (1995) and Bala & Goyal (1998). For instance, Galeotti & Goyal (2009) relate optimal marketing strategies, both under *word of mouth communication* and adoption externalities, to the characteristics of the random network. Under the assumption that consumers only inform their neighbors if they themselves purchase the product, Campbell (2013) proposes a dynamic model of optimal pricing and advertising in random networks where information diffusion is endogenously generated. More recently, Fainmesser & Galeotti (2016) build on the influential insights about optimal perfect discrimination of Candogan et al. (2012) and Bloch & Quérou (2013) to explore pricing strategies when consumers are heterogenous with respect to their influence abilities, and to obtain key implications on welfare. Our paper departs from those contributions in two respects. First, the novel form of second-degree discrimination with advertising that our paper proposes is not present in those works. Secondly, our model does not consider that the information about the advertised good flows through the network depending on how consumers are linked. While we assume that this information is transmitted publicly to all free version adopters, the role of the network is to provide the consumers with the incentives for their purchasing decisions depending on the relative sizes of the externalities.

Finally, perhaps the paper closest to ours in its scope is Gramstad (2016) wherein the monopolist allows consumers to choose from a menu of differentiated products in the presence of local externalities. These externalities are such that a consumer benefits from the consumption of a connection (either direct or indirect) only if they purchase the same product. While Gramstad (2016) explores the role of the network structure in optimal pricing in a context where consumers may choose between different versions of a product, as we do, there are important differences in both approaches. The main difference is that the influence of advertising and, therefore, of a mechanism through which the firm may raise its profits from an advertised product, are absent in his analysis. In addition, we consider that the size of the network externality depends only on the version chosen by the consumer himself and not on the versions chosen by his neighbors. As a consequence, in line with the classical second-degree price discrimination, his analysis focuses more on segmented markets along the network whereas we maintain a common market, with different versions of the product, throughout the network.

The rest of the paper is organized as follows. Section 2 proposes our new taxonomy of second-order degree discrimination through "versioning" and lays out the model. While Section 3 offers some preliminary insights, Section 4 describes the paper's main results. Section 5 provides some examples of our model, Section 6 discusses equilibrium existence and its unicity, and Section 7 concludes. The proofs of Propositions 1–4 are relegated to the Appendix.

2 The Model

A monopolist sells two (different and totally unrelated) products to a unit mass of consumers, indexed by $i \in [0, 1]$, that are embedded in a (large and complex) social network. The monopolist produces $z \geq 0$ units of a good, at a marginal cost c_z, and any discrete quantities of two versions of a service, at no cost, that is provided through the social network.

We can broadly interpret the links of the network as connections over some social media platform, informative communications, online trading connections, friendship or working relations.[12] The social network allows consumers to interact locally with respect to their consumptions (only) of the service. In particular, the consumption of the service exhibits a local (positive) network effect: a consumer's utility from (any version of) the service increases as his neighbors increase their consumptions. These externalities capture the idea that the total utility from making use of the service is positively related to the number of neighbors who are using it as well. In practice, these externalities take the form of informational gains (e.g., weather forecast, traffic monitoring, news services, or review and rating services), collaborative gains (e.g., online gaming or collaborative projects), and commercial or personal benefits from being able to interact or make transactions with a higher number of people (e.g. second hand selling, exercise tracking, or dating services).

One version of the service is offered with advertising about the good (*free version* or *ads version*) while the other is offered without advertising (*premium version* or *no-ads version*). By offering two different versions of the service, the firm is able to implement a second-degree discrimination policy where the consumers decide which version they adopt.

Each consumer has a unit demand for the good and a unit demand for the service. A consumer i is willing to pay up to ω_i for a single unit of the good and up to θ_i for a single unit of the service. If his expected valuation either of the good or the service exceeds not the corresponding price, then he purchases none of the respective product. The consumers' valuations for the good ω_i are independently drawn from a uniform distribution[13] $U[0,1]$ and their valuations for the service θ_i are independently drawn according to some (common) distribution from a support $\Theta \subset \mathbb{R}_{++}$, with $\underline{\theta} = \inf \Theta$. The two products are totally unrelated and, therefore, the valuations ω_i and θ_i are assumed to be independent from each other.

2.1 Advertising the Good

The literature on advertising has focused on three approaches which are typically labelled as *persuasive*, *complementary*, and *informative* (e.g., Bagwell (2007)). Advertising is persuasive when it raises the consumers' propensity to pay for the good being advertised and it is complementary when its consumption is complementary to that of the good. On the other hand, advertising is informative when it affects the consumers' knowledge, or perceptions, of the

[12] In principle, we regard the social network as a graph more general than the one generated only by those links facilitated by the service provider. The social network could include the links provided by the monopolist, links provided by other firms of different industries, and both formal and informal existing links such as those of working relations, family, or friendship.

[13] Assuming that the valuations of the good are uniformly distributed simplifies the analysis and the exposition of the results. The main results, though, follow qualitatively under more general specifications of how these valuations are distributed.

good features. While the general effects of the three types of advertising are rather similar, in the sense that all of them affect the consumers' tastes for the good, they are conceptually different and might lead to slightly different implications. In particular, persuasive and complementary advertising always raise individual demands[14] whereas informative ads need not do so since consumers might learn that their tastes are indeed not well suited to the good characteristics.[15]

In this paper, we consider informative advertising and, therefore, the role of advertising is to influence the consumers' knowledge of their own tastes for the good. The monopolist's advertising strategy has two dimensions: an *advertising level* and an *informative degree* of advertising. Choosing the advertising level is costly to the firm and it determines the valuation for the good induced to the consumers when communication through ads is fully informative. The informative degree of advertising is costless and it determines the quality of the information transmission through ads.[16] Since this paper's goal is to explore second-degree discrimination with advertising, the analysis focuses on equilibria where the informative degree is indeed fully informative. Therefore, for the class of equilibria that we explore, while free version adopters become completed persuaded that the product quality corresponds to that which was (costly) selected by the firm, those consumers who instead purchase the premium version retain their own priors about the quality of the good.

More specifically, the valuation of the good is uncertain to the consumers (as well as to the monopolist) and, prior to his purchasing decision, each consumer i observes not the true value of ω_i but some public signal or *advertising level* $\omega_l \in (0,1)$, which is selected by the firm at a marginal cost c_ω. Here, ω_l can be interpreted as the public observation of some posted information about the good quality, such as the one obtained from a commercial, a marketing sample, or from some other selling activity. More formally, ω_l is the valuation of the good that the firm induces to any consumer when it transmits credibly, without any noise, the information conveyed by its ads. In addition to the advertising level, the monopolist chooses an *informative degree* $a \in [0,1]$ for its advertising activity. The mechanism through which consumers update their initial beliefs is very simple. The advertising level and the informative degree are related to the consumers' valuations of the good according to the rule: $\omega_l = \omega$, with probability a, and ω_l is an independent draw from $U[0,1]$, with probability $1-a$. Then,

[14] In some of the pioneering models of the complementary advertising view, such as Stigler & Becker (1977) and Becker & Murphy (1993), the final effects on demand are ambiguous. The reason behind this, however, lies not in the consumers preferences for the good but in the fact that these models usually incorporate some costs to the consumers from their exposition to the ads. We incorporate this type of costs as well in the current paper to propose the second-degree discrimination scheme.

[15] In principle nothing prevents that, after receiving the ads, the consumers' final valuations of the good may fall short of their initial expectations.

[16] While the advertising level summarizes the quality of the product that the firm wishes to transmit to the audience, the informative degree determines the quality of the channels through which such an image of the product is transmitted. For example, the firm can decide, on the one hand, about hiring one or another public figure to serve as commercial ambassador of its good and, on the other hand, it must choose the design of its advertisements, which affects the way in which the intended message is communicated.

each consumer i obtains the posterior expectation of his valuation for the good by applying Bayes rule:

$$E[\omega_i \,|\, \omega_l] = a\omega_l + (1-a)(1/2). \tag{1}$$

Higher values of a indicate more informative advertising. If $a = 1$, the consumers believe that the advertised level ω_l is their actual valuation for the good. If $a = 0$, the consumers obtain no information whatsoever from the advertising activity and retain their priors. This formulation builds on the settings proposed by Lewis & Sappington (1994) and Johnson & Myatt (2006) to analyze informative advertising. In their approaches, though, the advertising level ω_l is exogenously taken whereas this paper assumes that the firm is able to choose it at a cost. In short, a describes the informative quality of the firm's advertising activity and ω_l represents the valuation of the good that the firm induces on the consumers when advertising is completely informative.

Finally, we assume that exposition to advertising generates a relatively small cost to the consumers. As mentioned in footnote 14, such type of costs from exposition to advertising have been commonly considered by the literature on advertising. We use $\psi(a)$ to describe the cost to any consumer from his exposing to an advertising activity with informative degree a. We assume that $\psi(0) = 0$ and that $\psi(a)$ increases in $a \in [0,1]$. This captures the idea that more informative advertising is associated to higher quantities of ads received by the consumer and, therefore, to higher costs from exposition. In this paper, we will assume arbitrarily small costs to the consumers from their exposition to ads to ensure that all consumers adopt one version of the service or the other.

2.2 The Strategies of the Firm and the Consumers

The monopolist and the consumers are engaged in a two-stage game. In the first stage, the monopolist chooses a *price* $p \geq 0$ *for the premium of the service*, a *price* $P \geq 0$ *for the good*, an *advertising level* $\omega_l \in (0,1)$, and an *informative degree* $a \in [0,1]$ for the advertising of the good, which it gives away through the free version of the service. In the second stage, the consumers observe the choices (p, P, ω_l, a) made by the monopolist and then make simultaneous *consumption decisions* about the service and the good $\big(z_0, z_a, x, y\big)$. Here, z_0 indicates the probability that a premium version adopter purchases the good and z_a, for $a \in (0,1]$, stands for the probability that a free version adopter (who receives ads according to some positive informative degree a) purchases the good. Also, each consumer i chooses a vector $(x_i, y_i) \in W = \big\{[0,1]^2 : 0 \leq x_i + y_i \leq 1\big\}$, where x_i and y_i indicate, respectively, the probabilities that i adopts the free and the premium version of the service. Therefore, $x = \lambda(\{i \in [0,1] : x_i = 1\})$ and $y = \lambda(\{i \in [0,1] : y_i = 1\})$ indicate the sizes (with respect to Lebesgue measure) of the sets of consumers who, respectively, adopt the free and the premium version of the service.

To focus on situations where each consumer always wishes to adopt some version of the service, Assumption 1 will restrict our set of equilibria to situations where *all* consumers at least want to adopt the free version of the service. Given this, we will then explore the conditions under which some consumers prefer the premium version to the free version.

2.3 The Social Network

The underlying social network is exogenously given and undirected. To capture the uncertainty that the monopolist and the consumers have about the architecture of the (large and complex) social network, we assume that it is stochastically generated. The monopolist and the consumers are uncertain about the complete configuration of the social network but they commonly known the stochastic process that generates it. All consumers have a (common) set of possible *neighbors* $\mathcal{B} = \Delta([0,1])$ in the network so that a *neighborhood* $B_i \in \mathcal{B}$ *for consumer* i is simply a Borel set in the overall set of consumers $[0,1]$. Accordingly, all consumers have a set $n \in N = [\underline{n}, \overline{n}] \subseteq \mathbb{R}_+$ of possible neighborhood sizes, or *degrees in the social network*. In principle, the set N could be unbounded as well and, in particular, $\overline{n}$ is allowed to tend to infinity in applications. The *degree distribution of the social network* is given by a twice continuously differentiable distribution $F(n)$, with a strictly positive density $f(n)$ over the support $[\underline{n}, \overline{n}]$. Thus, the degree of a fraction $F(n) = \int_{\underline{n}}^{n} f(m)dm$ of consumers exceeds not n.

We make use of the canonical *configuration model* to formalize how the random graph is generated.[17] The idea here is that each consumer i with degree n_i gets randomly linked to a set of size n_i of other consumers according to a weighted uniform distribution where the weights are determined by the corresponding degrees n_j of the consumers in the remaining sample. Therefore, if i gets linked with j, then the density according to which consumer j has another n neighbors is

$$h(n) = \frac{f(n)n}{\int_{\underline{n}}^{\overline{n}} mf(m)dm}.$$

Let $x(n) = \lambda(\{i \in [0,1] : x_i = 1 \,|\, n_i = n\})$ and $y(n) = \lambda(\{i \in [0,1] : y_i = 1 \,|\, n_i = n\})$ denote the fraction of consumers that, respectively, adopt the free and the premium version of the service, conditioned on their degrees being equal to n. The expected consumption of consumer i's neighbors can then be obtained as

$$\int_{j \in B_i} E[x_j + y_j \,|\, n_j = n]h(n)dn = n_i \int_{\underline{n}}^{\overline{n}} [x(n) + y(n)]h(n)dn = n_i K,$$

where

$$K = \frac{\int_{\underline{n}}^{\overline{n}} n[x(n) + y(n)]f(n)dn}{\int_{\underline{n}}^{\overline{n}} nf(n)dn} \in [0,1] \tag{2}$$

[17] The *configuration model* was originally developed by Bender & Canfield (1978) and, since then, has been extensively used in a number models with random graphs, such as Bollobás (2001), Newman et al. (2001), Jackson & Yariv (2007), Fainmesser & Galeotti (2016), and Shin (2016), among many others. A nice discussion of the configuration model, and of its relation with other random graph models, is provided by Jackson (2008).

is the *expected consumption of any neighbor in the social network* under the degree distribution described by F and f.

Finally, we will make use of the *degree independence assumption*, in order to guarantee that the only relevant information about the network for each consumer is his degree.[18]

Using the previous observations, we then specify the expected utility to consumer i, given the monopolist's choices (P, p, ω_l, a), as

$$U_i\big(x, y, z_a\big) = x_i\Big[\theta_i - \psi(a) + \gamma_x n_i K\Big] + y_i\Big[\theta_i - p + \gamma_y n_i K\Big] + z_a\Big[E[\omega_i \,|\, \omega_l] - P\Big], \tag{3}$$

where the parameters $\gamma_x, \gamma_y > 0$ describe the presence of local (positive) network externalities. A consumer's utility raises by an amount γ_x, when he adopts the free version of the service, or by an amount γ_y, when he adopts the premium version, for each unit of the service consumed by his neighbors, regardless of the version that the neighbors adopt. We assume that $\gamma_y > \gamma_x$, which can be viewed formally as a single crossing condition for the two types of service adopters where the network externality to the premium version adopters exceeds that of the free version adopters. This assumption captures the idea that the premium version allows consumers to enjoy the externalities of the service to a greater extent than in the case of the free version. This seems to be the case in most real-world services that are provided through social networks with both a free and a premium version. The existence of equilibria in our model where network externalities matter is only guaranteed if the degree of the externality varies across the two versions of the service. In the sequel, we will use $\beta = \gamma_y - \gamma_x > 0$ to denote the discrepancy between the network externalities or the *externality premium*.

2.4 The Hazard Rate Function Associated with the Random Network

We now describe an indicator that can be used to measure interesting features of the random social network. The *hazard rate function* of the random social network with distribution degree F is the function on $[\underline{n}, \overline{n}]$ defined as

$$r(n) = \frac{f(n)}{1 - F(n)}.$$

Intuitively, the function $r(n)$ gives us the probability that a randomly selected consumer has approximately n links,[19] conditioned on his actual neighborhood size being no less than n. The hazard rate function becomes a very interesting indicator of the degree distribution if we interpret the social network as a collection of neighborhood sizes that evolve dynamically according to some stochastic law. To grasp better the intuition here, suppose that the firm's

[18] The *degree independence assumption* states that the nodes of the network regard their shared links as independently chosen from the random network. This is a quite common assumption in the literature on random networks and has been used, among others, by Jackson & Yariv (2007), Galeotti et al. (2010), Fainmesser & Galeotti (2016), and Shin (2016).

[19] For our continuous distribution case, a number of links in the interval $(n - \varepsilon, n + \varepsilon)$, for $\varepsilon > 0$ sufficiently small.

records indicate that, up to a certain period t, there are no neighborhoods in the social network of size less than some minimum level n. Then, $r(n)$ can be interpreted as the odds that, in a subsequent period $t+1$, consumers have a number of links approximately equal to such a former lower bound n. Therefore, using this dynamic interpretation, an increasing hazard rate indicates that if the average neighborhood size increases over time, then it becomes very likely to have a reversal in this growing tendency because at each $t+1$ consumers tend to have a number of neighbors around the minimum size that was already achieved in the previous period t. Decreasing hazard rates imply higher probabilities that in the future consumers have neighborhood sizes that diverge from the current minimum size making it more likely that the network maintains its minimum neighborhood size as it grows.

Those intuitive implications have recently received quite appealing formal microeconomic foundations by Shin (2016). Under the assumptions on the random network generating process used in our model, Shin (2016) (Proposition 2) shows that increasing hazard rates follow if and only if a node is less likely to form additional new links as his degree increases. On the other hand, with decreasing hazard rates, the degree of a randomly chosen node in the network becomes arbitrarily large as the average neighborhood size increases (Shin (2016), Proposition 3).

Both the degree distribution and the hazard rate function are very useful to establish relations between different random social networks.

Definition 1. *The degree distribution F^A First Order Stochastically Dominates (FOSD) the degree distribution F^B if $F^A(n) \leq F^B(n)$ for each $n \in [\underline{n}, \overline{n}]$, with strict inequality for some degree n. The degree distribution F^A Hazard Rate Dominates (HRD) the degree distribution F^B if $r^A(n) \leq r^B(n)$ for each $n \in [\underline{n}, \overline{n}]$, with strict inequality for some degree n.*

The *HRD* order is stronger than the classical *FOSD* over distributions. This well-known result about the relation existing between the two stochastic orders is stated formally in the following Lemma.

Lemma 1. *Let F^A and F^B be two degree distributions over the set $[\underline{n}, \overline{n}]$, then for each $n \in [\underline{n}, \overline{n}]$, we have*

$$r^A(n) \leq r^B(n) \;\Rightarrow\; F^A(n) \leq F^B(n).$$

Moreover, the implication above holds with strict inequality for some $n \in [\underline{n}, \overline{n}]$.

Using the result in Lemma 1 above, we obtain a direct implication of the *HRD* ranking on comparisons between network densities according a density measure which is commonly known as the *degree density*. The *degree density* of a social network with degree distribution F is the average number of neighbors across individuals, $d_F = \int_{\underline{n}}^{\overline{n}} n f(n) dn$. Then, consider two degree distributions F^A and F^B such that F^A *HRD* F^B. It follows from Lemma 1 that F^A *FOSD* F^B so that $d_{F^A} \geq d_{F^B}$. In short, *hazard rate dominance* implies higher degree density of the random network.

We will consider the consumers' cost from exposition to ads $\psi > 0$ as a fixed primitive of the model. In some parts of the paper we will make use of the transformation $F_\beta(p) = F\left(\frac{p-\psi}{\beta}\right)$ for $p \in [\beta\underline{n} + \psi, \beta\overline{n} + \psi]$, for each size of the externality premium $\beta > 0$. Accordingly, we will also use the transformations $f_\beta(p) = \left(\frac{1}{\beta}\right) f\left(\frac{p-\psi}{\beta}\right)$ and

$$r_\beta(p) = \left(\frac{1}{\beta}\right) r\left(\frac{p-\psi}{\beta}\right) = \frac{f_\beta(p)}{1-F_\beta(p)}.$$

These transformations allow us to relate changes in the price of the service with the degree distribution. If $r_\beta(p)$ increases with p, this conveys the intuitive message of a large joint probability of having relatively high prices for the service and, at the same time, that consumers form fewer new links. A decreasing $r_\beta(p)$ indicates that, as the price rises, consumers become more likely to form new links. As a consequence, the amount of premium version adopters that switch to the free version following a price raise is relatively smaller when hazard rates are decreasing, compared to the case of increasing hazard rates. Under decreasing hazard rates, premium version adopters expect to have more neighbors in the future and benefit to a larger extent from the premium externality. In contrast, with increasing hazard rates, premium version adopters believe that their numbers of neighbours and, therefore, their benefits from the premium externality, will decrease.

3 Preliminaries: Optimal Decisions

We restrict attention to pure-strategy perfect-Bayes Nash equilibria of the game described. Furthermore, our analysis focuses on equilibria where consumers always adopt some version of the service so that $x^* + y^* = 1$ in equilibrium. To do this, we assume that the cost from exposing to ads satisfy the following condition with respect to the valuation of the service, the network externality from the free version, and the support of the degree distribution.

Assumption 1. *The cost of exposition to ads ψ satisfies $\psi \leq \underline{\theta} + \gamma_x \underline{n}\varepsilon$ for some arbitrary $\varepsilon > 0$.*

Assumption 1 above requires that the cost from exposing to ads be not excessively high so that, starting from some arbitrary aggregate consumption of the service $x(n) + y(n) \geq \varepsilon > 0$ for each neighborhood size $n \in [\underline{n}, \overline{n}]$ (i.e., an aggregate consumption uniformly bounded from below across degrees by some, possibly negligible, $\varepsilon > 0$), the net valuation of the premium version of the service of each consumer is strictly positive. Notice from the expression in (2) that Assumption 1 implies that $K^* = 1$. This is very intuitive: if all consumers adopt some version of the service for each of their possible degrees (i.e., $x^*(n) + y^*(n) = 1$ for each $n \in [\underline{n}, \overline{n}]$), then expected service consumption of a consumer i's neighbor is precisely the size of his neighborhood, regardless of the degree distribution of the network.

The remaining of this section describes some optimal choices by the firm and the consumers in equilibrium.

3.1 The Firm's Optimal Decisions

We first analyze the optimal choices of the firm on advertising. Regarding the informative degree of advertising, the monopolist wishes to choose some $a^* \in [0,1]$ so as to maximize its profits from the sales of the good. The information that the consumers obtain about their own valuation of the good depends on whether they adopt or not the free version of the service and on the optimal choice of the firm regarding advertising. This, in turn, influences the optimal price P that the firm sets for the good.

For any information level $a \in [0,1]$, let z_a^* denote the fraction of consumers who decide to purchase the good in equilibrium and let P_a^* denote the price of the good that the firm would optimally set conditioned on a. Although we are conditioning the optimal price of the good on the informative degree a, this paper does not consider price discrimination through the price P of the good and, therefore, the firm is not allowed to set a price P_a^* as a function of a. As a consequence, the firm will ultimately choose its optimal price P^* by averaging according to the proportions of consumers who receive different information levels a. The price of the good must satisfy $E[\omega \,|\, \omega_l] \geq P$ so that the monopolist optimally chooses $P^* = E[\omega \,|\, \omega_l]$. Then, for any information level $a \in (0,1]$ and price P_a^*, the fraction of consumers who decide to purchase the good is

$$\begin{aligned} z_a^* &= \mathbb{P}\Big(E[\omega \,|\, \omega_l] \geq P_a^*\Big) = \mathbb{P}\Big(a\omega_l + (1-a)(1/2) \geq P_a^*\Big) \\ &= \frac{a\overline{\omega} + (1-a)(1/2) - P_a^*}{a}, \end{aligned}$$

since, ex-ante, ω_l is uniformly distributed on the interval $[0,1]$. Then, by substituting the optimal price $P_a^* = a\omega_l + (1-a)(1/2)$ in the expression above, we obtain $z_a^* = \frac{\overline{\omega}-\omega_l}{\overline{\omega}-\underline{\omega}}$ for each $a \in (0,1]$. On the other hand, for the extreme situation $a = 0$, it follows from the assumption that ω is uniformly distributed on the interval $[0,1]$ that $z_0^* = 1/2$ with $P_0^* = 1/2$.

Since our goal in this paper is to study second-degree discrimination where some consumers may avoid paying a price for the service by receiving ads instead, we assume that the monopolist is restricted to choosing a positive informative degree $a > 0$. Otherwise, no consumer would receive any information whatsoever from ads and there would be no second-degree discrimination by construction. Then, for $a \in (0,1]$, the monopolist's profits from the sales of the good to the adopters of the free version of the service are given by

$$\pi(a,\omega_l) = \big[a\omega_l + (1-a)(1/2) - c_z\big]\,(1-\omega_l) - c_\omega \omega_l. \tag{4}$$

These profits are concave in ω_l and the first order condition with respect to ω_l yields the optimal advertising level

$$\omega_l^* = \frac{a - (1-a)(1/2) + c_z - c_\omega}{2a}.$$

In addition, the function $\pi(a, \omega_l)$ above is linear, and hence convex, in a. Therefore, by considering $a = 1$ in the expression above, we observe that if the inequality

$$\omega_l^* = \frac{1 + c_z - c_\omega}{2} > \frac{1}{2} \tag{5}$$

is satisfied, then profits are maximized at $a^* = 1$ for the case where the informative degree a is allowed to vary within the interval $(0, 1]$. The implication that, with informative advertising, the monopolist optimally wishes to choose either completely uninformative or fully informative advertising is now a well-known result in the advertising literature.[20] Thus, under the condition stated in (5), the only case where the monopolist discriminates across consumers through advertising is when it chooses $a^* = 1$. Otherwise, no consumer receives any ads whatsoever, regardless of their choices with respect to the service version. Let us refer to equilibria where the monopolist optimally chooses $a^* = 1$ as *discriminatory equilibria*. Given our research questions, we will restrict attention only to discriminatory equilibria. We will also impose a further condition on costs to ensure that the firm always makes nonnegative profits from its sales of the product. Given the particular distribution for the valuations of the good that we consider, it can be directly verified from the previous arguments that while condition (i) of the following assumption ensures that $\omega_l^* \in (0, 1)$ and, at the same time, that the condition in (5) holds, condition (ii) is sufficient to guarantee that $P_a^* \geq c_z$ for each $a \in [0, 1]$.

Assumption 2. *The marginal costs of producing the good and of choosing the advertising level, c_z and c_ω, satisfy:*

(i) $0 < c_z - c_\omega < 1$ *and*

(ii) $c_z \leq 1/2$.

We emphasize that our focus on discriminatory equilibria necessarily implies that the optimal advertising level set by the firm must be higher than the consumers' prior, i.e., $\omega_l^* \in (1/2, 1)$ in any discriminatory equilibrium. Finally (perhaps with some abuse of notation), throughout the remaining of the paper, let us simply denote by $\psi = \psi(1)$ the cost from exposition to advertising which is associated to the fully informative advertising degree.

4 Main Results

In order to analyze the optimal price that the firm sets for the good, let us use $y^*(p)$ to indicate the equilibrium size of the set of premium version adopters as a function of the price p chosen

[20] In fact, the function $\pi(a, \omega_l)$ in (4) above falls into the class of profit functions analyzed by Lewis & Sappington (1994) and Johnson & Myatt (2006) for informative advertising. In particular, Lewis & Sappington (1994) (Propositions 1 and 2) and Johnson & Myatt (2006) (Proposition 4) show that, even for more general production costs, the type of profit functions that we are analyzing in this paper turn out to be the maximum of convex functions so that they are themselves convex with respect to $a \in [0, 1]$. Therefore, the monopolist optimally chooses either $a^* = 0$ or $a^* = 1$. This is a well-known result in the advertising literature with the interpretation that the monopolist wishes to target either a "niche" or a "mass" market.

by the monopolist for the (premium version of) the service. Given the optimal choices by the consumers on service adoption, and the optimal choice (in the discriminatory equilibrium) $a^* = 1$ by the firm, the optimal price for the good must satisfy

$$\begin{aligned}P^* &= y^*(p)P_0^* + [1 - y^*(p)]P_1^* \\ &= y^*(p)(1/2) + [1 - y^*(p)]\omega_l^* \\ &= \omega_l^* - (\omega_l^* - 1/2)y^*(p).\end{aligned}$$

Recall that we are considering that the firm does not discriminate between the service adopters by charging them different prices for the good. This is consistent with the fact that the good and the service are totally unrelated products and, therefore, with the more general interpretation of our model where the good be provided instead by a different firm. For the discriminatory equilibrium, the fraction of consumers who buy the good is given by

$$\begin{aligned}z^* &= y^*(p)z_0^* + \big[1 - y^*(p)\big]z_1^* \\ &= y^*(p)(1/2) + \big[1 - y^*(p)\big](1 - \omega_l^*) \\ &= (1 - \omega_l^*) + (\omega_l^* - 1/2)\, y^*(p).\end{aligned}$$

Therefore, the monopolist's profits as a function of the price that it sets for the service are given by $\Pi(p) = \Pi_1(p) + \Pi_2(p)$, where

$$\begin{aligned}\Pi_1(p) &= (P^* - c_z)z^* - c_\omega\omega_l^* \\ &= [(\omega_l^* - c_z) - (\omega_l^* - 1/2)y^*(p)] \cdot [(1 - \omega_l^*) + (\omega_l^* - 1/2)\, y^*(p)] - c_\omega\omega_l^*\end{aligned} \tag{6}$$

are the firm's profits from its sales of the good to both types of service adopters and $\Pi_2(p) = py^*(p)$ are the firm's profits from its sales of the service to the service adopters. In particular, notice that the total profits of the firm can be expressed as a quadratic expression with respect to the proportion of premium version adopters. In particular,

$$\begin{aligned}\Pi(p) =& (\omega_l^* - c_z)(1 - \omega_l^*) - c_\omega\omega_l^* \\ &+ \Big[p + (2\omega_l^* - 1 - c_z)(\omega_l^* - 1/2)\Big]y^*(p) - (\omega_l^* - 1/2)^2\big[y^*(p)\big]^2.\end{aligned}$$

Finally, by using the expression obtained earlier for the optimal advertising level in a discriminatory equilibria, we obtain the following expression for the firm's profits.

$$\Pi(p) = \frac{(1 - c_z)^2 + c_\omega^2 - 2c_\omega(1 + c_z)}{4} + \left[p - \frac{c_\omega(c_z - c_\omega)}{2}\right] y^*(p) - \frac{(c_z - c_\omega)^2}{4}\big[y^*(p)\big]^2. \tag{7}$$

The following Proposition characterizes (interior) equilibria where both version of the service are adopted.

Proposition 1. *Suppose that Assumptions 1 and 2 hold. Then, in any discriminatory equilibrium with adopters of the premium version of the service, i.e., where* $y^*(p^*) > 0$ *and*

$p^* > \psi > 0$, *it follows that: (i) the firm chooses its advertising level as a function of its production and advertising costs,*

$$\omega_l^* = \frac{1 + c_z - c_\omega}{2}, \tag{8}$$

(ii) the firm chooses the price of (the premium version of) the service as a function of the premium externality, the cost to the consumers from the exposition to ads, the production and advertising costs, and the degree distribution of the social network,

$$r_\beta(p^*)\left[p^* - \frac{c_z(c_z - c_\omega)}{2} + \frac{(c_z - c_\omega)^2}{2}F_\beta(p^*)\right] = 1, \tag{9}$$

provided that the second order condition

$$\frac{r'_\beta(p^*)}{\left[r_\beta(p^*)\right]^2} + \left[1 + \frac{(c_z - c_\omega)^2}{2}f_\beta(p^*)\right] \geq 0,$$

holds, (iii) the consumers choose which version of the service as a function of its price and the degree distribution,

$$y^*(p^*) = 1 - F_\beta(p^*), \tag{10}$$

and (iv) the firm chooses the price of the good according to

$$P^* = \frac{1 + (c_z - c_\omega)F_\beta(p^*)}{2}. \tag{11}$$

Condition (ii) of Proposition 1 above gives us the key requirement that characterizes any interior optimal price for the service $p^* > \psi$. Yet, note that the support of the degree distribution may impose additional constraints on such a price. In particular, since $n \in [\underline{n}, \overline{n}]$, the optimal price must also satisfy the condition

$$\beta\underline{n} + \psi \leq p^* \leq \beta\overline{n} + \psi. \tag{12}$$

Thus, these constraints allow for the possibility of obtaining corner solutions, which are related to the support of the degree distribution. Intuitively, when the corner solution $p^* = \beta\underline{n} + \psi$ is obtained, the firm would like to provide the service to neighboorhoods whose size be smaller than the actual minimum size. In this case, by using the expression in (10) for the equilibrium proportion of the premium version adopters, we observe that the firm would like to achieve $y^*(p^*) = 1$ or, in other words, to provide only the premium version of the service. The analogous intuitive interpretation follows for the case where the corner solution $p^* = \beta\overline{n} + \psi$ is obtained. In this case, the firm would like to provide the service to neighborhoods whose size be larger than the maximum size actually available in the social network and it would like to achieve $y^*(p^*) = 0$, so that only the free version with ads would be served. In addition, if we allow $\overline{n}$ to tend to infinity, then the situation where the firm optimally wishes to serve only the free version arises if the firm's profits are always increasing in p. Of course, studying whether or not any of these types of solutions follow in particular applications either when the condition in (12) is not satisfied or when $p^* \to +\infty$ requires us to study the entire shape of the the corresponding profit function $\Pi(p)$ in the interval $[\beta\underline{n} + \psi, \beta\overline{n} + \psi]$.

A raise in the price p that the monopolist sets for the service generates the following effects on its total profits:

1. Higher price p changes the profits from the service sales $\Pi_2(p) = py^*(p)$. The direction of this change depends on the shape of the degree distribution of the social network. In particular, an infinitesimal increase in p raises locally the profits from the service sales if and only if $r_\beta(p) < 1/p$.

2. Higher price p always raises the optimal price $P^* = \frac{1+(c_z-c_\omega)F_\beta(p^*)}{2}$ that the monopolist sets for the good.

3. Higher price p lowers the fraction $y^*(p)$ of consumers who adopt the premium version of the service and, therefore, it lowers the fraction

$$z^* = \left(\frac{1 - c_z + c_\omega}{2}\right) + \left(\frac{c_z - c_\omega}{2}\right) y^*(p)$$

of consumers who purchase the good.

The result of the combination of the effects enumerated above is not obvious in general. The following straightforward corollary gives us the conditions that characterize the sign of the derivative $d\Pi(p)/dp$.

Corollary 1. *Suppose that Assumptions 1 and 2 hold. Then, for each price p of the service such that $y^*(p) > 0$ and $p > \psi > 0$, it follows that $d\Pi(p)/dp > 0$ if and only if the condition*

$$r_\beta(p)\left[p - \frac{c_z(c_z - c_\omega)}{2} + \frac{(c_z - c_\omega)^2}{2} F_\beta(p)\right] < 1$$

is satisfied.

Notice that the existence of a solution p^* where the monopolist wishes to serve only the free version of the service always follows when the condition stated in Corollary 1 is satisfied for each $p \in (\beta\underline{n}+\psi, \beta\overline{n}+\psi)$. This solution can either take the form of a corner solution $p^* = \beta\overline{n}+\psi$, if we consider a finite maximum neighborhood size, or of an indeterminate explosive price, if we allow $\overline{n} \to +\infty$. We observe that, other things equal, decreasing hazard rates make it easier for condition in Corollary 1 to be met, compared to the case of increasing hazard rates. This type of solutions describes a situation where the monopolist is mainly interested in making profits only through advertising. Recently, many firms (such as Twitter, YouTube, Facebook, or Google, just to mention a few of the most prominent ones) had adhered to this second scenario. Such firms regard potential expansions of the social network as beneficial for their businesses because they allow them to increase considerably its audience for advertisements.

Interestingly, many empirical studies suggest that decreasing hazard rates are present in most real-world complex and relatively sparse social networks (see, e.g., Barabási & Albert (1999), Clauset et al. (2009), Stephen & Toubia (2009) or Ugander et al. (2011)). Finally, given that we are considering a fairly general class of degree distributions, we are not able to obtain sufficient conditions under which the monopolist always wishes to serve only the free version. For degree distributions with monotone decreasing hazard rates, we discuss in Section 6 necessary and sufficient conditions under which this type of equilibria is obtained.

The following proposition establishes that an increase of the premium externality size β always leads to an increase of the optimal price p^*. This is very intuitive since premium version adopters always benefit from higher premium externalities so that they are willing to pay higher prices when the premium externality raises.

Proposition 2. *Suppose that Assumptions 1 and 2 hold. Then, in any discriminatory equilibrium with adopters of the premium version of the service, i.e., where $y^*(p^*) > 0$ and $p^* > \psi > 0$, it follows $\partial p^*/\partial \beta \geq 0$.*

We turn now to explore the implications on the optimal service price p^* of a decrease in the advertising cost c_ω. As can be directly observed from (8), lower advertising costs lead to higher optimal levels of advertising in our model. While this is very intuitive, it is less clear whether inducing higher valuations of the good to the consumers who purchase the free version will ultimately lead to higher prices for the premium version or not. Other things equal, raising the price of the service may lower profits due to a reduction in the service sales to the premium version adopters. As indicated earlier, this will be the case when $r_\beta(p^*) > 1/p^*$. However, raising the price of the service also increases the proportion of the free version adopters, which raises the profits from the good sales. In particular, notice from the expression in (6) that

$$\frac{\partial \Pi_1}{\partial y^*} = (\omega_l^* - 1/2)\Big[(c_z - c_\omega)[1 - y^*] - c_z\Big] < 0 \quad \forall\, y^* \in [0, 1].$$

In words, lower proportions of premium version adopters (and, hence, higher proportions of free version adopters) always raise the profits from the good sales. Nevertheless, this raise in profits is not always increasing with the optimal adverting level. From the expression in (6), it can be verified that

$$\frac{\partial^2 \Pi_1}{\partial y^* \partial \omega_l^*} = (2\omega_l^* - 1)\Big[1 - c_z - 2y^*\Big] < 0 \quad \Leftrightarrow \quad y^* > \frac{1 - c_z}{2},$$

so that we need that the current proportion of premium version adopters be sufficiently high in order to obtain a positive second order effect on the profits from the good sales. The following proposition gives us the condition on the current proportion of service adopters under which the positive effect on the good sales dominates the effect on the service sales.

Proposition 3. *Suppose that Assumptions 1 and 2 hold. Then, in any discriminatory equilibrium with adopters of the premium version of the service, i.e., where $y^*(p^*) > 0$ and*

$p^* > \psi > 0$, an increase of the optimal advertising level leads (locally) to an increase of the optimal service price if and only if the condition

$$F_\beta(p^*) \geq \frac{c_z}{2(c_z - c_\omega)}$$

is satisfied at equilibrium.

In intuitive terms, we observe that, when advertising becomes less costly, raising the price of the service will be profitable for the firm if the social network has already allowed for a large enough proportion of free version adopters at the current price.

We now explore the central issue of how the random network structure influences the price p^* that the monopolist sets for the service. The *HRD* ranking between distributions plays a crucial role in our model to determine the relations that may exist between the prices that the monopolist sets for the service for two different random networks.

Proposition 4. *Suppose that Assumptions 1 and 2 are satisfied. Consider two degree distributions F^A and F^B over $[\underline{n}, \overline{n}]$ such that F^A HRD F^B. Suppose that the cost from exposition to advertising ψ and the premium externality β remain fixed across the two random networks. Let p_A^* and p_B^* be equilibrium prices for the service under the degree distributions F^A and F^B, respectively, such that $p_A^*, p_B^* \in (\beta\underline{n} + \psi, \beta\overline{n} + \psi)$. Consider the cutoff price of the service $\widehat{p}$ specified by $r_\beta^B(\widehat{p}) = r_\beta^A(p_A^*)$. Then,*

(i) if both hazard rate functions r_β^A and r_β^B are (weakly) decreasing in $[\beta\underline{n} + \psi, \beta\overline{n} + \psi]$, the equilibrium prices p_A^* and p_B^* can satisfy either (a) $p_A^* < \widehat{p} \leq p_B^*$, if the cutoff price $\widehat{p}$ exists in the interval $(p_A^*, \beta\overline{n} + \psi)$, or (b) $p_A^* \geq p_B^*$, regardless of the cutoff price $\widehat{p}$.

(ii) if both hazard rate functions r_β^A and r_β^B are (weakly) increasing in $[\beta\underline{n} + \psi, \beta\overline{n} + \psi]$, the equilibrium prices p_A^* and p_B^* must always satisfy $p_A^* \geq p_B^*$.

While the *HRD* criterion remains ambiguous as to the relation between equilibrium prices when hazard rates are decreasing, it does provide a clear-cut condition when hazard rates are increasing. For social networks which feature increasing hazard rates, *hazard rate dominance* implies, other things equal, higher equilibrium prices for the service.

If a degree distribution F^A *HRD* another F^B, it follows that $r_\beta^A(\frac{p-\psi}{\beta}) \leq r_\beta^B(\frac{p-\psi}{\beta})$ for each $p \in [\beta\underline{n} + \psi, \beta\overline{n} + \psi]$. The intuitive implication of this inequality is that, for each fixed price p of the service, the probability that a randomly chosen consumer enjoys in the future more neighbours than its current degree is higher under F^A than under F^B. Therefore, other things equal, premium version adopters are more willing to accept an increase in the service price under F^A than under F^B. Furthermore, when both hazard rates r_β^A and r_β^B are increasing, a raise in the price p makes it more likely for consumers, both under F^A and F^B, to have fewer links in the future so that moving along the graphs of r_β^A and r_β^B does not make consumers

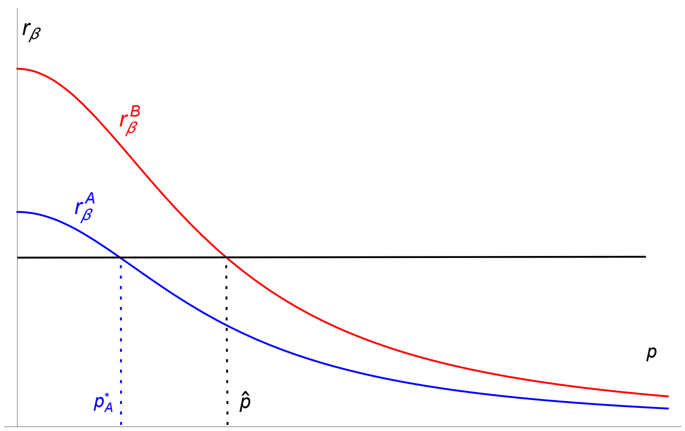

Figure 1: F^A *HRD* F^B: Decreasing Hazard Rates.

more willing to accept higher prices in any way. However, when both r_β^A and r_β^B are decreasing, moving along their graphs as p increases does make consumers more willing to accept higher prices. Suppose that the slope of r_β^B is much steeper than the slope of r_β^A. Then, we could have situations where a raise in the price p makes it (at least locally) more likely for consumers under F^B to have more links in the future than under F^A (in spite of the order given by the *harzard rate dominance* criterion). This would make consumers under F^B more willing to accept higher prices. This is the intuitive reason for the ambiguous result provided in Proposition 4 (i). In addition, in Section 6 we will argue that multiple equilibria can exist if the underlying hazard rates are decreasing.

The relation that must hold between the service equilibrium prices for two different degree distributions is illustrated in Figures 1 and 2 for the cases, respectively, of decreasing and increasing hazard rates. Consider the degree distribution F^A and let us begin with some equilibrium price of the service p_A^* for such a distribution. Then, to conduct the comparative statics exercise provided by Proposition 4, it is useful to identify the cutoff price $\widehat{p}$ of the service such that $r_\beta^B(\widehat{p}) = r_\beta^A(p_A^*)$, provided that such a price exists in the interval $(\beta\underline{n} + \psi, \beta\overline{n} + \psi)$. This price $\widehat{p}$ enables us to compare the equilibrium condition for the degree distribution F^B with that of the distribution F^A and determine the relation that any equilibrium price p_B^* must satisfy with respect to the price p_A^*.

Although the details of the proof are relegated to the Appendix, let us use Figures 1 and 2 to illustrate briefly how these results work. For the case where both hazard rates r_β^A and r_β^B are (weakly) decreasing, it can be verified that application of the equilibrium condition stated in (9) leads to that two pairs (F^A, r_β^A, p_A^*) and (F^B, r_β^B, p_B^*) can satisfy the equilibrium requirement if either (a) $p_B^* > p_A^*$ and, at the same time, $r_\beta^B(p_B^*) \leq r_\beta^A(p_A^*)$, or (b) $p_A^* \geq p_B^*$ and, at the same time, $r_\beta^B(p_B^*) \geq r_\beta^A(p_A^*)$. Thus, as depicted in Figure 1, we observe that, for condition (a) to hold, we need that $p_B^* \geq \widehat{p}$ whereas condition (b) can be automatically satisfied, without any additional requirements.

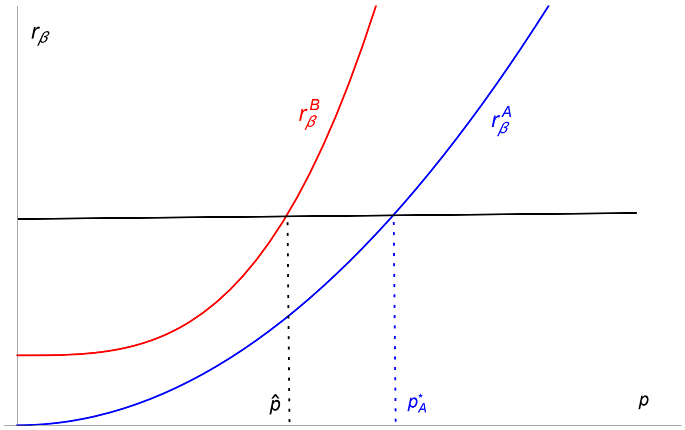

Figure 2: F^A *HRD* F^B: Increasing Hazard Rates.

On the other hand, for the case where both hazard rates r_β^A and r_β^B are (weakly) increasing, it can be verified that application of the equilibrium condition stated in (9) leads to that two pairs (F^A, r_β^A, p_A^*) and (F^B, r_β^B, p_B^*) can satisfy the equilibrium requirement if either (a) $p_A^* \geq p_B^*$ and, at the same time, $r_\beta^B(p_B^*) \geq r_\beta^A(p_A^*)$, or (b) $p_A^* \geq p_B^*$ and, at the same time, $r_\beta^B(p_B^*) < r_\beta^A(p_A^*)$. From Figure 2, we observe that (a) can hold only if $\widehat{p} \leq p_B^* \leq p_A^*$ whereas (b) can hold if $p_B^* < \widehat{p} \leq p_A^*$.

Finally, recall that if a degree distribution F^A *HRD* another F^B, then the random network associated with F^A has higher degree density than the network associated with F^B. Thus, from the result in Proposition 4 (ii) above, we observe that, under *HRD* for the case of increasing hazard rates, higher price service is obtained alongside with higher degree density.

5 A Few Examples

As an antidote to the abstractness of our model, we now present some examples that make use of common degree distributions in the social networks literature. Our goal with these examples is to illustrate the monopolist's optimal discrimination strategy, as well as to obtain intuitions about how our main results work.

Example 1 (**Constant Hazard Rate: The Exponential Distribution).** *Suppose that* $c_\omega = 1/10$ *and* $c_z = 5/10$ *so that Assumption 2 is satisfied. Consider the exponential distribution given by*

$$F_\beta(p) = 1 - e^{\rho\left(1+\frac{\psi-p}{\beta\underline{n}}\right)} \quad \textit{for each} \;\; p \geq \beta\underline{n} + \psi,$$

where $\rho > 0$. *The corresponding density and hazard rate function are*

$$f_\beta(p) = \left(\frac{\rho}{\beta\underline{n}}\right) e^{\rho\left(1+\frac{\psi-p}{\beta\underline{n}}\right)}, \quad r_\beta(p) = \frac{\rho}{\beta\underline{n}}.$$

The exponential degree distribution is often used to capture the formation of links according to

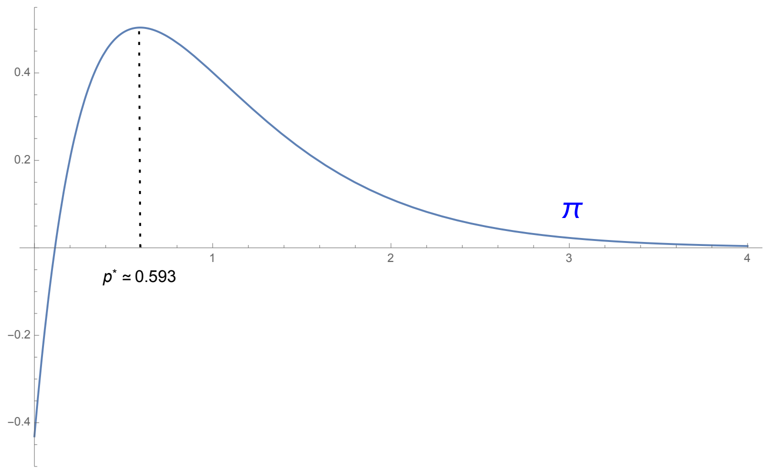

Figure 3: Firm's profits for the exponential degree distribution.

uniform randomness in growing random networks[21] *Since* $r'_\beta(p) = 0$, *the second order condition for the equilibrium price of the service is trivially satisfied. Then, from the expression in (9), we obtain that the price that the firm sets for the service must satisfy the equality*

$$100p^* - 8e^{\rho\left(1+\frac{\psi-p^*}{\beta \underline{n}}\right)} = \frac{2\rho + 100\beta\underline{n}}{\rho}. \tag{13}$$

We further pick $\rho = 1$, $\underline{n} = 0.5$, $\beta = 1$, *and* $\psi = 0.05$ *so that Assumption 1 is satisfied. Then, using the equilibrium condition in (13), we obtain a unique equilibrium price* $p^* \approx 0.593$ *with optimal profits* $\Pi(p^*) \approx 0.482$. *In particular, the function* $\Pi(p)$ *takes the form depicted in Figure 3.*

Finally, we would like to study the equilibrium prices obtained from two different exponential degree distributions F^A_β *and* F^B_β, *parameterized, respectively, by* ρ_A *and* ρ_B *such that* $\rho_A < \rho_B$, *so that* F^A_β HRD F^B_β. *From the results of Proposition 4, we should obtain* $p^*_A \geq p^*_B$. *To verify this result using this example, let us use* B *to denote the exponential distribution introduced above so that* $\rho_B = 1$ *and* $p^*_B \approx 0.593$. *Now, consider another exponential distribution with* $\rho_A = 1/2$. *For this second exponential distribution, we obtain that* $p^*_A \approx 1.068$ *with* $\Pi(p^*_A) \approx 0.6$. *The firm's profit functions for these two exponential degree distributions are depicted in Figure 4.*

Example 2 (**Increasing Hazard Rate: The Erlang Distribution).** *As in the previous example, suppose that* $c_\omega = 1/10$ *and* $c_z = 5/10$. *Consider the Erlang distribution given by*

$$F_\beta(p) = 1 - \left(\frac{p - \psi + \beta}{\beta}\right) e^{-\left(\frac{p-\psi}{\beta}\right)} \quad \textit{for each} \;\; p \geq \psi,$$

[21] See, e.g., Jackson (2008), Chapter 5, for an insightful description of the use of the exponential degree distribution in growing random networks.

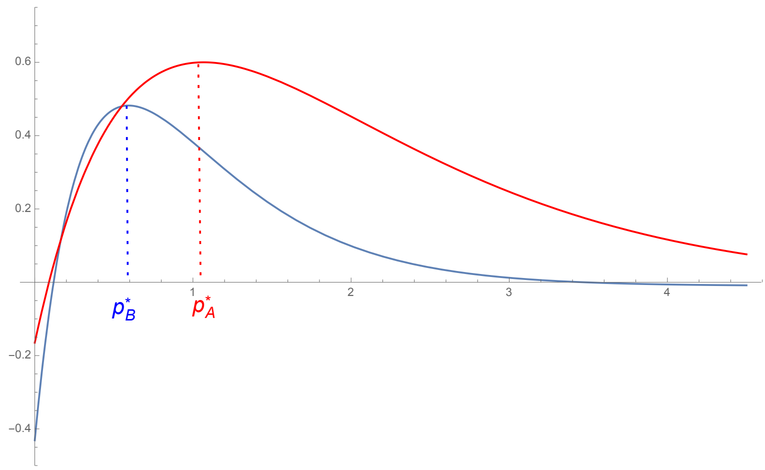

Figure 4: Firm's profits for two different exponential degree distributions.

where $\rho > 0$. *The corresponding density and hazard rate function are*

$$f_\beta(p) = \left(\frac{p-\psi}{\beta^2}\right) e^{-\left(\frac{p-\psi}{\beta}\right)}, \quad r_\beta(p) = \frac{p-\psi}{\beta(p-\psi+\beta)}.$$

Since $r'_\beta(p) = \frac{1}{\beta(p-\psi+\beta)^2} > 0$, *the second order condition for the equilibrium price of the service is satisfied. Then, from the expression in (9), we obtain that the price that the firm sets for the service must satisfy the equality*

$$100p^* - 8\left(\frac{p^*-\psi+\beta}{\beta}\right) e^{-\left(\frac{p^*-\psi}{\beta}\right)} = \frac{100\beta(p^*-\psi+\beta)}{p^*-\psi} + 2.$$

We take $\beta = 2$, *and* $\psi = 0.01$ *so that Assumption 1 is satisfied. This yields a unique equilibrium price* $p^* \approx 3.283$ *with optimal profits* $\Pi(p^*) \approx 1.654$. *In particular, given these parameter values, the function* $\Pi(p)$ *takes the form depicted in Figure 5.*

Let us now rewrite the Erlang distribution that we have introduced above as

$$F^B(n) = 1-(n+1)e^{-n}, \quad \text{with} \quad r^B(n) = \frac{1}{n+1}, \quad \text{for each} \quad n \geq 0,$$

with $p_B^* \approx 3.283$ *and* $\Pi(p_B^*) \approx 1.654$. *Then, consider another Erlang distribution given by*

$$F^A(n) = 1-\left(\frac{n^2}{2}+n+1\right)e^{-n}, \quad \text{with} \quad r^A(n) = \frac{n^2}{n^2+2n+2}, \quad \text{for each} \quad n \geq 0.$$

It follows that $F^A(n)$ HRD $F^B(n)$. *Therefore, from the results in Proposition 4, we should obtain that* $p_A^* \geq p_B^*$. *Indeed, we obtain* $p_A^* \approx 4.583$ *with* $\Pi(p_A^*) \approx 2.712$. *The firm's profit functions for these two Erlang degree distributions are depicted in Figure 6.*

Example 3 (Decreasing Hazard Rate: The Power Law Distribution). *Suppose that* $c_\omega = 1/10$ *and* $c_z = 2/10$ *so that Assumption 2 is satisfied. Consider the power law distribution*

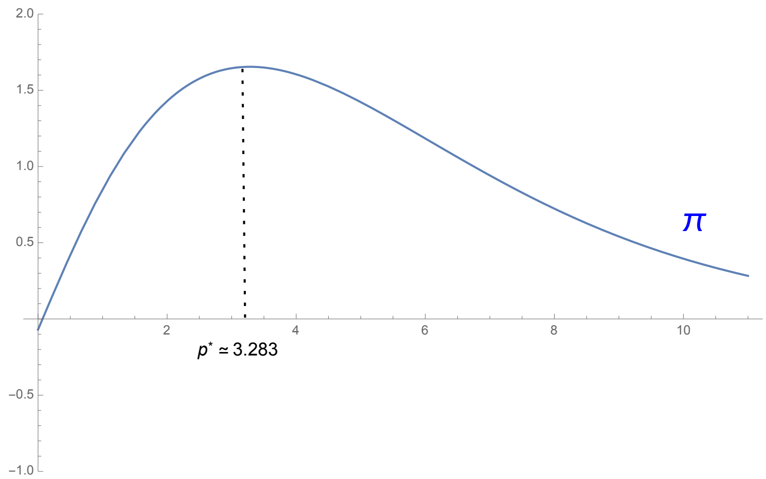

Figure 5: Firm's profits for the Erlang degree distribution.

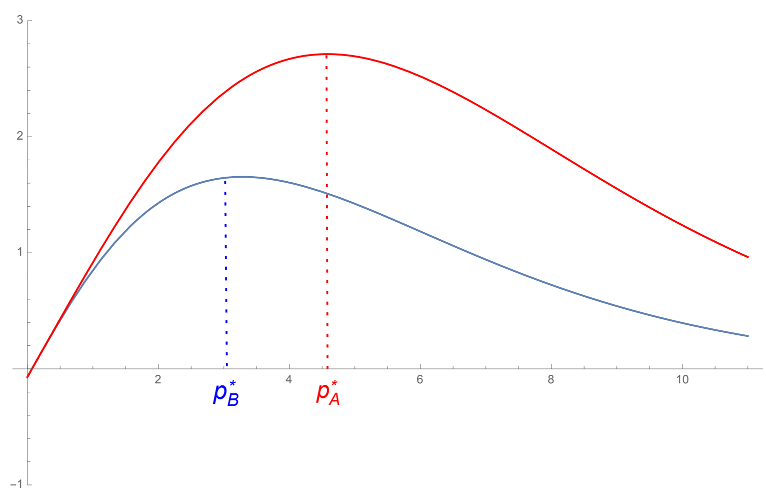

Figure 6: Firm's profits for two different Erlang degree distributions.

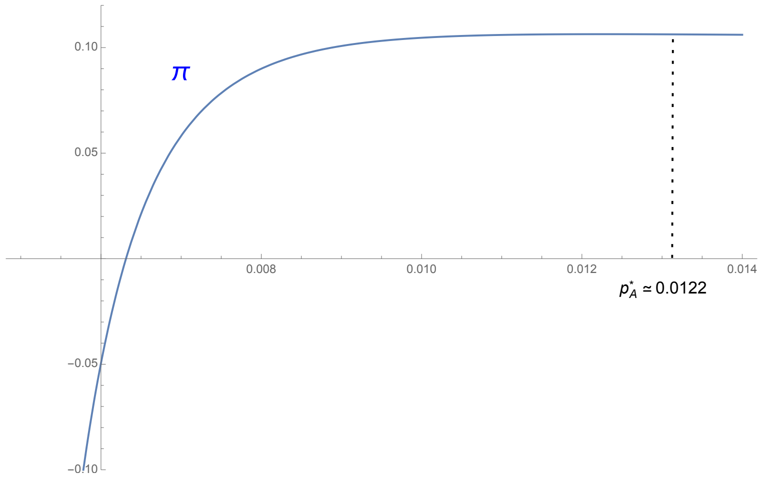

Figure 7: Firm's profits for the power law degree distribution.

given by

$$F_\beta(p) = 1 - \left(\frac{\beta \underline{n}}{p-\psi}\right)^{\alpha-1} \quad \text{for each} \quad p \geq \beta\underline{n} + \psi, \quad \text{where} \quad \alpha > 1.$$

The corresponding density and hazard rate function are

$$f_\beta(p) = \frac{(\alpha-1)(\beta\underline{n})^{\alpha-1}}{(p-\psi)^\alpha}, \quad r_\beta(p) = \frac{\alpha-1}{(p-\psi)}.$$

Although $r'_\beta(p) = -(\alpha-1)/(p-\psi)^2 < 0$ *in this random social network, it can be checked that the second order condition for the equilibrium price of the service is satisfied, at least for some reasonable values of* α*. In particular, one obtains*

$$\frac{r'_\beta(p^*)}{\left[r_\beta(p^*)\right]^2} = \frac{\alpha-2}{\alpha-1}$$

so that, for $\alpha > 2$*, we can guarantee that the required second order condition always hold. Then, from the expression in (9), we obtain the following expression for the optimal price that the firm sets for the service*

$$200p^* - \left(\frac{\beta\underline{n}}{p^*-\psi}\right)^{\alpha-1} - 200\left(\frac{p^*-\psi}{\alpha-1}\right) = 1.$$

Consider $\underline{n} = 0.01$*,* $\beta = 1$*, and* $\psi = 0.001$ *so that Assumption 1 is satisfied. Furthermore, take* $\alpha = 4$ *and denote by* F^A_β *this particular power law distribution. Then, we obtain a unique equilibrium price* $p^*_A \approx 0.0122$ *with optimal profits* $\Pi(p^*_A) \approx 0.106$*. In particular, given these parameter values, the function* $\Pi(p)$ *takes the form depicted in Figure 7.*

Finally, consider another power law distribution where $\alpha = 6$ *and let us use* F^B_β *to denote this second distribution. It can be directly verified that the previous power law distribution with*

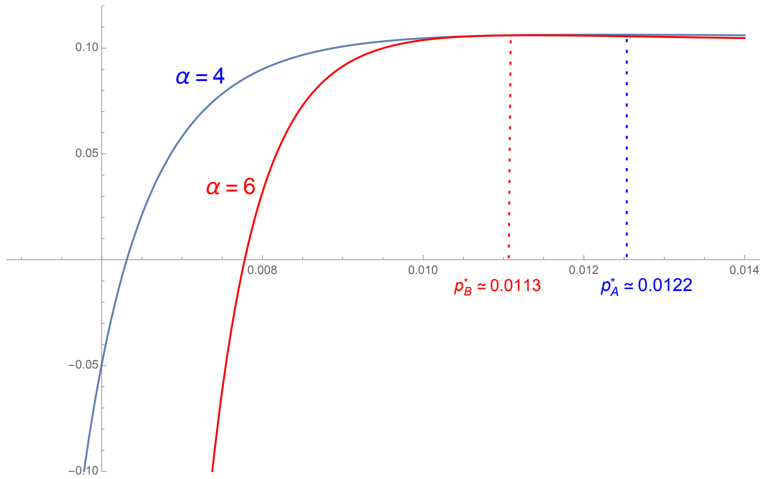

Figure 8: Firm's profits for two power law degree distributions ($\alpha = 4$ and $\alpha = 6$).

$\alpha = 4$ *(*F_β^A*)* HRD *this new one with* $\alpha = 6$ *(*F_β^B*). By applying our equilibrium condition in (9), we now obtain the equilibrium price for the service* $p_B^* \approx 0.0113$ *with optimal profits* $\Pi(p_B^*) \approx 0.106$*. The firm's profit functions under these two different power law degree distribution are depicted in Figure 8.*

From Figures 7 and 8, we observe that the monopolist's profits are not far from being always increasing in the price of the service. Our careful choice of parameters has made it possible to obtain interior solutions in this example. However, slight perturbations of the picked parameters would lead in this example to situations where the monopolist's profits are in fact always increasing in the price of the service so that no interior solution is achieved. For the a general power law distribution, it can verified that the condition stated in Corollary 1 is satisfied if and only if the following inequality holds.

$$(p - \psi)^{\alpha - 1} \Big[2(\alpha - 2)p + 2\psi - (\alpha - 1)c_\omega (c_z - c_\omega) \Big] < (\alpha - 1)(c_z - c_\omega)^2 (\beta \underline{n})^{\alpha - 1}.$$

For values $\alpha > 2$ *of the parameter, the inequality above is satisfied for prices of the service which are relatively low with respect to production and advertising costs.*

6 Equilibrium Existence and Multiplicity

Under our assumptions, there is always a solution for the monopolist's decision problem, provided that the second order condition in equation (9) is satisfied and the support $[\underline{n}, \overline{n}]$ of the degree distribution is bounded. This is the case even for more general cases where Assumption 1 needs not hold and, as a consequence, no service adopters may exist in equilibrium. Even for these more general cases, the existence of equilibrium in our model is guaranteed if the set of choices W is compact and convex and, furthermore, if each consumer i's objective function is quasiconcave on $(x_i, y_i) \in W$. Formally, these requirements ensure that there exists an equilibrium where each consumer chooses pure strategies. In our model, the first requisite above is trivially satisfied by the construction of W. As to the second requirement, notice that each U_i is linear on (x_i, y_i), which implies the desired quasiconvavity.

Unfortunately, we cannot always guarantee uniqueness of equilibrium in our model (even for interior equilibria). The main source of equilibrium multiplicity comes from the fact that the hazard rate of the degree distribution be either non-monotone or decreasing. For (weakly) increasing hazard rates, though, interior equilibrium, if it exists, is unique. Consider the function $\Psi(p)$ specified as

$$\Psi(p) = r_\beta(p)\left[p - \frac{c_z(c_z - c_\omega)}{2} + \frac{(c_z - c_\omega)^2}{2}F_\beta(p)\right] \tag{14}$$

so that $\Psi(p^*) = 1$ is the equilibrium condition specified in (9) for the service price. Since F and f are continuous functions, $\Psi(p)$ is continuous in $p \in [\beta\underline{n}+\psi, \beta\overline{n}+\psi]$ as well. In addition, let us specify the bounds on the hazard rate $\underline{r}_\beta = r_\beta(\beta\underline{n}+\psi)$ and $\overline{r}_\beta = r_\beta(\beta\overline{n}+\psi)$, provided that they are well-defined.[22] We observe that the term in brackets in the specification of the function $\Psi(p)$ in (14) is always increasing in p. Then, given our earlier observations, direct application of the *intermediate value Theorem* leads to the following result.

Proposition 5. *Suppose that assumptions 1 and 2 hold and that either* $r_\beta(\beta\overline{n}+\psi) < \infty$ *or* $\lim_{\overline{n}\to+\infty} r_\beta(\beta\overline{n}+\psi) < \infty$ *(for the case of unbounded support of the degree distribution). Suppose that the hazard rate function associated to the random network is monotone (either increasing or decreasing), then there exists a unique equilibrium price* $p^* \in (\beta\underline{n}+\psi, \beta\overline{n}+\psi)$ *if and only if the following condition holds*

$$\underline{r}_\beta\left[\beta\underline{n}+\psi - \frac{c_z(c_z - c_\omega)}{2}\right] < 1 < \overline{r}_\beta\left[\beta\overline{n}+\psi - \frac{c_\omega(c_z - c_\omega)}{2}\right]. \tag{15}$$

Notice first that if the hazard rate r_β is increasing in p, this ensures that the function $\Psi(p)$ specified in (14) is monotone in p and that any interior equilibrium price p^* is unique.[23] Secondly, if r_β is increasing in p, then the condition (15) stated in Proposition 5 is always satisfied for a fairly broad class of random networks. From Assumption 2 it follows that

$$\left[\beta\underline{n}+\psi - \frac{c_z(c_z - c_\omega)}{2}\right] < \left[\beta\overline{n}+\psi - \frac{c_\omega(c_z - c_\omega)}{2}\right].$$

Also, recall that, by Assumption 1, ψ is arbitrarily small. Therefore, if r_β is increasing in p, for condition (15) to hold, we only need to consider a support $[\underline{n}, \overline{n}]$ such that $\underline{n}$ be relatively

[22] Unlike a density function, the tail of a hazard rate function needs not converge to zero; it can increase, decrease, converge to some constant, etc. For the case of unbounded support of the degree distribution, let $\overline{r}_\beta = \lim_{\overline{n}\to+\infty} r_\beta(\beta\overline{n}+\psi)$, provided that such a limit exists.

[23] Using a model different to the one proposed here but where a monopolist also chooses a price for a good that is distributed through a network with positive degree externalities, Shin (2016) obtains uniqueness of the optimal price when the underlying hazard rate is increasing. While increasing hazard rates directly ensure the required single-crossing property in his model, we also need an additional form of single-crossing property which, in our model, takes the form of the assumed strictly positive premium externality.

small and $\overline{n}$ be relatively large. Most applications fall into this class of degree distributions. Thirdly, if r_β is decreasing in p, then the result obtained in Corollary 1, under which the firm's total profits are always increasing in p is obtained if and only if

$$\min\left\{\underline{r}_\beta\left[\beta\underline{n}+\psi-\frac{c_z(c_z-c_\omega)}{2}\right],\overline{r}_\beta\left[\beta\overline{n}+\psi-\frac{c_\omega(c_z-c_\omega)}{2}\right]\right\}<1.$$

Since, $\underline{r}_\beta > \overline{r}_\beta$ when r_β is decreasing in p, the condition stated above can be obtained for a fairly reasonable class of degree distributions. As indicated earlier, decreasing hazard rates are often associated with profits that always increase in the service price.

In addition, notice that monotonicity of the function $\Psi(p)$ specified in (14) needs not follow either when $r_\beta(p)$ is non-monotone or when it is decreasing in p. This constitutes the only source of multiplicity if we restrict attention to equilibria where each consumer chooses pure strategies.

Finally, it follows from a perturbation argument over the two parameters $(\psi, \beta) \in \mathbb{R}^2_{++}$, that the set of equilibria is *generically* a finite one. This perturbation argument allows us to eliminate equilibria both where the consumers choose mixed strategies (i.e., the non-generic cases where $n_i = (p^* - \psi)/\beta K^*$ for some $i \in [0, 1]$) and where the condition in the first requirement of equation (9) might lead to a continuum set of solutions p^*.

7 Concluding Remarks

While advertising is an ubiquitous aspect in economic life, important to understand the functioning of many markets, the pricing behavior of firms that operate in social networks has become a central area of interest for economists. The growth of online and assorted technologies has made it possible for firms to implement advertising and discrimination strategies that were not available earlier. These new possibilities have motivated an array of important questions about how advertising and prices interact with the architecture, and the dynamic evolution, of social networks. In this article, we have suggested and investigated a new taxonomy of second-degree discrimination in social networks which is implemented through two versions of a product, one of them being offered with advertising.

We have obtained clear-cut predictions of how fundamentals such as the difference in sizes of the network externalities, or the advertising costs, affect the optimal discrimination strategy. The comparative statics of optimal pricing with respect to *hazard rate dominance* are very different depending on whether hazard rates are increasing or decreasing. When hazard rates are increasing, *hazard rate dominance* always leads to higher prices and, therefore, to smaller proportions of premium version adopters. When hazard rates are decreasing, the *hazard rate dominance* criterion does not provide clear implications and, furthermore, there might exist multiple equilibrium prices.

On the other hand, important implications about the firm's behavior follow from decreasing hazard rates. In some cases, decreasing hazard rates lead to that the firm's total profits are always increasing in the service price which, in turn, imply that the firm wishes to serve only the free version with ads. A number of prominent real-world service providers seem to adjust to this pattern and our model provides a foundation for the popular claims that many service providers regard potential expansions of the networks as valuable because of the induced increase in their audiences for advertising. Beyond that, this model says when firms benefit from increasing their audiences for ads, depending on the shapes of the underlying hazard rate functions.

There are different directions in which our model could be extended. First, we have considered the case of monopoly and it would be very interesting to explore the consequences of having several service providers competing in a common social network. In this respect, our results nonetheless continue to be compelling if, for most practical situations, we believe that there is indeed sufficient service differentiation and/or that different service providers do not actually serve a significant number of common links (i.e., there is a certain degree of network differentiation across service providers). Secondly, our framework assumes that the only factor that determines the size of the network externalities is the degree of the nodes. Thus, we have not incorporated important features present in real-world social networks (such as clustering or assortativity) which, in practice, could influence the consumption externalities. For example, the externality could be larger when common neighbors are shared or when the links are formed between consumers with some similar characteristics. Finally, our analysis has ignored the fact that the links could be used to transmit information about the characteristics of the advertised good. This information transmission would interact (either in a complementary or substitutive way) with the information distributed through formal advertising. This allows for interesting interactions between the formal advertising by the firm and the *world of mouth* advertising through the social network. In particular, for review and rating services, including this possibility into the analysis seems very important.

While our framework is broadly applicable, it seems to capture empirical observations about how real-world business models operate in their provision of online services through social networks. The implications of our model could be useful for conducting empirical research and of interest for practitioners in the area.

Appendix

Proof of Proposition 1. Note that there are only two necessary conditions that remain to be obtained. One of them is with respect to the optimal price that the firm sets for the service, p^*, and the other one is with respect to the size of the set of premium version service adopters, $y^*(p)$. From the specification of the monopolist's profits in 7, provided that monopolist chooses $p > \psi > 0$ and that there are premium version service version adopters at the price p $(y^*(p) > 0)$, we obtain:

$$\frac{d\Pi(p)/dp}{y^*(p)} = 1 + \left[p - \frac{c_\omega(c_z - c_\omega)}{2}\right]\frac{dy^*(p)/dp}{y^*(p)} - \frac{(c_z - c_\omega)^2}{2} dy^*(p)/dp. \tag{16}$$

On the other hand, from the consumers' preference specification in 3, we obtain, under Assumptions 1 and 2,

$$\begin{aligned} y^*(p) &= \mathbb{P}\Big(\theta - \psi + \gamma_x n \leq \theta - p + \gamma_y n\Big) \\ &= \mathbb{P}\left(n \geq \frac{p-\psi}{\beta}\right) = 1 - F_\beta(p), \end{aligned}$$

which gives us the specification of the optimal size of the set of premium version adopters under a price of the service p stated in Proposition 1 (iii). In turn, this specification of the optimal size of consumers $y^*(p)$ implies $\frac{dy^*(p)/dp}{y^*(p)} = -r_\beta(p)$ and $dy^*(p)/dp = r_\beta(p)\big[F_\beta(p) - 1\big]$. Therefore, the expression in (16) above, together with the first order condition of the monopolist's problem when it sets the optimal price for the service $p^* > \psi > 0$, leads to that $d\Pi(p^*)/dp = 0$ if and only if

$$1 - r_\beta(p^*)\left[p^* - \frac{c_z(c_z - c_\omega)}{2} + \frac{(c_z - c_\omega)^2}{2} F_\beta(p^*)\right] = 0$$

holds in equilibrium, an equality that can be directly rewritten as the equilibrium condition stated in Proposition 1 (ii).

As for the second order condition in 1 (ii), notice that, for $y^*(p^*) > 0$, we have $\frac{d^2\Pi(p^*)}{dp^2} \leq 0$ if and only if

$$r'_\beta(p^*)\left[p^* - \frac{c_z(c_z - c_\omega)}{2} + \frac{(c_z - c_\omega)^2}{2} F_\beta(p^*)\right] + r_\beta(p^*)\left[p^* + \frac{(c_z - c_\omega)^2}{2} f_\beta(p^*)\right] \geq 0.$$

Then, using the equilibrium condition for the price p^*, we obtain that, for $y^*(p^*) > 0$, the required second order condition can be written as

$$\frac{r'_\beta(p^*)}{r_\beta(p^*)} + r_\beta(p^*)\left[p^* + \frac{(c_z - c_\omega)^2}{2} f_\beta(p^*)\right] \geq 0,$$

or, equivalently,

$$\frac{r'_\beta(p^*)}{\big[r_\beta(p^*)\big]^2} + \left[p^* + \frac{(c_z - c_\omega)^2}{2} f_\beta(p^*)\right] \geq 0,$$

as stated. ∎

Proof of Proposition 2. Define the function

$$\Gamma\big(p^*(\beta), \beta\big) = r_\beta(p^*)\Big[p^* - \frac{c_z(c_z - c_\omega)}{2} + \frac{(c_z - c_\omega)^2}{2} F_\beta(p^*)\Big],$$

so that $\Gamma\big(p^*(\beta), \beta\big) = 1$ is the equilibrium condition for the optimal price that the monopolist sets for the service, as stated in Proposition 1 (ii). Application of the *implicit function Theorem* allows us to obtain

$$\frac{\partial p^*}{\partial \beta} = -\frac{\partial \Gamma / \partial \beta}{\partial \Gamma / \partial p^*}$$

for $\partial \Gamma / \partial p^* \neq 0$. It can then be verified that

$$\begin{aligned}\frac{\partial \Gamma}{\partial \beta} = -\left(\frac{1}{\beta}\right)\bigg\{&\big[r_\beta(p^*) + (p^* - \psi) r'_\beta(p^*)\big]\Big[p^* - \frac{c_z(c_z - c_\omega)}{2} + \frac{(c_z - c_\omega)^2}{2} F_\beta(p^*)\Big] \\ &+ \frac{(c_z - c_\omega)^2}{2}(p^* - \psi) r_\beta(p^*) f_\beta(p^*)\bigg\}.\end{aligned}$$

On the other hand, it can be verified that

$$\begin{aligned}\frac{\partial \Gamma}{\partial p^*} =& r'_\beta(p^*)\Big[p^* - \frac{c_z(c_z - c_\omega)}{2} + \frac{(c_z - c_\omega)^2}{2} F_\beta(p^*)\Big] \\ &+ r_\beta(p^*)\Big[1 + \frac{(c_z - c_\omega)^2}{2} f_\beta(p^*)\Big].\end{aligned}$$

Furthermore, using the equilibrium condition for the price of the service stated in Proposition 1 (ii), the expressions above can be rewritten, respectively, as

$$\frac{\partial \Gamma}{\partial \beta} = -\left(\frac{1}{\beta}\right)\left[1 + (p^* - \psi)\frac{r'_\beta(p^*)}{r_\beta(p^*)} + \frac{(c_z - c_\omega)^2}{2}(p^* - \psi) r_\beta(p^*) f_\beta(p^*)\right].$$

and

$$\frac{\partial \Gamma}{\partial p^*} = \frac{r'_\beta(p^*)}{r_\beta(p^*)} + r_\beta(p^*)\Big[1 + \frac{(c_z - c_\omega)^2}{2} f_\beta(p^*)\Big].$$

Thus, if we set

$$A(p^*) = \frac{(c_z - c_\omega)^2}{2} r_\beta(p^*) f_\beta(p^*) > 0,$$

then the local change of the equilibrium price of the service p^* as the size of the premium externalities β varies is given by

$$\frac{\partial p^*}{\partial \beta} = \left(\frac{1}{\beta}\right)\frac{1 + (p^* - \psi)\left[\frac{r'_\beta(p^*)}{r_\beta(p^*)} + A(p^*)\right]}{r_\beta(p^*) + \left[\frac{r'_\beta(p^*)}{r_\beta(p^*)} + A(p^*)\right]}. \tag{17}$$

Note first that we obtain $\left[\frac{r'_\beta(p^*)}{r_\beta(p^*)} + A(p^*)\right] \geq 0$ if and only if the condition

$$r'_\beta(p^*) + \frac{(c_z - c_\omega)^2}{2}\big[r_\beta(p^*)\big]^2 f_\beta(p^*) \geq 0$$

is satisfied. But this inequality is in fact directly satisfied from the equilibrium second order condition obtained in Proposition 1 (ii). Therefore, it follows that $\frac{\partial p^*}{\partial \beta} \geq 0$. ∎

Proof of Proposition 3. In order to analyze the effects on the equilibrium price of the service p^* of a local change in the advertising cost c_ω, we define the function

$$\Gamma\big(p^*(c_z), c_z\big) = r_\beta(p^*)\Big[p^* - \frac{c_z(c_z - c_\omega)}{2} + \frac{(c_z - c_\omega)^2}{2}F_\beta(p^*)\Big],$$

so that $\Gamma\big(p^*(c_z), c_z\big) = 1$ is the equilibrium condition for the optimal price that the monopolist sets for the service, as stated in Proposition 1 (ii) Also, we use the expression $A(p^*) = \frac{(c_z - c_\omega)^2}{2} r_\beta(p^*) f_\beta(p^*) > 0$ which was specified in the proof of Proposition 2. Then, it can be verified that

$$\frac{\partial \Gamma}{\partial c_\omega} = r_\beta(p^*)\Big[c_z/2 - (c_z - c_\omega)F_\beta(p^*)\Big].$$

Hence, by using the *implicit function Theorem*, we obtain

$$\frac{\partial p^*}{\partial c_\omega} = \frac{r_\beta(p^*)\Big[(c_z - c_\omega)F_\beta(p^*) - c_z/2\Big]}{r_\beta(p^*) + \Big[\frac{r'_\beta(p^*)}{r_\beta(p^*)} + A(p^*)\Big]}. \tag{18}$$

From the second order condition that must hold in equilibrium, which was obtained in Proposition 1 (ii), the denominator of the expression (18) above is positive. Therefore, since $r_\beta(p^*) \geq 0$, it follows that $\frac{\partial p^*}{\partial c_\omega} \geq 0$ if and only if

$$F_\beta(p^*) \geq \frac{c_z}{2(c_z - c_\omega)},$$

as stated. ∎

Proof of Proposition 4. Consider two degree distributions F^A and F^B such that $r^A(n) \leq r^B(n)$ for all $n \in [\underline{n}, \overline{n}]$, with strict inequality for some n. As stated in Lemma 1, this in turn implies that $F^A(n) \leq F^B(n)$ for all $n \in [\underline{n}, \overline{n}]$, with strict inequality for some n. Let the parameters ψ and β remain constant across the two random networks. Suppose that p^*_A and p^*_B are equilibrium prices for the service under the degree distributions, respectively, F^A and F^B. Also, suppose that $p^*_A, p^*_B \in (\beta\underline{n} + \psi, \beta\overline{n} + \psi)$. Application of the equilibrium condition stated in Proposition 1 (ii) to both triplets $(r^A_\beta, F^A_\beta, p^*_A)$ and $(r^B_\beta, F^B_\beta, p^*_B)$ leads to

$$\frac{r^A_\beta(p^*_A)}{r^B_\beta(p^*_B)} = \frac{2p^*_B + (c_z - c_\omega)^2 F^B_\beta(p^*_B) - c_z(c_z - c_\omega)}{2p^*_A + (c_z - c_\omega)^2 F^A_\beta(p^*_A) - c_z(c_z - c_\omega)}. \tag{19}$$

Consider first the case where (i) both hazard rate functions r^A_β and r^B_β are (weakly) decreasing in $[\beta\underline{n} + \psi, \beta\overline{n} + \psi]$. We obtain:

(ia) Let $p^*_A < p^*_B$. Since the functions F^A_β, F^B_β are nondecreasing and, in addition, $F^A_\beta(p) \leq F^B_\beta(p)$ for all $p \in [\beta\underline{n} + \psi, \beta\overline{n} + \psi]$ (with strict inequality for some p), then it must be the case that $F^B_\beta(p^*_B) > F^A_\beta(p^*_A)$. Therefore, the right-hand side of equation (19) must exceed one. This is compatible with p^*_A and p^*_B being equilibrium prices only if $r^A_\beta(p^*_A) > r^B_\beta(p^*_B)$. Since we are considering that r_β^A and r_β^B are decreasing, this inequality can only be obtained if there exists a cutoff price $\widehat{p} \in (\beta\underline{n} + \psi, \beta\overline{n} + \psi)$ such that $r_\beta^B(\widehat{p}) = r_\beta^A(p_A^*)$ and $p_A^* < \widehat{p}$. If this price $\widehat{p}$ exists, then p_A^* and p_B^* are equilibrium prices such that $p_A^* < \widehat{p} \leq p_B^*$.

(ib) Let $p_A^* \geq p_B^*$. Since we are considering that the functions r_β^A, r_β^B are (weakly) decreasing and, in addition, $r_\beta^A(p) \leq r_\beta^B(p)$ for all $p \in [\beta\underline{n} + \psi, \beta\overline{n} + \psi]$ (with strict inequality for some p), then it must be the case that $r_\beta^B(p_B^*) \geq r_\beta^A(p_A^*)$. Therefore, the left-hand side of equation (19) is less than one. Using the right-hand side of equation (19), it can be verified that these equilibrium prices p_A^* and p_B^* must satisfy the following condition

$$F_\beta^B(p_B^*) - F_\beta^A(p_A^*) \leq \frac{2(p_A^* - p_B^*)}{(c_z - c_\omega)^2}.$$

The condition above does not contradicts any of the previous requirements for these price relation to hold.

Consider now the case where (ii) both hazard rate functions r_β^A and r_β^B are (weakly) increasing in $[\beta\underline{n} + \psi, \beta\overline{n} + \psi]$. We obtain:

(iia) Let $p_A^* < p_B^*$. Since the functions F_β^A, F_β^B are nondecreasing and, in addition, $F_\beta^A(p) \leq F_\beta^B(p)$ for all $p \in [\beta\underline{n} + \psi, \beta\overline{n} + \psi]$ (with strict inequality for some p), then it must be the case that $F_\beta^B(p_B^*) > F_\beta^A(p_A^*)$. In addition, since the functions r_β^A, r_β^B are (weakly) increasing and, furthermore, $r_\beta^A(p) \leq r_\beta^B(p)$ for all $p \in [\beta\underline{n} + \psi, \beta\overline{n} + \psi]$ (with strict inequality for some p), then it must be the case that $r_\beta^B(p_B^*) > r_\beta^A(p_A^*)$. Therefore, the left-hand side of equation (19) is less than one. Using the right-hand side of equation (19), it can be verified that this is compatible with p_A^* and p_B^* being equilibrium prices only if

$$F_\beta^B(p_B^*) - F_\beta^A(p_A^*) < \frac{2(p_A^* - p_B^*)}{(c_z - c_\omega)^2} < 0.$$

But this leads to a contradiction since, as indicated above, it must be the case that $F_\beta^B(p_B^*) > F_\beta^A(p_A^*)$.

(iib) Let $p_A^* \geq p_B^*$. Suppose that there exists a price $\widehat{p} \in (\beta\underline{n} + \psi, \beta\overline{n} + \psi)$ such that $r_\beta^B(\widehat{p}) = r_\beta^A(p_A^*)$ and $\widehat{p} \leq p_A^*$. Then, for $\widehat{p} \leq p_B^* \leq p_A^*$, we have $r_\beta^B(p_B^*) \geq r_\beta^A(p_A^*)$ so that the left-hand side of equation (19) is no larger than one. Using the right-hand side of equation (19), it can be verified that these equilibrium prices p_A^* and p_B^* must in addition satisfy the following condition

$$F_\beta^B(p_B^*) - F_\beta^A(p_A^*) \leq \frac{2(p_A^* - p_B^*)}{(c_z - c_\omega)^2}.$$

Notice that equilibrium prices p_A^* and p_B^*, with $p_A^* \geq p_B^*$, that satisfy the inequality above can also be obtained even if the price $\widehat{p}$ does not exist in the interval $(\beta\underline{n} + \psi, \beta\overline{n} + \psi)$. In this case, we always obtain the required condition $r_\beta^B(p_B^*) \geq r_\beta^A(p_A^*)$.

Finally, consider the case where cutoff price $\widehat{p}$ exists but $p_B^* \leq \widehat{p}$. This leads to that $r_\beta^B(p_B^*) \leq r_\beta^A(p_A^*)$ so that the left-hand side of equation (19) is no less than one. Using the right-hand side of equation (19), it can then be verified that these equilibrium prices p_A^* and p_B^*, with $p_A^* \geq p_B^*$, are still compatible with equilibrium when the following condition is satisfied.

$$F_\beta^B(p_B^*) - F_\beta^A(p_A^*) \geq \frac{2(p_A^* - p_B^*)}{(c_z - c_\omega)^2} > 0.$$

Note that this condition can only be obtained if there exists a price $\widetilde{p} \in (\beta\underline{n} + \psi, \beta\overline{n} + \psi)$ such that $F_\beta^B(\widetilde{p}) = F_\beta^A(p_A^*)$ and $\widetilde{p} \leq p_A^*$. Also, this condition does not contradicts any of the previous requirements for these price relation to hold. If such a cutoff price $\widetilde{p}$ exists, then it must be the case that $\widetilde{p} \leq p_B^* \leq p_A^*$ for these prices to be compatible with equilibrium. ∎

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