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Mass Production and Competition

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Abstract

I evaluate the optimality properties of a two sector market economy consisting of a mass production sector with market power and a competitive sector with small scale production. Adam Smith's results hold: the presence of market power renders production and innovation inefficient, as higher prices for mass produced goods deviate resources from inputs to profits. Aggregate production, wages and innovation in both sectors are suboptimal. The higher the large scale sector market power, the more important small scale sector innovation is for salary levels. An approximation to the first best in production and innovation can be achieved by a product-specific market power tax rewarding production rather than profit rates, with zero taxes levied at equilibrium. Innovation, but not production, can be optimized by taxing profits and subsidizing innovation. The concentration of property inherent in the large scale sector also helps to explain the inequality pointed out by Piketty in "Capital in the Twenty'First Century".

These results hold for leading countries or for countries lagging in levels or in growth rates. In lagging countries differing only in institutional and fixed productivity effects, the small scale sector is relatively more backward than in leading countries. This provides an explanation for the existence of large informal and other excluded sectors. Pro-poor growth reduces market power in the large scale sector and promotes technologies in the small scale sector. La concentracion de la propiedad inherente del sector de producción masiva también ayuda a explicar la desigualdad que señala Piketty en "Capital in the Twehty'First Century".

Keywords: Welfare, market power, inequality, optimality, mass production.

Resumen

Evalúo las propiedades de optimalidad de una economía de mercado de dos sectores, un sector de producción masiva con poder de mercado y otro competitivo con producción en pequeña escala. Los resultados de Adam Smith se mantienen: la presencia de poder de mercado hace que la producción e innovación sean ineficientes. La presencia de precios altos en los bienes producidos en masa desvían los recursos productivos de insumos a ganancias. La producción agregada, los salarios y la innovación son subóptimos en ambos sectores. Entre mayor sea el poder de mercado del sector de gran escala, más importante es la innovación del sector de pequeña escala para los niveles salariales. Una aproximación al óptimo de mercado en producción e innovación puede lograrse mediante un impuesto al poder de mercado en líneas de producción que premie la producción en lugar de las tasas de ganancia, con cero impuestos gravados en el equilibrio. La innovación, pero no de la producción, puede optimizarse por medio de un impuesto a las ganancias y un subsidio a la innovación. La concentración de la propiedad inherente en el sector de gran escala también ayuda a explicar la desigualdad señalada Piketty en "Capital en el siglo Twenty'First".

Estos resultados se mantienen para países l,íderes y para países rezagados en niveles o en las tasas de crecimiento. En países seguidores que difieran sólo en instituciones o en efectos fijos de productividad, el sector de la pequeña escala es relativamente más atrasado que en los países líderes. Esto proporciona una explicación para la existencia de grandes sectores informales o excluidos en países subdesarrollados. Un crecimiento pro-pobre debe reducir el poder de mercado en el sector de gran escala y promover la absorción de tecnología en el sector de pequeña escala.

Palabras clave: **Bienestar, poder de mercado, desigualdad, optimalidad, producción en gran escala.**

Introduction

I examine the optimality properties of a market economy with mass production. To do this I construct a model with a continuum of goods that are produced in one of two ways. The first is by mass production, with a single producer for each good, who therefore has market power. The second is by small scale production in a competitive sector, with an infinite number of small producers for each good. I show that the presence of market power renders production in this economy inefficient.

Next I include innovation. In the mass production sector, for simplicity I consider that the producer has an innovation advantage over her competitors. She innovates continuously and produces for the whole market. I let her market power be endogenous, increasing with her innovation investment, but also limited by the competition of other potential producers. In the small scale production sector any worker has access to the small scale technologies of production and can potentially run a firm as well as innovate to improve small scale production. Thus all firms in the small scale production sector also innovate continuously. As before, the presence of market power renders innovation inefficient. When market power is increasing in innovation (and therefore a fixed markup is not assumed), this reduces the distortion of innovation due to market power, but also reduces the participation of wages.

Innovation is modeled as a function of three inputs. The first consists of positive externalities from the leading technological edge. For simplicity I assume that these spillovers between countries occur between the large scale production sectors, and that in each country additional spillovers occur from the large scale to the small scale sectors. These spillovers generate an advantage of backwardness and therefore convergence. The second is current, implemented knowledge and skills. These can generate a disadvantage of backwardness and therefore divergence. The third is a material input, which occurs in proportion to the combination of the first two inputs. For simplicity, innovation is modelled with certainty and with myopic perfect foresight (into the infinitesimal future), itself a technical contribution of the paper. The presence of both advantages and disadvantages of backwardness implies the existence of multiple steady states in technological change that model development and two types of underdevelopment, one lagging in levels and the second in growth rates. The inefficiency of the market economy in the presence of mass production, for both production and innovation, holds across the full spectrum of development and underdevelopment.

The next question is whether a benevolent government can improve the functioning of a market economy with mass production through a combination of taxes and subsidies. I show that the first best can be approximating by taxing excess profit rates, therefore rewarding *production* rather than *profit rates* and discouraging market power. In the equilibrium this regime has no taxes. Innovation can be optimized by taxing profits and subsidizing innovation in both the large and the small scale sectors, with a balanced budget.[1]

[1]The combination of high profit taxes and support for science (as well as human capital formation) in the US during the Great Prosperity approximately give this combination.

While the assumption of market power is standard in the literature on innovation, following is data supporting both this assumption and the concept of a two sector economy with large and small scale production. According to the US Census Bureau, in 2011 89.8% of the 5,684,424 US firms had 20 employees or less, employed 17.9% of the US workforce, and paid 14.2% of the payroll. (Thus 10.2% of firms had more than 20 employees, employed 82.1% of the workforce and paid 85.8% of the payroll.) Looking at larger firms, 0.3% of the firms had 500 or more employees, employed 51.5% of the workforce and paid 58% of the payroll.[2] Market concentration has been the norm in US production during the 20th Century. From 1935 to 1992, the average production of the four largest firms in 459 industries was 38.4% of all shipments.[3] In 2002, the 201 largest manufacturing companies accounted for 57.5% of manufacturing value added.[4] Consistently with market power, Hall (1988) shows in a study of US industry that marginal cost is often well below price. Industrial concentration has also been a salient feature of globalization. In 2007, 89.3% of global FDI inflows consisted of mergers and acquisitions (UNCTAD, 2008). By 2008, the world's top 100 non-financial transnational corporations produced 14.1% of global output (ibid). Nevertheless, market structure is a complex subject. The literature on industrial organization, market structure, and game theory is extensive and distinguishes itself by its complexity. Standard theoretical and empirical solutions for the main problems are still to come: "Why are some industries dominated worldwide by a handful of firms? Why is the size distribution of firms within most industries highly skewed? Questions of this kind have attracted continued interest among economists for over a half Century" (Sutton, 2007).

The large scale sector appeared in the US in the late 19th Century with the consolidation of the Second Industrial Revolution. Lipton's (2006) first wave of mergers and acquisitions took place from 1893 to 1904. This period saw the birth of the main steel, telephone, oil, mining, railroad and other giants of the basic manufacturing and transportation industries. The late 19th Century was also the time when a wave of mergers radically transformed the banking sectors of Boston and Providence (Lamoreaux, 1991). The Sherman Antitrust Act, meant to prevent the destruction of competition through the formation of cartels and monopolies, dates to 1890, indicating the same time period in general terms and also an existing tendency for economic consolidation based on strategic competition. Lipton's (2006) second merger wave, from 1919 to 1929, saw the emergence of the major automobile manufacturers, featuring vertical integration that in the case of Ford reached all the way to the iron and coal mines.

In the next section I develop the model for a market economy with mass production and prove its inefficiency. In the following section I extend this result to the case of innovation. The analysis of taxes and subsidies for improving efficiency is included in each of the sections. Finally I conclude.

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[2]See http://www2.census.gov/econ/susb/data/2011/us_6digitnaics_2011.xls read 4/25/2014.

[3]See data in http://www.census.gov/epcd/www/concentration92-47.xls, read 4/21/2014. (It would be very useful for summary tabular data by firm size to be available on employment, value added and profits together.)

[4]http://www.census.gov/prod/ec02/ec0231sr1.pdf, read 4/27/2014.

1. The Model

Consider an economy with two sectors L and S that produce a continuum of tradeable goods indexed by $\eta \in [0, N]$, where each η refers to a product and $N \geq 1$ is a measure of sectors that we will keep exogenous. Large scale sector goods $\eta \in \Theta_L = [0, N_L)$ use a mass production technology and are therefore modelled with all production concentrated on a single large producer that is able to make a profit, while small scale sector goods $\eta \in \Theta_S = [N_L, N_L + N_S]$, with $N_L + N_S = N$, are produced on the small scale, with constant returns to scale, therefore modelled with infinitely many small, identical, competitive producers. For now I take N_L and N_S as exogenously determined by two types of innovation, the creation of product variety and the creation of new or even of general purpose technologies for mass production. However, in each sector technological change follows a distinct endogenous dynamic of technological change due to the different kinds of competition structure. For simplicity I abstract from innovation uncertainty and assume that innovation is symmetric within each sector L and S. Thus I am all assuming goods $\eta \in \Theta_j$ in each sector j have the same technological level A_{jt}, $j \in \{L, S\}$.

Innovation occurs as follows. In the large scale sector L there is for each good $\eta \in \Theta_L$ a single, infinitely lived innovator who invests in innovation and becomes a national monopolist, producing in the presence of a competitive fringe. Innovation is cheaper for the producing incumbent than for the competitive fringe, and she therefore has an innovation advantage. Her monopoly therefore persists indefinitely. By contrast, in the small sector S anybody can innovate, so as to reap the productive benefits of new technologies, namely the availability of returns to production factors, in this model labor.

I assume that small producers can produce any good, while large producers can only produce goods in sector Θ_L for which mass production technologies are available that are more productive than small scale technologies.

Because incentives for innovation are higher in sector L, $A_{Lt} > A_{St}$ will be maintained.

I assume innovation as in Mayer-Foulkes (2014). At each time t the economy is fully defined by the state variables, A_{Lt}, and A_{St}.

1.1. **Consumption.** Let the instantaneous consumer utility $U = U(C_t)$ depend on a subutility function C_t for an agent consuming $c_t(\eta)$ units of goods $\eta \in [0, 1]$, according to the Cobb-Douglass function

$$\ln(C_t) = \ln N + \frac{1-\theta}{N} \int_0^N \ln(c_t(\eta))\, d\eta, \tag{1.1}$$

where $0 < \theta < 1$. As we shall see, given a constant budget and goods at a constant price, this utility function expresses Cobb-Douglass preferences for variety.

1.2. **Two kinds of producers.** Let us examine the two sectors of production, L and S, in more detail.

Applying technologies of mass production requires producing for a sizeable proportion of the economy. Producers are therefore not small. The markets for goods produced using large scale technologies therefore exhibit market power and profits, as in monopolistic competition. At the same time these profits provide incentives for innovation, as a means to increase profits and maintain market power. In this sense, market power is an endogenous function of innovation. Rather than assuming monopoly, it is customary to assume the existence of a competitive fringe that can enter the market at a lower level of productivity but nevertheless limits the markup that the incumbent can obtain. Here I assume this competition lies in the large scale production sector. I limit myself to innovation as the source of market power, though fixed costs and increasing returns to scale are also present in mass production. In constructing the model I attempted to use these in addition to innovation, but both gave rise to mathematics that were too complex for the present purpose.[5]

By contrast, small-scale production occurs in small firms that I will assume are price takers. Nevertheless, these firms will also invest to improve their productivity. However, the returns to this investment will not be profits but labor productivity. The two processes of innovation will be thought to be qualitatively different, again because the first operates on a large scale and the second in the small.

The previous two paragraphs explain why, while the large and small-scale production sectors are quite different, their production functions can for simplicity both be represented as Cobb Douglass functions. The two sectors will only be distinguished by their competitive context. Nevertheless, I still assume that the sectors can differ in their labor intensity, generally the small-scale sector being more labor intensive.

Definition 1. *The production function for goods $\eta \in \Theta_j$ in sector $j \in \{L, S\}$ is:*

$$y_{jt}(\eta) = \frac{1}{\varepsilon_j} \left[x_{jt}(\eta)\right]^{\alpha_j} \left[q_j A_{jt} l_{jt}(\eta)\right]^{\beta_j}, \quad j \in \{L, S\}, \tag{1.2}$$

where $\alpha_j + \beta_j = 1$, $\varepsilon_j = \alpha_j^{\alpha_j} \beta_j^{\beta_j}$, $j \in \{L, S\}$, and $\beta_S > \beta_L$ when the small scale sector is more labor intensive.■

Here $y_{jt}(\eta)$ represents the quantity produced of good $\eta \in \Theta_j$ and $x_{jt}(\eta)$ a composite good input, representing the input of a continuum of goods according to the definition following below. q_j is a fixed productivity factor representing the effects of such non-technological factors as geography, institutions and policies influencing each sector's total factor productivity. This might under certain circumstances differ in the large and small scale sectors. A_{jt} is the technological level in each sector. $l_{jt}(\eta)$ is the quantity of labor input. The introduction of scaling factors ε_j facilitates the comparison of Cobb-Douglass functions with different α_j.

The composite good flow $x_{jt}(\eta)$ complementing labor can in part be considered as a flow of capital services. Being complementary to the flow of labor $l_{jt}(\eta)$ it allows for the determination of a wage without introducing an additional state variable for capital.

[5]It is worth noting that in the case of fixed costs two equilibria arise for the two sector economy developed here, as in Murphy, Shleifer, and Vishny (1989). Also, increasing returns to scale are related to the ratio of employment demand between the two sectors and therefore to wage levels.

Definition 2. *The composite good x represents the combined input of all goods $\eta \in [0,1]$ according to the kernel:*

$$\ln x_t = \ln N + \frac{1-\theta}{N}\int_0^N \ln x_t(\omega)d\omega. \blacksquare \tag{1.3}$$

This has the same kernel as the utility function, and for a constant budget makes variety more productive. I let the price of the composite good be the numeraire.

1.3. **Purchases of composite good.** Suppose a producer has a budget B for purchasing inputs x_{jt}^L, x_{jt}^S to form the composite good input, and assume large and small scale sectors goods each have a common price p_{Lt}, p_{St}. Since the composite good kernel is Cobb Douglass, the optimal input allocation dedicates the same budget to the purchase of each type of good η. This budget is $\frac{B}{N}$, so the quantity bought of each type of good is $x_{jt}^L = \frac{B}{p_{Lt}N}$, $x_{jt}^S = \frac{B}{p_{St}N}$. Hence the quantity of composite good produced is given by

$$\ln x_t = \ln N + (1-\theta)\left(\xi \ln \frac{B}{p_{Lt}N} + (1-\xi)\ln\frac{B}{p_{St}N}\right), \tag{1.4}$$

where

$$\xi = \frac{N_L}{N}, \text{ so } 1-\xi = \frac{N_S}{N}, \tag{1.5}$$

which implies

$$x_t = N^\theta B^{1-\theta}\left(p_{Lt}^{\xi}p_{St}^{1-\xi}\right)^{-(1-\theta)}. \tag{1.6}$$

Given a budget $p_{Lt}^{\xi}p_{St}^{1-\xi}$, the amount of composite good produced is $x_t = N^\theta$, showing that variety improves the productivity of goods bought with a constant budget. Since the composite good is the numeraire, this costs N^θ, so

$$p_{Lt}^{\xi}p_{St}^{1-\xi} = N^\theta. \tag{1.7}$$

A similar result holds for the purchase of consumption goods.

1.4. **Choice of inputs.** Let w be the domestic wage level. When small producers minimize costs, they choose a ratio of composite good input to labor:

$$\frac{x_{St}}{l_{St}} = \frac{\alpha_S w_t}{\beta_S}. \tag{1.8}$$

It follows that the production cost and price p_{St} of each unit of good $\eta \in \Theta_S$ is constant in η,

$$k_{St} = \frac{\alpha_S w_t l_{St}}{r\beta_S}. \tag{1.9}$$

$$\frac{y_{St}}{rk_{St} + w_t l_{St}} = \left[\frac{1}{r}\right]^{\alpha_j}\left[\frac{q_j A_{jt}}{w_t}\right]^{\beta_j} \tag{1.10}$$

$$p_{St} = r^{\alpha_j}\left[\frac{w_t}{q_S A_{St}}\right]^{\beta_S}, \tag{1.11}$$

and the production quantities are

$$y_{St} = \frac{1}{\beta_S} w_t^{\alpha_S} \left[q_S A_{St}\right]^{\beta_S} l_{St}. \tag{1.12}$$

The level of production is given by the aggregate expenditure level on this good, $z_t = p_{St} y_{St}$. The Cobb Douglass structure of the composite good input and similar preferences for consumption will make z_t constant across sectors of all types. Because this sector is competitive, expenditure equals costs and

$$x_{St} = \alpha_S z_t, \quad w_t l_{St} = \beta_S z_t. \tag{1.13}$$

In the case of large producers, I consider that each domestic sector has two types of potential competitors. The first type of competitors are small-scale producers, who can produce good η using a technological level A_{St}. Hence it will always be necessary that $p_{Lt} \leq p_{St}$, mass production just being feasible at equality. The second type of competitor is a potential industrial competitor with a lower technological level $\chi_t^{-1} A_{Lt}$, with $\chi_t > 1$, who is just unwilling to enter the market at zero profit. This competitor also produces on a large scale and supplies the full market. The incumbent will keep to a maximum price level just at the feasibility level for her competitor. We can think that other potential industrial competitors have even lower technologies for the production of this particular good η.

The level of production considered by both the incumbent and her competitor are given by the aggregate expenditure level on this good, $z_t = p_{Lt}(\eta)\, y_{Lt}(\eta)$, which as we have seen is constant across sectors of all types.

As we see below the maximum markup that the incumbent can use will be $\chi_t^{\beta_L}$. Unless we are considering a transition for which mass-production comes into existence, the usual case will be when under the full markup $\chi_t^{\beta_L}$ nevertheless $p_{Lt} \leq p_{St}$. The markup is a measure of the incumbent's market power. I will endogenize χ_t below by assuming that the relative advantage of the producer depends on her technological level. The formulation, introduced together with endogenous technological change, will permit a direct comparison with the case of constant χ_t.

Writing subindex C for the incumbent's industrial competitor, since $p_{Ct} y_{Ct} = z_t$, and preliminarily dropping η from the notation as we consider a single sector, the competitor's price is given by minimizing

$$\min \frac{z_t}{\frac{1}{\varepsilon_L} x_{Ct}^{\alpha_L} \left[q_L \chi_t^{-1} A_{Lt} l_{Ct}\right]^{\beta_L}} \text{ s.t. } x_{Ct} + w_t l_{Ct} = z_t. \tag{1.14}$$

It is not hard to see that cost minimizing inputs satisfy

$$\frac{x_{Ct}}{l_{Ct}} = \frac{\alpha_L w_t}{\beta_L}, \tag{1.15}$$

and that therefore the competitor would use inputs $l_{Ct} = \beta_L \frac{z_t}{w_t}$, $x_{Ct} = \alpha_L z_t$, and arrive at a production level $y_{Ct} = z_t \left[\frac{q_L \chi_t^{-1} A_{Lt}}{w_t}\right]^{\beta_L}$. It follows that the incumbent sets the competitor's potential price

$$p_{Lt} = p_{Ct} = \left[\frac{w_t}{q_L \chi_t^{-1} A_{Lt}}\right]^{\beta_L}. \tag{1.16}$$

Now the incumbent maximizes profits by setting this price and supplying the demanded quantity $y_{Lt} = z_t/p_{Lt}$. To minimize production costs she also selects inputs according to the ratio

$$\frac{x_{Lt}}{l_{Lt}} = \frac{\alpha_L w_t}{\beta_L}. \tag{1.17}$$

The incumbent produces quantity $y_{Lt} = y_{Ct}$ above, but requires less inputs than her competitor. It follows that

$$w_t l_{Lt} = \beta_L \chi_t^{-\beta_L} z_t, \quad x_{Lt} = \alpha_L \chi_t^{-\beta_L} z_t. \tag{1.18}$$

Hence the incumbent's profits will be income minus costs,

$$\pi_{Lt} = (1 - \chi_t^{-\beta_L}) z_t. \tag{1.19}$$

Because the production function, wages and prices are constant in sectors η of the same type, so also are the quantities $x_{jt}(\eta)$, $l_{jt}(\eta)$, $y_{jt}(\eta)$, so the variable η can now be dropped from the notation.

1.5. **The wage level.** The wage level can now be obtained by substituting (1.16), (1.11) in (1.7).

$$N^\theta = p_{Lt}^\xi p_{St}^{1-\xi} = \left[\frac{w_t}{q_L \chi_t^{-1} A_{Lt}}\right]^{\beta_L \xi} \left[\frac{w_t}{q_S A_{St}}\right]^{\beta_S(1-\xi)}. \tag{1.20}$$

Now defining the mean relative labor participations γ_L, γ_S in each sector, and a mean variety impact factor $\hat{\theta}$,

$$\gamma_L = \frac{\beta_L \xi}{\beta_L \xi + \beta_S (1-\xi)}, \quad \gamma_S = 1 - \gamma_L, \quad \hat{\theta} = \frac{\theta}{\beta_L \xi + \beta_S (1-\xi)} \tag{1.21}$$

it follows that

$$w_t = N^\theta \left[q_L \chi_t^{-1} A_{Lt}\right]^{\gamma_L} \left[q_S A_{St}\right]^{\gamma_S}. \tag{1.22}$$

Substituting back in (1.16), (1.11) and simplifying,

$$p_{St} = N_t^{\hat{\theta}\beta_S} \left[\frac{q_L \chi_t^{-1} A_{Lt}}{q_S A_{St}}\right]^{\gamma_L \beta_S}, \tag{1.23}$$

$$p_{Lt} = N_t^{\hat{\theta}\beta_L} \left[\frac{q_S A_{St}}{q_L \chi_t^{-1} A_{Lt}}\right]^{\gamma_S \beta_L}. \tag{1.24}$$

Note $\gamma_L \beta_S + \gamma_S \beta_L = \frac{\beta_S \beta_L}{\beta_L \xi + \beta_S (1-\xi)}$, so

$$\frac{p_{St}}{p_{Lt}} = \left(N^{\theta(\beta_S - \beta_L)} \left[\frac{q_L \chi_t^{-1} A_{Lt}}{q_S A_{St}}\right]^{\beta_S \beta_L}\right)^{\frac{1}{\beta_L \xi + \beta_S (1-\xi)}}. \tag{1.25}$$

This quantity has to be greater than 1 for large-scale production to outcompete small-scale production and therefore be feasible.

1.6. **Market clearing for goods and labor.** Let the population of the economy be $\mathcal{L}$. Suppose $\mathcal{L}_L$ and $\mathcal{L}_S$ are the employment levels in sectors L and S, with $\mathcal{L}_L + \mathcal{L}_S = \mathcal{L}$. Then specific sector employment levels l_{Lt}, l_{St} satisfy:

$$N_L l_{Lt} = \mathcal{L}_L, \quad N_S l_{St} = \mathcal{L}_S, \quad N_L l_{Lt} + N_S l_{St} = \mathcal{L} \tag{1.26}$$

and the market clearing condition is $\mathcal{L}_L + \mathcal{L}_S = \mathcal{L}$. Now, since the participation of labor is $w_t l_{St} = \beta_S z_t$ in sectors S, see (1.13), and $w_t l_{Lt} = \beta_L \chi_t^{-\beta_L} z_t$ in sectors L, see (1.18), it follows that

$$\frac{l_{Lt}}{l_{St}} = \frac{\beta_L \chi_t^{-\beta_L}}{\beta_S}. \tag{1.27}$$

Hence, we can solve

$$l_{St} = \frac{\beta_S \mathcal{L}/N}{\xi \beta_L \chi_t^{-\beta_L} + (1-\xi)\beta_S}, \tag{1.28}$$

$$l_{Lt} = \frac{\beta_L \chi_t^{-\beta_L} \mathcal{L}/N}{\xi \beta_L \chi_t^{-\beta_L} + (1-\xi)\beta_S}. \tag{1.29}$$

1.7. **Income.** From wages and employment income now follows. Using equation (1.13) and (1.22),

$$z_t = \frac{1}{\beta_S} w_t l_{St} = \frac{N^{-(1-\theta)} \left[q_L \chi_t^{-1} A_{Lt}\right]^{\gamma_L} \left[q_S A_{St}\right]^{\gamma_S} \mathcal{L}}{\xi \beta_L \chi_t^{-\beta_L} + (1-\xi)\beta_S}. \tag{1.30}$$

Note that aggregate net income is $Z_t = \alpha_S \int_0^N z_t d\eta = N z_t$. The average wage participation is

$$\frac{w_t \mathcal{L}}{N z_t} = \xi \beta_L \chi_t^{-\beta_L} + (1-\xi)\beta_S. \tag{1.31}$$

As ξ rises, wage participation drops because $\beta_L \chi_t^{-\beta_L} - \beta_S < \beta_L - \beta_S < 0$. Wage participation in the large scale sector is lower than in the small scale sector, one reason for Schumacher's "Small is Beautiful" (1973).

To work out net income, write down the income identity breaking down gross income into wages, profits and expenditure in intermediate goods,

$$W_t + \Pi_t + X_t = Z_t, \quad Y_t = W_t + \Pi_t = Z_t - X_t, \tag{1.32}$$

where

$$W_t = w_t \mathcal{L}, \quad \Pi_t = N\xi \pi_{Lt} = (1 - \chi_t^{-\beta_L})\xi N z_t, \quad X_t = \left[\xi \alpha_L \chi_t^{-\beta_L} + (1-\xi)\alpha_S\right] N z_t. \tag{1.33}$$

Net aggregate income can be written out in two useful ways,

$$Y_t = Z_t - X_t = \left[1 - \xi \alpha_L \chi_t^{-\beta_L} - (1-\xi)\alpha_S\right] N z_t \tag{1.34}$$

$$= \Upsilon(\chi_t) N^\theta \left[q_L A_{Lt}\right]^{\gamma_L} \left[q_S A_{St}\right]^{\gamma_S} \mathcal{L} \tag{1.35}$$

where $\Upsilon(\chi_t)$ expresses the various static impacts of market power,[6]

$$\Upsilon(\chi_t) = \frac{\xi \beta_L + \xi \alpha_L \left(1 - \chi_t^{-\beta_L}\right) + (1-\xi)\beta_S}{\xi \beta_L \chi_t^{-\beta_L} + (1-\xi)\beta_S} \chi_t^{-\gamma_L}. \tag{1.36}$$

Proposition 1. *Aggregate net income is decreasing in market power:* $\Upsilon(1) = 1$ *and for* $\chi \geq 1$, $\Upsilon'(\chi) < 0$.

[6] For an example, let $\beta_S = \beta_L = \frac{2}{3}$, $\chi_t^{-\beta_L} = .8$, $\xi = \frac{1}{2}$. Then $\gamma_L = \frac{1}{2}$, $\chi_t = 0.8^{-3} = 1.953$, $\chi_t^{-\gamma_L} = 0.716$ and $\Upsilon(\chi_t) = 0.835$. If instead $\beta_S = 0.733$, $\gamma_L = 0.4373$, $\chi_t^{-\gamma_L} = 0.746$ and $\Upsilon(\chi_t) = 0.861$.

Proof. For the purposes of the proof, let $A=\xi\beta_L$, $B=(1-\xi)\beta_S$, $C=\xi\alpha_L$, $\beta=\beta_L$, $\gamma=\gamma_L$. Then $\beta=\frac{A}{A+C}$ and $\gamma=\frac{\beta_L\xi}{\beta_L\xi+\beta_S(1-\xi)}=\frac{A}{A+B}$. Now

$$\begin{aligned}
&\chi^{\gamma+1}\left(A+B\chi^{\beta}\right)^2\frac{d}{d\chi}\left(\frac{A+C\left(1-\chi^{-\beta}\right)+B}{A\chi^{-\beta}+B}\chi^{-\gamma}\right)=\\
=\ & AC\gamma+(A+B)(A+C)(\beta-\gamma)\chi^{\beta}-B(A+B+C)\gamma\chi^{2\beta}\\
=\ & \frac{A^2C+A(A+B)(B-C)\chi^{\beta}-AB(A+B+C)\chi^{2\beta}}{A+B}\\
=\ & \frac{A^2C\left(1-\chi^{\beta}\right)+AB(A+B)\left(1-\chi^{\beta}\right)\chi^{\beta}-ABC\left(1+\chi^{\beta}\right)\chi^{\beta}}{A+B}\\
\leq\ & 0.
\end{aligned}$$

In particular

$$\Upsilon(1)=1,\quad \Upsilon'(1)=-\frac{2ABC}{(A+B)^3}=-\frac{\xi^2(1-\xi)\alpha_L\beta_L\beta_S}{(\xi\beta_L+(1-\xi)\beta_S)^3}<0.\blacksquare$$

1.8. **Endogenous market power.** Now I let the endogeneity of market power in the mass production sector be explicit. Since market power in this sector is one of the objectives of innovation, I suppose that an incumbent's market power is a function of the ratio of her productivity to the average productivity level, so that

$$\chi_t=\chi_0\left[\frac{q_LA_{Lt}}{q_SA_{St}}\right]^{\kappa}. \tag{1.37}$$

Hence from (1.30)

$$z_t = \frac{N^{-(1-\theta)}\chi_0^{-\gamma_L}\left[q_LA_{Lt}\right]^{(1-\kappa)\gamma_L}\left[q_SA_{St}\right]^{\gamma_S+\kappa\gamma_L}\mathcal{L}}{\xi\beta_L\chi_0^{-\beta_L}\left[\frac{q_SA_{St}}{q_LA_{Lt}}\right]^{\beta_L\kappa}+(1-\xi)\beta_S}, \tag{1.38}$$

$$w_t = N^{\theta}\chi_0^{-\gamma_L}\left[q_LA_{Lt}\right]^{(1-\kappa)\gamma_L}\left[q_SA_{St}\right]^{\gamma_S+\kappa\gamma_L}, \tag{1.39}$$

$$\frac{w_t\mathcal{L}}{z_tN} = \xi\beta_L\chi_0^{-\beta_L}\left[\frac{q_SA_{St}}{q_LA_{Lt}}\right]^{\beta_L\kappa}+(1-\xi)\beta_S, \tag{1.40}$$

$$l_{St} = \frac{\beta_S\mathcal{L}/N}{\xi\beta_L\chi_0^{-\beta_L}\left[\frac{q_SA_{St}}{q_LA_{Lt}}\right]^{\beta_L\kappa}+(1-\xi)\beta_S}. \tag{1.41}$$

1.9. **Market power and efficiency.** Following are stated the static distortions due to the presence of market power.

Theorem 1. *Market power distorts the described two sector economy as follows:*

1) Aggregate income Y_t is decreasing in market power.

2) Aggregate profit and profit per sector are increasing in market power.

3) Wages and aggregate wage participation are decreasing in market power.

4) The aggregate wage to profit ratio is decreasing in market power and in ξ.

5) Employment intensity l_{Lt} in the large scale sector is decreasing in market power, while employment intensity l_{St} in the small scale sector is increasing in market power.

Proof. 1) Aggregate income on each sector is decreasing in market power.

$$\frac{\partial}{\partial\chi_t}\ln z_t = -\frac{\gamma_L}{\chi_t} + \frac{\xi\beta_L^2\chi_t^{-\beta_L-1}}{\xi\beta_L\chi_t^{-\beta_L} + (1-\xi)\beta_S} \tag{1.42}$$

$$< -\frac{\gamma_L}{\chi_t} + \frac{\xi\beta_L^2\chi_t^{-\beta_L-1}}{\xi\beta_L\chi_t^{-\beta_L} + (1-\xi)\beta_S\chi_t^{-\beta_L}} \tag{1.43}$$

$$= -\frac{\gamma_L(1-\beta_L)}{\chi_t} < 0. \tag{1.44}$$

Finally, $\frac{\partial}{\partial\chi_t}\ln(Y_t) = \frac{\partial}{\partial\chi_t}\ln(Nz_t) = \frac{\partial}{\partial\chi_t}\ln z_t$.

2) Aggregate profit and profit per sector are increasing in market power,

$$\frac{\partial}{\partial\chi_t}\ln\frac{N\xi\pi_{Lt}}{Nz_t} = \frac{\partial}{\partial\chi_t}\ln\frac{\pi_{Lt}}{z_t} = \frac{\partial}{\partial\chi_t}\ln\left(1-\chi_t^{-\beta_L}\right) = \frac{\beta_L\chi_t^{-\beta_L-1}}{1-\chi_t^{-\beta_L}} > 0. \tag{1.45}$$

3) Wages and aggregate wage participation are decreasing in market power.

$$\frac{\partial}{\partial\chi_t}\ln w_t = -\frac{\gamma_L}{\chi_t} < \frac{\partial}{\partial\chi_t}\ln z_t < 0. \quad \text{Hence} \quad \frac{\partial}{\partial\chi_t}\ln\frac{w_t\mathcal{L}}{Nz_t} < 0. \tag{1.46}$$

4) Using (1.33) and then (1.31), $\frac{W_t}{\Pi_t} = \frac{w_t\mathcal{L}}{(1-\chi_t^{-\beta_L})\xi Nz_t} = \frac{\xi\beta_L\chi_t^{-\beta_L}+(1-\xi)\beta_S}{(1-\chi_t^{-\beta_L})\xi}$. Hence, first dividing numerator and denominator by ξ,

$$\frac{\partial}{\partial\xi}\left(\frac{W_t}{\Pi_t}\right) = -\frac{1}{\xi^2}\frac{\beta_S}{1-\chi_t^{-\beta_L}} < 0. \tag{1.47}$$

Writing $\varkappa = \chi_t^{\beta_L}$ for the markup, $\frac{W_t}{\Pi_t} = \frac{\xi\beta_L+(1-\xi)\beta_S\varkappa}{(\varkappa-1)\xi}$, so, using (1.21),

$$\frac{\partial}{\partial\varkappa}\left(\frac{W_t}{\Pi_t}\right) = \frac{(1-\xi)\beta_S(\varkappa-1) - (\xi\beta_L + (1-\xi)\beta_S\varkappa)}{(\varkappa-1)^2\xi} = -\frac{\beta_L}{(\varkappa-1)^2\gamma_L} < 0. \tag{1.48}$$

5) Employment intensity l_{St} in the small scale sector is increasing in market power,

$$\frac{\partial}{\partial\chi_t}\ln l_{St} = \frac{\xi\beta_L^2\chi_t^{-\beta_L-1}}{\xi\beta_L\chi_t^{-\beta_L} + (1-\xi)\beta_S} > 0. \tag{1.49}$$

Hence, by (1.26), market power distorts employment in the large scale sector downwards.■

1.10. **Wages, market power and the size of the large scale sector.** Let us examine how the size of the large scale sector ξ affect wages in the presence of market power χ_t.

Theorem 2. *When the size of the large scale sector increases, wages respond as follows:*

$$\frac{\partial\ln w_t}{\partial\xi} = \frac{\beta_L\beta_S\ln\left(\chi_0^{-1}\left[\frac{q_LA_{Lt}}{q_SA_{St}}\right]^{1-\kappa}\right)}{(\beta_L\xi + \beta_S(1-\xi))^2}. \tag{1.50}$$

This is non-positive for high enough market power, when $\chi_0 \geq \left[\frac{q_LA_{Lt}}{q_SA_{St}}\right]^{1-\kappa}$, a condition consistent with $\frac{p_{St}}{p_{Lt}} \geq 1$. For high enough market power, wages depend only on the technological level of the *small scale sector. This occurs for example under the following two similar conditions, 1) large scale producers can raise prices to the level of the small scale sector, $p_{Lt} = p_{St}$, or 2) endogenous market power χ_t is given by the ratio of large scale to small scale productivities, $\chi_0 = \kappa = 1$. In both these cases wages depend only on the small scale sector technology, as follows:*

$$(1.51) \qquad 1)\ w_t = N^{\theta\left(1-\frac{(\beta_S-\beta_L)\xi}{\beta_S(\beta_L\xi+\beta_S(1-\xi))}\right)} q_S A_{St}, \quad 2)\ w_t = N^{\theta} q_S A_{St}.$$

Proof. First, note $\frac{\partial\gamma_L}{\partial\xi} = \frac{\beta_L\beta_S}{(\beta_L\xi+\beta_S(1-\xi))^2} > 0$, $\frac{\partial\gamma_S}{\partial\xi} = -\frac{\partial\gamma_L}{\partial\xi} < 0$. Hence

$$\begin{aligned} \frac{\partial \ln w_t}{\partial\xi} &= \frac{\partial\gamma_L}{\partial\xi}\ln\left[q_L\chi_t^{-1}A_{Lt}\right] + \frac{\partial\gamma_S}{\partial\xi}\ln\left[q_S A_{St}\right] = \frac{\beta_L\beta_S\left[\ln\left[q_L\chi_t^{-1}A_{Lt}\right]-\ln\left[q_S A_{St}\right]\right]}{\left(\beta_L\xi+\beta_S(1-\xi)\right)^2} \\ (1.52) \qquad &= \frac{\beta_L\beta_S\ln\left(\chi_0^{-1}\left[\frac{q_L A_{Lt}}{q_S A_{St}}\right]^{1-\kappa}\right)}{\left(\beta_L\xi+\beta_S(1-\xi)\right)^2}. \end{aligned}$$

This is nonpositive if market power is large enough, when $\frac{q_L\chi_t^{-1}A_{Lt}}{q_S A_{St}} \leq 1$. This condition is consistent with $\frac{p_{St}}{p_{Lt}} \geq 1$ since $N \geq 1$, see (1.25). There are two relevant scenarios which imply this condition. If $\frac{p_{St}}{p_{Lt}} = 1$, by (1.25) $\frac{q_L\chi_t^{-1}A_{Lt}}{q_S A_{St}} = N^{-\frac{\theta(\beta_S-\beta_L)}{\beta_S\beta_L}} < 1$ which yields the result. If $\chi_0 = \kappa = 1$, the condition holds at equality. The expressions for w_t follow from (1.21), (1.22) and (1.37).■

1.11. **A market power tax.** The results show that the presence of market power implies an inefficiency in production levels and wages. Therefore if incentives can be found for producers not to diminish production to raise their prices and profits, aggregate economic efficiency will rise.

Let us suppose that a series of conditions not modelled here imply the social, economic or political convenience of some socially designated positive profit rate for large scale production, which however is lower than can be obtained in an unregulated market. I define a market power tax, specific to a product line, whose incentives are for the producer to decrease her exercise of market power up to the socially designated profit rate. Hence at equilibrium the market power tax produces no charge and instead has the effect of increasing production. The effect of the market power tax is to improve both efficiency and equity.[7]

We have seen that several conditions define a maximum markup factor $\chi_t^{\beta_L}$ by which the large scale producer can increase her prices above unit cost. For any feasible markup $\varkappa \in [1, \chi_t^{\beta_L}]$ profits will be $\pi_{Lt} = \left(1-\varkappa^{-1}\right) z_t$. Note the profit to input rate is $\frac{1-\varkappa^{-1}}{\varkappa^{-1}} = \varkappa - 1$.

Let $\tau(\varkappa)$ be the tax schedule

$$(1.53) \qquad \tau(\varkappa) = \begin{cases} \tau_0\left(\varkappa-\varkappa_0\right)+\phi_L^{\pi} & \varkappa \geq \varkappa_0, \\ \phi_L^{\pi} & \varkappa < \varkappa_0. \end{cases}$$

Besides the constant profit tax rate ϕ_L^{π}, above the profit rate $\varkappa_0 - 1$, where $\varkappa_0 \in (1, \chi_t^{\beta_L})$, taxes rise with markup rates. The result is that from this point on profits are higher for higher production levels rather than higher gross profits.

[7] At this point other reasons for taxation are not being treated, including all types of public and social goods and equity. However, a more efficient and equitable society has less unsatisfied needs and may therefore need less taxes.

Theorem 3. *Under a tax schedule $\tau(\varkappa)$ the economy can approximate the first best for which $\chi_t = 1$. If $\tau_0 > \frac{1}{\varkappa_0(\varkappa_0 - 1)}$, the economy behaves as if market power has lowered to $\chi_0 = \varkappa_0^{1/\beta_L}$. In this example the marginal tax on profits as the markup increases at $\varkappa_0$ is less than 1 so long as the profit rate is less than 61.8%. Define instead tax schedule (1.53) using $\varkappa_0 = 1$. To avoid the tax large scale production adjusts to a markup $\varkappa^*(\tau_0) = \sqrt{1 + \frac{1}{\tau_0}}$, which also tends to 1 as $\tau_0 \to \infty$.*

Proof. Below $\varkappa_0$, since $\varkappa_0 < \chi_t^{\beta_L}$, the incentives are to raise prices to increase profits. Above $\varkappa_0$, the derivative with respect to $\varkappa$ of $(1 - \tau(\varkappa))\pi_{Lt}(\varkappa) = (1 - \tau_0(\varkappa - \varkappa_0))(1 - \varkappa^{-1}) z_t$ is negative if

$$0 > -\tau_0 \left(1 - \varkappa^{-1}\right) + \left(1 - \tau_0(\varkappa - \varkappa_0)\right)\varkappa^{-2} \tag{1.54}$$

$$\Leftrightarrow \quad 0 > -\tau_0 \left(\varkappa^2 - \varkappa\right) + 1 - \tau_0(\varkappa - \varkappa_0) \tag{1.55}$$

For $\varkappa_0 > 1$ to satisfy the inequality we need $\tau_0\left(\varkappa_0^2 - \varkappa_0\right) > 1$, that is, $\tau_0 > \frac{1}{\varkappa_0(\varkappa_0 - 1)}$. The inequality remains valid for $\varkappa > \varkappa_0$ since the next derivative with $\varkappa$, $-\tau_0(2\varkappa - 1) - \tau_0 < 0$ for these values. Observe that the marginal tax on profits at $\varkappa_0$ is

$$\left.\frac{\frac{d}{d\varkappa}\left[(1 - \tau(\varkappa))\pi_{Lt}(\varkappa)\right]}{\pi_{Lt}(\varkappa)}\right|_{\varkappa = \varkappa_0} = \frac{\varkappa_0^{-2}}{1 - \varkappa_0^{-1}} < 1 \tag{1.56}$$

when $\varkappa_0^2 - \varkappa_0 - 1 > 0$, that is, so long as $\varkappa_0 < \frac{1}{2}\sqrt{5} + \frac{1}{2} = 1.618$, which will only stop holding in this stylized case when the profit rate is above 61.8%.

Now let $\varkappa_0 = 1$. Then the derivative of $(1 - \tau(\varkappa))\pi_{Lt}(\varkappa)$ is negative if

$$0 > -\tau_0\left(\varkappa^2 - \varkappa\right) + 1 - \tau_0(\varkappa - 1) = 1 - (1 + \varkappa)\tau_0(\varkappa - 1), \tag{1.57}$$

that is, for $\varkappa > \varkappa^*(\tau_0) = \sqrt{1 + \frac{1}{\tau_0}}$. ■

In what follows, given a tax schedule $\tau(\varkappa)$ defined by (1.53) the notation considers that market power is fixed at $\chi_t = \varkappa_0^{1/\beta_L}$, and that taxes on profits in the large scale sector are ϕ_L^π.

2.

2. Technological Change

I define a process of endogenous change for the technological levels A_{Lt}, A_{Nt} in this two sector economy. The process described applies to both a technological leader creating new technologies and to a lagging country implementing or adopting technologies from a technological leader. This framework can be extended in further work to the context of trade and FDI.

I define a myopic decision maker who has perfect foresight as her time horizon Δt tends to zero. This is both more realistic (there *is* no perfect foresight!) and simpler. It eliminates the need for a second set of variables predicting the prices of all goods (forever!) that is required in perfect foresight models. In addition, in this model a scale effect occurs in innovator's incentives through the impact of the future relative size of the small and large scale sectors. A model with perfect myopic foresight simplifies the treatment of this scale effect by bringing it to the current time.

2.1. **Innovation in the large scale sector.** As mentioned above, there is in each mass production sector a single, infinitely lived innovator who can produce an innovation for the next period. I consider a myopic innovator who maximizes profits in the short term Δt by choosing innovation inputs. Then I let $\Delta t \to 0$ and obtain a continuos time model. By contrast in the small scale sector each producer will be a worker-innovator who maximizes her earnings by maximizing his productivity, also in the short term Δt.

The effectiveness of innovation investment of the product η entrepreneur has three components. The first is derived from knowledge and is proportional to the skill level $S_{Lt} = A_{Lt}$ that she has been able to accumulate in production, which we assume is the technological level of her firm. This generates a disadvantage of backwardness. The second component consists of positive externalities from the nascent leading technological edge, $\left((1+\sigma_L)\bar{A}_{Lt} - A_{Lt}\right)\Delta t$. The term $(1+\sigma_L) A_{1t}$ represents this technologically multiplying impact, presenting itself in diverse forms as nascent possibilities, for example embodied in the use of other firm's embodied technologies at time $t+\Delta t$. The difference with A_{Lt} measures how far back our innovating firm, situated in a leading or a lagging country, is from these nascent possibilities. However, the effectiveness of these combined inputs is inversely proportional to the level of the nascent possibilities, the fishing out effect. The third component is a material input $v\Delta t$. Innovation occurs with certainty combining these components to obtain a technological level $A_{t+\Delta t}$ according to:

$$A_{t+\Delta t} = A_{Lt} + \mu_L \left(\frac{\left((1+\sigma_L)\bar{A}_{Lt} - A_{Lt}\right) S_{Lt}\Delta t}{\bar{A}_{Lt+\Delta t}} \right)^{\tau_L} (v\Delta t)^{1-\tau_L}, \tag{2.1}$$

where $\mu_L, \sigma_L > 0$, $0 < \tau_L < 1$.

This means that, as in Howitt and Mayer-Foulkes (2005) the impact of innovator's skill on the technological change that a firm can obtain is proportional first, to the skill level, and second to its distance to the nascent technological frontier. In addition, this skill impact combines with material inputs according to a Cobb-Douglass function. The parameter μ_L represents the innovation productivity of the combined inputs.

The firm expects its potential industrial competitor to have a productivity $\frac{A_{Lt+\Delta t}}{\chi_t}$. Therefore if it innovates to a technological level $A_{t+\Delta t}(v)$ resulting from an instantaneous level of innovation investment v, the incumbent's market power, the price mark up it can make, will be $\frac{\chi_t A_{t+\Delta t}(v)}{A_{Lt+\Delta t}}$. Using myopic perfect foresight, so that a given firm correctly expects the new technological levels $A_{jt+\Delta t}$, $j \in \{S, L\}$, her profits level will be:

$$\pi_{t+\Delta t} = (1 - \left(\frac{\chi_t A_{t+\Delta t}(v)}{A_{Lt+\Delta t}}\right)^{-\beta_L}) z_{t+\Delta t}, \tag{2.2}$$

where according to (1.37)

$$\chi_t = \chi_0 \left[\frac{q_L A_{t+\Delta t}(v)}{q_S A_{St+\Delta t}}\right]^{\kappa}. \tag{2.3}$$

Hence the profit maximizing rate of innovation investment is obtained by maximizing:

$$\max_v e^{-\delta\Delta t} (1-\phi_L^{\pi}) (1 - \left(\chi_0 \frac{q_L^{\kappa} A_{t+\Delta t}(v)^{1+\kappa}}{q_S^{\kappa} A_{St+\Delta t}^{\kappa}}\right)^{-\beta_L}) z_{t+\Delta t} - (1-\phi_L^{\iota}) v\Delta t, \tag{2.4}$$

where $e^{-\delta\Delta t}$ is the discount factor, and $\phi_L^\pi, \phi_L^\iota \in (0,1)$ represents a profit tax and an innovation subsidy, positive or negative proxies for all distortions and policies affecting profits and the incentives to innovate.

The first order condition can be written as follows, where after differentiating we substitute $A_{t+\Delta t} = A_{Lt+\Delta t}$ since firms in sector L are symmetric ex-post and have perfect myopic foresight:

$$\frac{e^{-\delta\Delta t}(1+\kappa)\beta_L(1-\phi_L^\pi)}{\chi_{t+\Delta t}^{\beta_L} A_{Lt+\Delta t}}\mu_L\left(\frac{\left((1+\sigma_L)\bar{A}_{Lt}-A_{Lt}\right)S_{Lt}\Delta t}{\bar{A}_{Lt+\Delta t}}\right)^{\tau_L} \times (1-\tau_L)(v\Delta t)^{-\tau_L} z_{t+\Delta t}\Delta t = (1-\phi_L^\iota)\Delta t.$$

Letting $\hat{\mu}_L = \frac{\beta_L(1-\tau_L)(1-\phi_L^\pi)}{(1-\phi_L^\iota)}\mu_L$, material inputs v are given by:

$$v\Delta t = \left(\frac{e^{-\delta\Delta t}\hat{\mu}_L(1+\kappa)}{\chi_{t+\Delta t}^{\beta_L}}\frac{z_{t+\Delta t}}{A_{Lt+\Delta t}}\right)^{\frac{1}{\tau_L}}\frac{\left((1+\sigma_L)\bar{A}_{Lt}-A_{Lt}\right)S_{Lt}\Delta t}{\bar{A}_{Lt+\Delta t}}. \tag{2.5}$$

Substituting this result in (2.1), and writing $\varsigma_j = \frac{1-\tau_j}{\tau_j}$, $j \in \{S,T\}$,

$$A_{t+\Delta t} = A_{Lt} + \mu_L\frac{\left((1+\sigma_L)\bar{A}_{Lt}-A_{Lt}\right)S_{Lt}\Delta t}{\bar{A}_{Lt+\Delta t}}\left(\frac{e^{-\delta\Delta t}\hat{\mu}_L(1+\kappa)}{\chi_{t+\Delta t}^{\beta_L}}\frac{z_{t+\Delta t}}{A_{Lt+\Delta t}}\right)^{\varsigma_L}. \tag{2.6}$$

Note that since $z_{t+\Delta t}$ depends on both A_{Lt} and A_{St} a relative scale effects is introduced that complicates the dynamics once technological change in both variables is considered. This aspect is simplified by using continuous myopic foresight. Note also that innovation is *decreasing* in market power χ_t, because, as can be seen by following the derivative above, the higher the market power, the relatively lower the input costs compared to profits and therefore the lower the impact of technological improvement on profit. Now set:

$$\tilde{\mu}_L = \mu_L\hat{\mu}_L^{\varsigma_L} = \left(\frac{\beta_L(1-\tau_L)(1-\phi_L^\pi)}{1-\phi_L^\iota}\right)^{\varsigma_L}\mu_L^{1+\varsigma_L}, \tag{2.7}$$

Taking the limit as $\Delta t \to 0$, and writing $A_{t+\Delta t} = A_{Lt+\Delta t}$,

$$\frac{d}{dt}A_{Lt} = \frac{\tilde{\mu}_L}{\chi_t^{\beta_L\varsigma_L}}\frac{\left((1+\sigma_L)\bar{A}_{Lt}-A_{Lt}\right)A_{Lt}}{\bar{A}_{Lt}}\left(\frac{(1+\kappa)z_t}{A_{Lt}}\right)^{\varsigma_L}. \tag{2.8}$$

2.2. **Innovation in the small scale sector.** Competitive producers in small scale sectors can innovate to reap the productive benefits of new technologies. I assume that a single worker running a firm can invest $v\Delta t$ units of material input to obtain a technological level $A_{t+\Delta t}$ given by an innovation function analogous to (2.1),

$$A_{t+\Delta t} = A_{St} + \mu_S\left(\frac{(1+\sigma_S)A_{Lt}-A_{St}}{A_{Lt+\Delta t}}S_{St}\Delta t\right)^{\tau_S}(v\Delta t)^{1-\tau_S}. \tag{2.9}$$

Here μ_S, σ_S, τ_S are parameters analogous to those in (2.1), and S_{St} is the skill level of the worker, which I consider equal to A_{St}. Here, however, the nascent technological edge for the small scale sector is the country's large scale technology A_{Lt}. The innovating worker/firm expects her competitors in the same sector to sell at an accurately expected price $p_{St+\Delta t}$, which satisfies (1.8), and therefore chooses $x_{t+\Delta t}$, v to maximize

$$\max \frac{e^{-\delta \Delta t} p_{St+\Delta t}}{\varepsilon_S} x_{t+\Delta t}^{\alpha_S} \left[q_S A_{t+\Delta t}\right]^{\beta_S} - x_{t+\Delta t} - (1-\phi_S^{\iota})\, v\Delta t.$$

Here again ϕ_S^{ι} is an innovation subsidy, positive or negative proxies for all distortions and policies affecting profits and the incentives to innovate. I do not include wage taxes for now, which would affect factor assignment in the small scale sector. Note

$$\frac{\partial A_{t+\Delta t}}{\partial v} = (1-\tau_S)\,\mu_S \left(\frac{(1+\sigma_S)\,A_{Lt} - A_{St}}{A_{Lt+\Delta t}} S_{St}\Delta t\right)^{\tau_S} (v\Delta t)^{-\tau_S}\,\Delta t, \tag{2.10}$$

so the first order conditions are:

$$\begin{aligned} \alpha_S e^{-\delta\Delta t} p_{St+\Delta t} \frac{1}{\varepsilon_S} x_{t+\Delta t}^{\alpha_S - 1} \left[q_S A_{t+\Delta t}\right]^{\beta_S} &= 1, \\ \hat{\mu}_S \beta_S e^{-\delta\Delta t} p_{St+\Delta t} \frac{1}{\varepsilon_S} x_{t+\Delta t}^{\alpha_S} \left[q_S A_{t+\Delta t}\right]^{\beta_S - 1} q_S \times & \\ \left(\frac{(1+\sigma_S)\,A_{Lt} - A_{St}}{A_{Lt+\Delta t}} S_{St}\Delta t\right)^{\tau_S} (v\Delta t)^{-\tau_S}\,\Delta t &= \Delta t, \end{aligned}$$

where $\hat{\mu}_S = \frac{1-\tau_S}{1-\phi_S^{\iota}}\mu_S$. Dividing the second condition by the first, noting that by (1.8) $x_{St+\Delta t} = \frac{\alpha_S w_{t+\Delta t}}{\beta_S}$, and simplifying for $v\Delta t$,

$$v\Delta t = \left(\frac{\hat{\mu}_S w_{t+\Delta t}}{A_{t+\Delta t}}\right)^{\frac{1}{\tau_S}} \frac{(1+\sigma_S)\,A_{Lt} - A_{St}}{A_{Lt+\Delta t}} S_{St}\Delta t. \tag{2.11}$$

Substituting in (2.9), taking A_{St} to the left hand side, dividing by Δt, taking the limit as $\Delta t \to 0$, noting the skill level in the small scale sector is $S_{St} = A_{St}$, using (1.22) and writing ex-post $A_{t+\Delta t} = A_{St+\Delta t}$,

$$\frac{d}{dt} A_{St} = \tilde{\mu}_S \frac{(1+\sigma_S)\,A_{Lt} - A_{St}}{A_{Lt}} A_{St} \left(q_S N^{\theta} \left[\frac{q_L \chi_t^{-1} A_{Lt}}{q_S A_{St}}\right]^{\gamma_L}\right)^{\varsigma_S}, \tag{2.12}$$

where

$$\tilde{\mu}_S = \mu_S^{1+\varsigma_S} \left(\frac{1-\tau_S}{1-\phi_S^{\iota}}\right)^{\varsigma_S}. \tag{2.13}$$

It is also possible to take into account a credit restriction for innovators in the small scale sector, who need to borrow if they desire to invest more than their wage in innovation. Following Aghion et al (2005), consider $v = w_t + d_t$ where d_t is debt that costs $R_t d_t$. Here R_t includes the costs of offering credit and the market power of the creditor. Financial development can be modelled with a parameter $c \in (0,1)$ that measures how expensive it is to defraud the creditor as a proportion of the credit. When the innovator is restricted,

$$v = w_t + d_t \leq \frac{1+R_t}{1+R_t - c} w_t.$$

After a similar derivation to the one above,

$$\frac{d\ln A_t}{dt} = \mu_S\left(\frac{(1+\sigma_S)A_{Lt}-A_{St}}{A_{Lt+\Delta t}}\right)^{\tau_S} \tag{2.14}$$

$$\times\left(\frac{1+R_t}{1+R_t-c}N_t^{\theta}\left[\frac{q_L}{\bar{\Xi}\bar{a}_t^{1-\kappa}}\right]^{\gamma_L}q_S^{\gamma_S}\right)^{1-\tau_S} \tag{2.15}$$

$$\frac{d}{dt}\ln\bar{A}_{St} \equiv \bar{H}_{St}(\bar{a}_t,R_t)=\mu_S\left(\frac{1+R_t}{1+R_t-c}\right)^{1-\tau_S}\left(N^{\theta}\frac{q_L^{\gamma_L}q_S^{\gamma_S}}{\bar{\Xi}^{\gamma_L}}\right)^{1-\tau_S} \tag{2.16}$$

$$\times(1+\sigma_S-\bar{a}_t)^{\tau_S}\bar{a}_t^{-(1-\kappa)\gamma_L(1-\tau_S)}, \tag{2.17}$$

so the effect is essentially equivalent to lowering μ_S.

2.3. **Steady state in the leading country.** For the leading country I write all variables with a bar on top, for example $\bar{A}_{jt}$, $j\in\{S,L\}$. Variables N, α_j, β_j, $j\in\{S,L\}$, χ_0, κ, and ξ are common to both countries.

Definition 3. *Define the relative state variables*

$$\bar{a}_t=\frac{\bar{A}_{St}}{\bar{A}_{Lt}},\quad \bar{q}=\frac{\bar{q}_S}{\bar{q}_L}.\blacksquare \tag{2.18}$$

In terms of these variables (1.37) can be written

$$\bar{\chi}_t=\chi_0[\bar{q}\bar{a}]^{-\kappa}=\bar{\Xi}\bar{a}_t^{-\kappa},\quad\text{with}\quad\bar{\Xi}=\chi_0\bar{q}^{-\kappa}. \tag{2.19}$$

Define also

$$\bar{\Omega}=N^{\theta}\bar{q}_S^{\gamma_S}\bar{q}_L^{\gamma_L}\bar{\Xi}^{-\gamma_L},\quad\bar{\Psi}(a_t)=\frac{\bar{\Xi}^{-\beta_L}\bar{a}_t^{\kappa(\gamma_L+\beta_L)+\gamma_S}}{\xi\beta_L\bar{\Xi}^{-\beta_L}\bar{a}_t^{\kappa\beta_L}+(1-\xi)\beta_S}, \tag{2.20}$$

$$\bar{H}_L(\bar{a}_t)\equiv\bar{\bar{\mu}}_L\left[(1+\kappa)\tfrac{\bar{\mathcal{L}}}{N}\bar{\Omega}\bar{\Psi}(\bar{a}_t)\right]^{\varsigma_L},\quad\bar{H}_S(\bar{a}_t)\equiv\bar{\bar{\mu}}_S\left[\bar{\Omega}\bar{a}_t^{-(1-\kappa)\gamma_L}\right]^{\varsigma_S}. \tag{2.21}$$

Income (1.30) can be written in the form

$$\frac{\bar{z}_t}{\bar{A}_{Lt}}=\frac{\bar{\Omega}\bar{a}_t^{\kappa\gamma_L+\gamma_S}\bar{\mathcal{L}}/N}{\xi\beta_L\bar{\Xi}^{-\beta_L}\bar{a}_t^{\kappa\beta_L}+(1-\xi)\beta_S}. \tag{2.22}$$

Substituting in (2.8), (2.12),

$$\frac{d}{dt}\ln\bar{A}_{Lt}=\sigma_L\bar{H}_L(\bar{a}_t),\quad\frac{d}{dt}\ln\bar{A}_{St}=(1+\sigma_S-\bar{a}_t)\bar{H}_S(\bar{a}_t),$$

Note that the case $\kappa=0$ $\bar{H}_L(\bar{a}_t)$ and $\bar{H}_S(\bar{a}_t)$ are simpler functions of $\bar{a}_t$,

$$\bar{H}_L(\bar{a}_t) \equiv \bar{\bar{\mu}}_L\left[\tfrac{\bar{\mathcal{L}}}{N}\bar{\Omega}\frac{\chi_0^{-\beta_L}}{\xi\beta_L\chi_0^{-\beta_L}+(1-\xi)\beta_S}\bar{a}_t^{\gamma_S}\right]^{\varsigma_L}, \tag{2.23}$$

$$\bar{H}_S(\bar{a}_t) \equiv \bar{\bar{\mu}}_S\bar{\Omega}^{\varsigma_S}\bar{a}_t^{-\gamma_L\varsigma_S}. \tag{2.24}$$

Hence the dynamics of the relative technological level between small and large scale production in the leading country are the following:

$$\frac{d}{dt}\ln\bar{a}_t=\bar{H}(\bar{a}_t)\equiv(1+\sigma_S-\bar{a}_t)\bar{H}_S(\bar{a}_t)-\sigma_L\bar{H}_L(\bar{a}_t). \tag{2.25}$$

Proposition 2. *Suppose $\bar{H}(1) < 0$, so the small scale sector of the leading country cannot overtake the large scale sector in technological level. Then the relative technological level $\bar{a}_t$ of the small to the large scale sector has a unique positive steady state, at which the growth rate of the leading sector's technological level is $\gamma = \sigma_L \bar{H}_L(\bar{a}^*)$, which is increasing in $\bar{a}^*$.*

Proof. Note the first term in $\bar{H}(\bar{a}_t)$ is a product of two decreasing positive terms, so is decreasing, while $H_{Lt}(\bar{a})$ is a monotonic function of a term of the form $T(a) = \frac{a^b}{ca^d+f}$ with $b > d$. Since $T'(a) = \frac{a^{b-1}}{(f+a^d c)^2}\left(bf + a^d c(b-d)\right) > 0$, $\bar{H}_L(\bar{a}_t)$ is increasing. Hence the difference $\bar{H}(\bar{a}_t)$ is decreasing. Since $\lim_{\bar{a}_t \to 0} \bar{H}_S(\bar{a}_t) = \infty$ and $\bar{H}_L(0) = 0$, $\lim_{\bar{a}_t \to 0} \bar{H}(\bar{a}_t) = \infty$. Hence since $\bar{H}(1) < 0$ there is a unique steady state $\bar{a}^* \in (0,1)$ given by $\bar{H}(\bar{a}^*) = 0$, or equivalently $\bar{H}_S(\bar{a}^*) = \bar{H}_L(\bar{a}^*)$. The statements on $\frac{d}{dt}\ln \bar{A}_{Lt}$ follow by definition and since $\bar{H}_L(\bar{a}_t)$ is increasing.■

2.4. **Steady state in the lagging country.** The lagging country's state is defined in relative terms with reference to the leading country as follows.

Definition 4. *Define the relative state variables*

$$a_t = \frac{A_{St}}{A_{Lt}}, \quad b_t = \frac{A_{Lt}}{\bar{A}_{Lt}}, \quad q = \frac{q_S}{q_L}. \blacksquare \tag{2.26}$$

Then (2.19) holds as before in terms of the lagging country variables. Substituting as before and letting H_j be the same functions as $\bar{H}_j$, but with the bars removed from the parameters, $j \in \{L, S\}$,

$$\frac{d}{dt}\ln A_{Lt} = (1 + \sigma_L - b_t) H_L(a_t), \quad \frac{d}{dt}\ln A_{St} = (1 + \sigma_S - a_t) H_S(a_t).$$

It now follows

$$\begin{aligned} \tfrac{d}{dt}\ln b_t = H^b(a_t, b_t, \bar{a}_t) &\equiv (1 + \sigma_L - b_t) H_L(a_t) - \sigma_L \bar{H}_L(\bar{a}_t), \\ \tfrac{d}{dt}\ln a_t = H(a_t, b_t, \bar{a}_t) &\equiv (1 + \sigma_S - a_t) H_S(a_t) - (1 + \sigma_L - b_t) H_L(a_t). \end{aligned} \tag{2.27}$$

Proposition 3. *Suppose the leading country is at a steady state $\bar{a}^* \in (0,1)$, that the lagging country's large scale sector cannot catch up with the leading country's, so $H^b(a_t, 1, \bar{a}^*) < 0$, and that the lagging country's small scale sector cannot catch up with its large scale sector, so $H(1, b_t, \bar{a}^*) < 0$ (where $0 \le a_t, b_t, \bar{a}^* \le 1$). Then 1) the relative technological level of the small to the large scale sector has a unique steady state $a^* \in (0,1)$, and 2) the relative technological level of the lagging country's large scale sector to the leading country's large scale sector has a unique steady state $b^* \in [0,1)$. There is divergence in levels between the lagging and leading large scale sectors if $b^* > 0$, and in growth rates if $b^* = 0$ and $(1+\sigma_L) H_L(a^*) < \sigma_L \bar{H}_L(\bar{a}^*)$.*

Proof. Let us examine the phase diagram of dynamical system (2.27). As before, $H_L(a_t)$ is positive and increasing, while $H_S(a_t)$ is positive and decreasing. Hence the locus $b_t = f^b(a_t)$ on which $0 = \frac{d}{dt}\ln b_t = H^b\left(a_t, f^b(a_t), \bar{a}_t\right) = 0$ on the (a_t, b_t) plane satisfies

$$f^{b\prime}(a_t) = \frac{(1 + \sigma_L - b_t) H_L'(a_t)}{H_L(a_t)} > 0, \tag{2.28}$$

and in addition $\frac{\partial}{\partial b_t}\frac{d\ln b_t}{dt} = -H_L(a_t) < 0$. The locus of $b_t = f^a(a_t)$ of $\frac{d}{dt}\ln a_t = H(a_t, b_t, \bar{a}_t) = 0$ satisfies

$$f^{a\prime}(a_t) = \frac{(1+\sigma_L - b_t)\,H_L'(a_t) + H_S(a_t) - (1+\sigma_S - a_t)\,H_S'(a_t)}{H_L(a_t)} > 0. \tag{2.29}$$

In addition $\frac{\partial}{\partial b_t}\frac{d\ln a_t}{dt} = H_L(a_t) > 0$. Moreover whenever two curves $H^b\left(a_t, f^b(a_t), \bar{a}_t\right) = const$ and $H(a_t, b_t, \bar{a}_t) = const$ intersect, their slopes satisfy

$$f^{a\prime}(a_t) - f^{b\prime}(a_t) = \frac{H_S(a_t) - (1+\sigma_S - a_t)\,H_S'(a_t)}{H_L(a_t)} > 0. \tag{2.30}$$

If there is an instersection of the two loci at which $\frac{d\ln b_t}{dt} = \frac{d\ln a_t}{dt}$, representing a steady state (a^*, b^*), it is unique. The combination of signs and slopes implies such a steady state is stable. In what cases might there not be an intersection? Since $\frac{d}{dt}\ln b_t$ is negative on $b_t = 1$, and $\frac{d}{dt}\ln a_t$ is negative on $a_t = 1$, it is only possible for an interior steady state not to exist if the boundary of the region $\left\{(a_t, b_t) : \frac{d}{dt}\ln a_t \le 0, \frac{d}{dt}\ln b_t \le 0\right\}$ intersects either of the axis. Note that for any given b_t, $H(a_t, b_t, \bar{a}^*) = 0$ has a unique solution as in the proof of Proposition 1, since this is a decreasing function of a_t with $\lim_{a_t\to 0} H(a_t, b_t, \bar{a}^*) = \infty$ and $H(1, b_t, \bar{a}^*) < 0$. Hence this boundary cannot intersect the b_t axis, so there is some a^* for which $H(a^*, 0, \bar{a}^*) = 0$ and also $\frac{d}{dt}\ln b_t < 0$, that is $(1+\sigma_L)\,H_L(a^*) < \sigma_L \bar{H}_L(\bar{a}^*)$. This is now a condition on the parameters and steady state values a^*, $\bar{a}^*$ that implies divergence in growth rates between the leading and lagging countries:

$$0 > \frac{d}{dt}\ln b_t = (1+\sigma_L)\,H_L(a^*) - \sigma_L \bar{H}_L(\bar{a}^*) \text{ implies } \frac{d}{dt}\ln A_{Lt} < \boldsymbol{\gamma}.\blacksquare \tag{2.31}$$

2.5. **Government incentives for innovation.** Is the private assignment of innovation resources optimal or can the government improve income growth by subsidizing innovation? Under what conditions can it pay for this by taxing profits?

In concordance with perfect myopic foresight, let the government maximize $Y_{t+\Delta t}$, deducting expenses in innovation incurred for increasing Y_t. Note that this optimization assumes exchange takes place in the presence of market power, so the question posed is only seeking a second best. Note also that in a steady state when growth is constant this is equivalent to maximizing the present value of income, $Y_t/\left(\delta - g_t^Y\right)$, where g_t^Y is the growth rate of Y_t.

The government maximizes

$$\max Y_{t+\Delta t} - [\xi v_L + (1-\xi)\, l_{St} v_S]\, N\Delta t, \tag{2.32}$$

where v_L is innovation investment in each large scale sector, v_S is innovation investment by each of the l_{St} workers in each small scale sector, subject to

$$\frac{A_{Lt+\Delta t} - A_{Lt}}{A_{Lt}} = \mu_L \left(\frac{(1+\sigma_L)\,\bar{A}_{Lt} - A_{Lt}}{\bar{A}_{Lt+\Delta t}}\Delta t\right)^{\tau_L} \left(\frac{v_L}{A_{Lt}}\Delta t\right)^{1-\tau_L}, \tag{2.33}$$

$$\frac{A_{St+\Delta t} - A_{St}}{A_{St}} = \mu_S \left(\frac{(1+\sigma_S)\,A_{Lt} - A_{St}}{A_{Lt+\Delta t}}\Delta t\right)^{\tau_S} \left(\frac{v_S}{A_{St}}\Delta t\right)^{1-\tau_S}. \tag{2.34}$$

These are the physical equations for technological change (2.1) and (2.9). In fact, taking the limit as $\Delta t \to 0$, the myopic optimization problem is equivalent to

$$\max \frac{dY_t}{dt} - N\left[\xi v_L + (1-\xi)\, l_{St} v_S\right] \tag{2.35}$$

subject to

$$\frac{d}{dt}\ln A_{Lt} = \mu_L \left(\frac{(1+\sigma_L)\,\bar{A}_{Lt} - A_{Lt}}{\bar{A}_{Lt}}\right)^{\tau_L} \left(\frac{v_L}{A_{Lt}}\right)^{1-\tau_L}, \tag{2.36}$$

$$\frac{d}{dt}\ln A_{St} = \mu_S \left(\frac{(1+\sigma_S)\,A_{Lt} - A_{St}}{A_{Lt}}\right)^{\tau_S} \left(\frac{v_S}{A_{St}}\right)^{1-\tau_S}. \tag{2.37}$$

Note that workers repeat innovation, or learning, so there is additional savings to be achieved by reducing such innovation repetition in the small scale sector.[8]

Now, using expression (1.35),

$$\frac{dY_t}{dt} = Y_t\left[\kappa \frac{\Upsilon'(\chi_t)}{\Upsilon(\chi_t)} \frac{d}{dt}\ln\left(\frac{A_{Lt}}{A_{St}}\right) + \frac{d}{dt}\ln\left(A_{Lt}^{\gamma_L} A_{St}^{\gamma_S}\right)\right]. \tag{2.38}$$

The first order conditions for (2.35) are:

$$0 = \frac{\partial \frac{dY_t}{dt}}{\partial v_L} - N\xi \Leftrightarrow \tag{2.39}$$

$$N\xi = \left[\frac{(1-\tau_L)\kappa Y_t}{v_L}\frac{\Upsilon'(\chi_t)}{\Upsilon(\chi_t)} + \frac{\gamma_L(1-\tau_L)Y_t}{v_L}\right]\mu_L\left(\frac{(1+\sigma_L)\bar{A}_{Lt}-A_{Lt}}{\bar{A}_{Lt}}\right)^{\tau_L}\left(\frac{v_L}{A_{Lt}}\right)^{1-\tau_L} \tag{2.40}$$

$$0 = \frac{\partial \frac{dY_t}{dt}}{\partial v_S} - N(1-\xi)\, l_{St} \Leftrightarrow \tag{2.41}$$

$$N(1-\xi)\, l_{St} = \left[-\frac{(1-\tau_S)\kappa Y_t}{v_S}\frac{\Upsilon'(\chi_t)}{\Upsilon(\chi_t)} + \frac{\gamma_S(1-\tau_S)Y_t}{v_S}\right]\mu_S\left(\frac{(1+\sigma_S)A_{Lt}-A_{St}}{\bar{A}_{Lt}}\right)^{\tau_S}\left(\frac{v_S}{A_{St}}\right)^{1-\tau_S} \tag{2.42}$$

Hence the government would assign innovation expenditures as follows:

$$\frac{v_L}{A_{Lt}} = \left(\mu_L \frac{(1-\tau_L)Y_t}{A_{Lt}}\left[\frac{\kappa}{N\xi}\frac{\Upsilon'(\chi_t)}{\Upsilon(\chi_t)} + \frac{\gamma_L}{N\xi}\right]\right)^{\frac{1}{\tau L}} \frac{(1+\sigma_L)\bar{A}_{Lt}-A_{Lt}}{\bar{A}_{Lt}} \tag{2.43}$$

$$\frac{v_S}{A_{St}} = \left(\mu_S \frac{(1-\tau_S)Y_t}{l_{St}A_{St}}\left[-\frac{\kappa}{(1-\xi)N}\frac{\Upsilon'(\chi_t)}{\Upsilon(\chi_t)} + \frac{\gamma_S}{N(1-\xi)}\right]\right)^{\frac{1}{\tau_S}} \frac{(1+\sigma_S)A_{Lt}-A_{St}}{\bar{A}_{Lt}} \tag{2.44}$$

When these are compared to (2.5) and (2.11), once $\Delta t \to 0$, the condition for obtaining the same resource assignment for innovation is:

$$\frac{\beta_L\left(1-\phi_L^{\pi}\right)(1+\kappa)}{\left(1-\phi_L^{\iota}\right)\chi_t^{\beta_L}}\frac{z_t}{A_{Lt}} = \frac{Y_t}{A_{Lt}}\left[\frac{\kappa}{N\xi}\frac{\Upsilon'(\chi_t)}{\Upsilon(\chi_t)} + \frac{\gamma_L}{N\xi}\right], \tag{2.45}$$

$$\frac{w_t}{\left(1-\phi_S^{\iota}\right)A_{St}} = \frac{Y_t}{l_{St}A_{St}}\left[-\frac{\kappa}{(1-\xi)N}\frac{\Upsilon'(\chi_t)}{\Upsilon(\chi_t)} + \frac{\gamma_S}{N(1-\xi)}\right]. \tag{2.46}$$

[8] The equivakence in perfect myopic foresight of maximizing profits at $t+\Delta t$ and taking the limit as $\Delta t \to 0$, or maximizing the rate of change of profits is proved by Mayer-Foulkes (2014).

These equalities are possible if:

$$\begin{aligned}
(2.47)\qquad \frac{1-\phi_L^{\pi}}{1-\phi_L^{\iota}} &= \frac{Y_t}{\chi_t^{-\beta_L}\xi N z_t}\frac{\gamma_L+\kappa\frac{\Upsilon'(\chi_t)}{\Upsilon(\chi_t)}}{\beta_L(1+\kappa)} \\
(2.48)\qquad &= \frac{(1-\xi)\beta_S+\xi\left(1-\alpha_L\chi_t^{-\beta_L}\right)}{\xi\chi_t^{-\beta_L}}\frac{\gamma_L+\kappa\frac{\Upsilon'(\chi_t)}{\Upsilon(\chi_t)}}{\beta_L(1+\kappa)} \\
(2.49)\qquad &\equiv \varphi_L,
\end{aligned}$$

where (1.34) was used. φ_L is value $\frac{1-\phi_L^{\pi}}{1-\phi_L^{\iota}}$ must take.

Using (1.34) again, as well as $w_t l_{St} = \beta_S z_t$,

$$\begin{aligned}
\frac{1}{1-\phi_S^{\iota}} &= \frac{Y_t}{N(1-\xi)\,l_{St}w_t}\left[\gamma_S-\kappa\frac{\Upsilon'(\chi_t)}{\Upsilon(\chi_t)}\right] \\
&= \frac{1-\xi\alpha_L\chi_t^{-\beta_L}-(1-\xi)\alpha_S}{(1-\xi)\beta_S}\left[\gamma_S-\kappa\frac{\Upsilon'(\chi_t)}{\Upsilon(\chi_t)}\right] \\
(2.50)\qquad &= \frac{(1-\xi)\beta_S+\xi\left(1-\alpha_L\chi_t^{-\beta_L}\right)}{(1-\xi)\beta_S}\left[\gamma_S-\kappa\frac{\Upsilon'(\chi_t)}{\Upsilon(\chi_t)}\right] \\
&\equiv \varphi_S;
\end{aligned}$$

where now φ_S is the value $\frac{1}{1-\phi_S^{\iota}}$ must take.

The following efficiency results for appropriate government incentives for innovation can now be stated, partly restricted to $\kappa = 0$ for simplicity.

Theorem 4. *Government taxes and subsidies can improve innovation incentives as follows.*

1) As market power tends to zero, when $\chi_0 \to 1$ and $\kappa \to 0$, privately assigned innovation tends to efficiency.

2) When the market power tax is applied, as $\varkappa_0 \to 1$, case 1) is approached in the limit.

3) If $\chi_t > 1$ is constant, and $\kappa = 0$, both values φ_L and φ_S are greater than 1. Thus innovation subsidies ϕ_L^{ι} and ϕ_S^{ι} both have positive optimal values. Also, in this case optimal innovation subsidy rates are higher than optimal taxes in the large scale sector, $\phi_L^{\iota} > \phi_L^{\pi}$, and higher than optimal innovation subsidy rates in the small scale sector, $\phi_L^{\iota} > \phi_S^{\iota}$.

4) When $\kappa > 0$, higher incentives to increase market power through innovation reduce the subsidies needed for innovation in the large scale sector and increase them in the small scale sector.

5) Suppose that large scale sector profits are quantitatively higher that optimal innovation costs for both sectors. Then taxes and subsidies ϕ_L^{π}, ϕ_L^{ι}, $\phi_S^{\iota} \in (0,1)$ exist for which the government's budget is balanced and innovation is optimal. If profits are not that high, a lump sum tax on wages is needed to obtain optimal innovation with a balanced budget.

Proof. 1) When $\chi_0 \to 1$, $\kappa \to 0$, assignments approach the case given by $\chi_t = 1$, $\kappa = 0$,

$$\begin{aligned}
(2.51)\qquad \frac{1-\phi_L^{\pi}}{1-\phi_L^{\iota}} &= \frac{1-\xi\alpha_L-(1-\xi)\alpha_S}{\xi}\frac{\gamma_L}{\beta_L} = \frac{1-\xi\alpha_L-(1-\xi)\alpha_S}{\beta_L\xi+\beta_S(1-\xi)} = 1, \\
(2.52)\qquad \frac{1}{1-\phi_S^{\iota}} &= 1,
\end{aligned}$$

so innovation is efficient.

2) When the incentives of a market power tax hold, $\chi_t^{\beta_L}$ is replaced by $\varkappa$, and thus in effect κ is replaced by zero. Thus in the limit the previous case applies.

3) In the case $\kappa = 0$, $\chi_0 > 1$,

$$\frac{1-\phi_L^{\pi}}{1-\phi_L^{\iota}} = \frac{\beta_L \xi + \beta_S (1-\xi) + \xi \alpha_L \left(1 - \chi_t^{-\beta_L}\right)}{\left(\beta_L \xi + \beta_S (1-\xi)\right) \chi_t^{-\beta_L}} = \frac{\xi \alpha_L \left(\chi_t^{\beta_L} - 1\right)}{\beta_L \xi + \beta_S (1-\xi)} + \chi_t^{\beta_L} > 1, \tag{2.53}$$

$$\frac{1}{1-\phi_S^{\iota}} = \frac{\beta_S (1-\xi) + \xi \left(1 - \alpha_L \chi_t^{-\beta_L}\right)}{\beta_L \xi + \beta_S (1-\xi)} = 1 + \frac{\xi \alpha_L \left(1 - \chi_t^{-\beta_L}\right)}{\beta_L \xi + \beta_S (1-\xi)} > 1. \tag{2.54}$$

Also, in this case $\frac{\chi_t^{\beta_L}}{1-\phi_S^{\iota}} = \frac{1-\phi_L^{\pi}}{1-\phi_L^{\iota}}$. Hence

$$\frac{1-\phi_L^{\pi}}{1-\phi_L^{\iota}} = \frac{\chi_t^{\beta_L}}{1-\phi_S^{\iota}} > 1 \Leftrightarrow \phi_L^{\iota} > \phi_L^{\pi}, \quad \frac{1-\phi_S^{\iota}}{1-\phi_L^{\iota}} = \frac{\chi_t^{\beta_L}}{1-\phi_L^{\pi}} > 1 \Leftrightarrow \phi_L^{\iota} > \phi_S^{\iota}.$$

4) It is easy to verify, based on $\Upsilon'(\chi_t) < 0$ (Proposition 1), that $\frac{d\varphi_L}{d\kappa} < 0$ and $\frac{d\varphi_S}{d\kappa} > 0$.

5) Observe that for any given value $\varphi_L > 1$, the function $\phi_L^{\pi} = f(\phi_L^{\iota}) = 1 - \varphi_L + \varphi_L \phi_L^{\iota}$ (for which $\frac{1-\phi_L^{\pi}}{1-\phi_L^{\iota}} = \varphi_L$) satisfies $f(\frac{\varphi_L - 1}{\varphi_L}) = 0$, $f(1) = 1$ and $f'(\phi_L^{\iota}) = \varphi_L > 1$. Setting also $\frac{1}{1-\phi_S^{\iota}} = \varphi_S$, the government surplus or deficit in establishing taxes and subsidies ϕ_L^{π}, ϕ_L^{ι}, ϕ_S^{ι} is given by

$$G(\phi_L^{\iota}) = N\xi f(\phi_L^{\iota})(1 - \chi_t^{-\beta_L}) z_t - N\left[\xi \phi_L^{\iota} v_L + (1-\xi) l_{St} \phi_S^{\iota} v_S\right]. \tag{2.55}$$

Let us evaluate this government surplus or deficit at $\phi_L^{\iota} = \frac{\varphi_L - 1}{\varphi_L}$ and $\phi_L^{\iota} = 1$. In the first case $\phi_L^{\pi} = 0$, while ϕ_L^{ι}, $\phi_S^{\iota} > 0$, so $G(\frac{\varphi_L - 1}{\varphi_L}) < 0$. In the second case

$$G(1) = N\xi(1 - \chi_t^{-\beta_L}) z_t - N\xi v_L - N(1-\xi) l_{St} \phi_S^{\iota} v_S \tag{2.56}$$

$$> N\xi(1 - \chi_t^{-\beta_L}) z_t - N\xi v_L - N(1-\xi) l_{St} v_S. \tag{2.57}$$

Since this quantity, aggregate profits minus innovation costs, is positive by assumption,

$$G'(\phi_L^{\iota}) = N\xi \varphi_L (1 - \chi_t^{-\beta_L}) z_t - N\xi v_L \tag{2.58}$$

$$> N\xi(1 - \chi_t^{-\beta_L}) z_t - N\xi v_L - N(1-\xi) l_{St} v_S \geq 0 \tag{2.59}$$

by the same assumption. Hence by the intermediate value theorem there exists $\phi_L^{\iota} \in (\frac{\varphi_L - 1}{\varphi_L}, 1)$ for which the government budget is balanced. At this value ϕ_L^{π}, ϕ_L^{ι}, $\phi_S^{\iota} \in (0,1)$. If instead $G(1) < 0$ a lump sum tax on wages is needed to obtain optimal innovation with a balanced budget.■

2.6. **Some comparisons between leading and lagging countries.** In the present model, differences between leading and lagging countries can derive from differences in the parameters, mainly q_L, q_S, χ_0, κ, ξ, N, μ_L, μ_S. For the purpose of the following examples, I concentrate only in differences in fixed productivity effect q_L, q_S, including institutional effects, expressed through Ω, see (2.20). Ω itself, considered from the static point of view, is an institutional measure that is higher the higher the fixed productivity effects q_L, q_S, and the lower the market power χ_t of the large scale sector.

Proposition 4. *Suppose the only parameter difference between the leading and lagging countries is $\frac{\Omega}{\bar{\Omega}}$. The steady states a^*, b^* of the lagging country can be considered as functions $a^* = a^*(\bar{a}^*, \frac{\Omega}{\bar{\Omega}})$ and $b^* = b^*(\bar{a}^*, \frac{\Omega}{\bar{\Omega}})$ of the steady state $\bar{a}^*$ of the leading country as well of the relative institutional development $\frac{\Omega}{\bar{\Omega}}$ of the lagging country. Then steady states a^* and b^* are increasing in both parameters,*

$$\frac{\partial a^*}{\partial \frac{\Omega}{\bar{\Omega}}} > 0, \quad \frac{\partial a^*}{\partial \bar{a}^*} > 0, \quad \frac{\partial b^*}{\partial \frac{\Omega}{\bar{\Omega}}} > 0, \quad \frac{\partial b^*}{\partial \bar{a}^*} > 0. \tag{2.60}$$

Proof. 1) At the steady state,

$$\begin{aligned} (1+\sigma_L - b^*)\, H_L(a^*) &= \sigma_L \bar{H}_L(\bar{a}^*), \\ (1+\sigma_S - a^*)\, H_S(a^*) &= \sigma_L \bar{H}_L(\bar{a}^*), \\ (1+\sigma_S - \bar{a}^*)\, \bar{H}_S(\bar{a}^*) &= \sigma_L \bar{H}_L(\bar{a}^*). \end{aligned} \tag{2.61}$$

Since $\mu_S = \bar{\mu}_S$, the second two equations imply

$$(1+\sigma_S - a^*)\, \Omega^{\varsigma_S} a^{*-(1-\kappa)\gamma_L \varsigma_S} = (1+\sigma_S - \bar{a}^*)\, \bar{\Omega}^{\varsigma_S} \bar{a}^{*-(1-\kappa)\gamma_L \varsigma_S}, \tag{2.62}$$

$$\text{or} \quad h(a^*) \left(\frac{\Omega}{\bar{\Omega}}\right)^{\varsigma_S} = h(\bar{a}^*), \tag{2.63}$$

where $h(a) = (1+\sigma_S - a)\, a^{-(1-\kappa)\gamma_L \varsigma_S}$ is a decreasing function. Let $a^* = a^*(\bar{a}^*, \frac{\Omega}{\bar{\Omega}})$ be the implicit function for a^* in terms of $\bar{a}^*$ and $\frac{\Omega}{\bar{\Omega}}$. Then the steady state a^* is increasing in both $\bar{a}^*$ and $\frac{\Omega}{\bar{\Omega}}$,

$$\frac{\partial a^*}{\partial \frac{\Omega}{\bar{\Omega}}} = -\frac{\varsigma_S h(a^*)}{h'(a^*)\, \frac{\Omega}{\bar{\Omega}}} > 0, \quad \frac{\partial a^*}{\partial \bar{a}^*} = \frac{h'(\bar{a}^*)}{h'(a^*) \left(\frac{\Omega}{\bar{\Omega}}\right)^{\varsigma_S}} > 0. \tag{2.64}$$

Now from the first equation in (2.61) we can write

$$b^* = 1 + \sigma_L - \frac{\sigma_L \bar{H}_L(\bar{a}^*)}{H_L\left(a^*(\bar{a}^*, \frac{\Omega}{\bar{\Omega}})\right)}. \tag{2.65}$$

Since H_L is increasing it follows that $\frac{\partial b^*}{\partial \frac{\Omega}{\bar{\Omega}}} > 0$, $\frac{\partial b^*}{\partial \bar{a}^*} > 0$.■

The result $\frac{\partial a^*}{\partial (\Omega/\bar{\Omega})} > 0$ implies that in a lagging country the small scale sector is relatively more backward than in leading countries. This can be understood as an explanation of the backwardness of the informal sector, as part of the small scale sector, in this case for fixed productivity effects independent of any argument related to tax evasion.

For an example of a comparison between innovation assignment in leading and lagging countries, with $\kappa = 0$, let us investigate the impact of an institutional difference $q_S^{\gamma_S} q_L^{\gamma_L} < \bar{q}_S^{\gamma_S} \bar{q}_L^{\gamma_L}$ on the optimal allocation of innovation investment in the large and small scale sectors, with all other parameters the same.

Proposition 5. *Suppose that $q_S^{\gamma_S} q_L^{\gamma_L} < \bar{q}_S^{\gamma_S} \bar{q}_L^{\gamma_L}$, and $\kappa = 0$. The lagging to leading country ratios of the optimal innovation investment allocations in the large and small scale sectors, compared each to their own technological levels, are the following:*

$$\frac{\frac{v_{Lt}^*}{A_{Lt}^*}}{\frac{\bar{v}_{Lt}^*}{\bar{A}_{Lt}^*}} = \frac{\Omega}{\bar{\Omega}} \left(\frac{a^*}{\bar{a}^*}\right)^{-\gamma_S}, \quad \frac{\frac{v_{St}^*}{A_{St}^*}}{\frac{\bar{v}_{St}^*}{\bar{A}_{St}^*}} = \frac{\Omega}{\bar{\Omega}} \left(\frac{a^*}{\bar{a}^*}\right)^{-\gamma_L}. \tag{2.66}$$

This holds for both the private and public allocations. Hence

$$\frac{\frac{v_{St}^*}{A_{St}^*}}{\frac{\bar{v}_{St}^*}{A_{St}^*}} / \frac{\frac{v_{Lt}^*}{A_{Lt}^*}}{\frac{\bar{v}_{Lt}^*}{A_{Lt}^*}} = \left(\frac{a^*}{\bar{a}^*}\right)^{\gamma_S - \gamma_L}. \tag{2.67}$$

Proof. 1) In the case $\chi_t = 1$, $\kappa = 0$,

$$\frac{Y_t}{A_{Lt}} = \Upsilon(\chi_0)\,\chi_0^{\gamma_L}\Omega\mathcal{L}a^{*\gamma_S}, \quad \frac{w_t}{A_{St}} = \Omega a^{*-\gamma_L}. \tag{2.68}$$

Hence, writing stars on government optimal trajectories at steady state,

$$\frac{\frac{v_{Lt}^*}{A_{Lt}^*}}{\frac{\bar{v}_{Lt}^*}{A_{Lt}^*}} = \frac{\left(\mu_L \frac{(1-\tau_L)Y_t}{A_{Lt}}\left[\frac{\kappa}{N\xi}\frac{\Upsilon'(\chi_t)}{\Upsilon(\chi_t)} + \frac{\gamma_L}{N\xi}\right]\right)^{\frac{1}{\tau_L}}(1+\sigma_L - b^*)}{\left(\bar{\mu}_L \frac{(1-\tau_L)\bar{Y}_t}{A_{Lt}}\left[\frac{\kappa}{N\xi}\frac{\Upsilon'(\bar{\chi}_t)}{\Upsilon(\bar{\chi}_t)} + \frac{\gamma_L}{N\xi}\right]\right)^{\frac{1}{\tau_L}}\sigma_L} \tag{2.69}$$

$$= \left(\frac{\Omega a^{*\gamma_S}}{\bar{\Omega}\bar{a}^{*\gamma_S}}\right)^{\frac{1}{\tau_L}}\frac{\bar{H}_L(\bar{a}^*)}{H_L(a^*)} = \frac{\Omega a^{*\gamma_S}}{\bar{\Omega}\bar{a}^{*\gamma_S}}, \tag{2.70}$$

where the σ_L's are eliminated using the first equation in (2.61), and since

$$\frac{\bar{H}_L(\bar{a}^*)}{H_L(a^*)} = \frac{\bar{\tilde{\mu}}_L\left((1+\kappa)\frac{\bar{\mathcal{L}}}{N}\bar{\Omega}\right)^{\varsigma_L}\left(\frac{\chi_0^{-\beta_L}}{\xi\beta_L\chi_0^{-\beta_L}+(1-\xi)\beta_S}\right)^{\varsigma_L}\bar{a}^{*\gamma_S\varsigma_L}}{\tilde{\mu}_L\left((1+\kappa)\frac{\mathcal{L}}{N}\Omega\right)^{\varsigma_L}\left(\frac{\chi_0^{-\beta_L}}{\xi\beta_L\chi_0^{-\beta_L}+(1-\xi)\beta_S}\right)^{\varsigma_L}a^{*\gamma_S\varsigma_L}} = \frac{\bar{\Omega}^{\varsigma_L}\bar{a}^{*\gamma_S\varsigma_L}}{\Omega^{\varsigma_L}a^{*\gamma_S\varsigma_L}}, \tag{2.71}$$

and also $\frac{1}{\tau_L} - \varsigma_L = 1$. The private optimal trajectories are the same because other than constants involved in the comparison of (2.5) and (2.43) sales z_t equal aggregate income Y_t.

On the other hand, since in this case $\bar{H}_S(a_t) = \bar{\tilde{\mu}}_S\bar{\Omega}^{\varsigma_S}\bar{a}_t^{-\gamma_L\varsigma_S}$, it follows from (2.61) that

$$\frac{1+\sigma_S - a^*}{1+\sigma_S - \bar{a}^*} = \frac{\bar{H}_S(\bar{a}^*)}{H_S(a^*)} = \frac{\bar{\Omega}^{\varsigma_S}}{\Omega^{\varsigma_S}}\left(\frac{a^*}{\bar{a}^*}\right)^{\gamma_L\varsigma_S}. \tag{2.72}$$

Applying this to (2.11), since from (1.22) and (2.20) $\frac{w_t}{A_{St}} = \Omega a^{*-\gamma_L}$ and similarly for the leading country, in the case of private optimal trajectories

$$\frac{\frac{v_{St}^*}{A_{St}^*}}{\frac{\bar{v}_{St}^*}{A_{St}^*}} = \frac{\left(\frac{\hat{\mu}_S w_t}{A_{St}}\right)^{\frac{1}{\tau_S}}(1+\sigma_S - a^*)}{\left(\frac{\hat{\mu}_S \bar{w}_t}{A_{St}}\right)^{\frac{1}{\tau_S}}(1+\sigma_S - \bar{a}^*)} = \left(\frac{\Omega}{\bar{\Omega}}\right)^{\frac{1}{\tau_S}-\varsigma_S}\left(\frac{a^*}{\bar{a}^*}\right)^{-\gamma_L\frac{1}{\tau_S}+\gamma_L\varsigma_S} = \frac{\Omega}{\bar{\Omega}}\left(\frac{a^*}{\bar{a}^*}\right)^{-\gamma_L} \tag{2.73}$$

The corresponding ratio of the government innovation assignments is the same because given the parameters held constant $\frac{Y_t}{w_t} = \frac{\bar{Y}_t}{\bar{w}_t}$, which makes the relevant ratios of conditions (2.11) and (2.44) equal. Finally, (2.67) follows by division from (2.66).■

If there are many large scale sectors, so $\beta_L\xi > \beta_S(1-\xi)$ and therefore $\gamma_L > \gamma_S$, and taking into account $\frac{\partial a^*}{\partial(\Omega/\bar{\Omega})} > 0$, see (2.60), (2.67) implies that relative to their own technological level, there is relatively less investment in small scale sectors in lagging countries than in leading countries. This can be understood to go together with the backwardness of the informal sector, as part of the small scale sector, independently of any argument related to tax evasion.

If instead the large scale sector is small, as in initial stages of industrialization, it would be the large scale sector which would lag further behind.

The following proposition shows how deeply innovation forces determine the steady state. Given the available innovation functions, relative technological levels and labor assignment between sectors will conform to parallel growth between them.

Proposition 6. *Suppose that* $q_S^{\gamma_S} q_L^{\gamma_L} < \bar{q}_S^{\gamma_S} \bar{q}_L^{\gamma_L}$, and $\kappa = 0$. . *The lagging to leading country ratios of aggregate output to the optimal private and public innovation investment allocations in the large and small scale sectors, are equal:*

$$\frac{\frac{Y_t}{v_{Lt}^*}}{\frac{\bar{Y}_t}{\bar{v}_{Lt}^*}} = \frac{\frac{Y_t}{v_{St}^*}}{\frac{\bar{Y}_t}{\bar{v}_{St}^*}} = 1. \tag{2.74}$$

Hence the ratio of the optimal private and public innovation investment allocation between the small scale and large scale sectors is equal in leading and lagging countries, and between leading and lagging countries the ratio of income is the same as the ratio of optimal sectoral innovation investments in each country:

$$\frac{v_{St}^*}{v_{Lt}^*} = \frac{\bar{v}_{St}^*}{\bar{v}_{Lt}^*}, \quad \text{and} \quad \frac{Y_t}{\bar{Y}_t} = \frac{v_{Lt}^*}{\bar{v}_{Lt}^*} = \frac{v_{St}^*}{\bar{v}_{St}^*} \tag{2.75}$$

Proof. Using (1.35), since $\frac{Y_t}{A_{Lt}} = \Upsilon(\chi_0) \mathcal{L} \chi_0^{-\gamma_L} \Omega a_t^{\gamma_S}$,

$$\frac{\frac{A_{Lt}^*}{Y_t}}{\frac{\bar{A}_{Lt}^*}{\bar{Y}_t}} = \frac{\bar{\Omega}\bar{a}^{*\gamma_S}}{\Omega a_t^{*\gamma_S}}, \quad \frac{\frac{A_{St}^*}{Y_t}}{\frac{\bar{A}_{St}^*}{\bar{Y}_t}} = \frac{\bar{\Omega}\bar{a}^{*-\gamma L}}{\Omega a_t^{*-\gamma_L}}. \tag{2.76}$$

Multiplying each of these by each equation in (2.66) we get the reciprocal of (2.74).■

Factual deviations from equalities (2.74), (2.75) point to additional differences between countries and sectors, depending on other variables, opening possibilities for empirical evaluation.

2.7. **Expansion of mass production.** In what follows I investigate the dependence of the steady states on the size ξ of the large scale sector. I consider now that this size may be different in the leading and lagging countries, so there are two variables, $\bar{\xi}$ and ξ. These effects all work through the impact of ξ on $\frac{z_t}{A_{Lt}}$ and therefore on innovation, and not through technological externalities.

Proposition 7. *Suppose that the number* $\bar{\xi}$ *of mass production sectors in the leading country increases. Then the growth rate* $\boldsymbol{\gamma}$ *of the mass production technological level rises while the relative technological levels* $\bar{a}^*$, b^*, a^* *in all other production sectors fall behind,*

$$\frac{\partial \boldsymbol{\gamma}}{\partial \bar{\xi}} > 0, \quad \frac{\partial \bar{a}^*}{\partial \bar{\xi}} < 0, \quad \frac{\partial b^*}{\partial \bar{\xi}} < 0, \quad \frac{\partial a^*}{\partial \bar{\xi}} < 0. \tag{2.77}$$

Suppose that the number ξ *of mass production sectors in the lagging country increases. Then the relative technological levels* b^* *of the mass production sector increases and the relative level* a^* *of small scale sector remains constant*

$$\frac{\partial b^*}{\partial \xi} > 0, \quad \frac{\partial a^*}{\partial \xi} = 0. \tag{2.78}$$

Proof. 1) In both the leading and lagging countries the parameters and functions χ_t, Ξ, Ω, Ω_L, Ω_S, Υ, are independent of ξ, see (2.21). Hence the only dependence on ξ of the functions H_L, H_S

for the leading and lagging countries is through the function Ψ. Observe

$$\frac{\partial \Psi(a_t)}{\partial \xi} = -\varsigma_L \Psi(a_t) \frac{\beta_L \chi_t^{-\beta_L} - \beta_S}{\xi \beta_L \chi_t^{-\beta_L} + (1-\xi)\beta_S} > 0, \tag{2.79}$$

because $\beta_L \Xi^{-\beta_L} a_t^{\kappa \beta_L} - \beta_S = \beta_L \chi_t^{-\beta_L} - \beta_S < 0$, since $\beta_L < \beta_S$ and $\chi_t > 1$. Hence $\frac{\partial H_L(a_t)}{\partial \xi} > 0$. Suppose $\bar{\xi}$ increases in the leading country. Then, since $\bar{H}_S' < 0$, $\bar{H}_L' > 0$,

$$\frac{\partial \bar{a}^*}{\partial \bar{\xi}} = \frac{\sigma_L \frac{\partial \bar{H}_L(\bar{a}^*)}{\partial \bar{\xi}}}{-\bar{H}_S(\bar{a}^*) + (1+\sigma_S - \bar{a}^*)\bar{H}_S'(\bar{a}^*) - \sigma_L \bar{H}_L'(\bar{a}^*)} < 0. \tag{2.80}$$

This also has an impact on the lagging country. Differentiating the first two equations in (2.61),

$$\begin{array}{c} -\frac{\partial b^*}{\partial \bar{\xi}} H_L(a^*) + (1+\sigma_L - b^*) H_L'(a^*) \frac{\partial a^*}{\partial \bar{\xi}} = \sigma_L \bar{H}_L'(\bar{a}^*) \frac{\partial \bar{a}^*}{\partial \bar{\xi}} + \sigma_L \frac{\partial \bar{H}_L(\bar{a}^*)}{\partial \bar{\xi}}, \\ -\frac{\partial a^*}{\partial \bar{\xi}} H_S(a^*) = \sigma_L \bar{H}_L'(\bar{a}^*) \frac{\partial \bar{a}^*}{\partial \bar{\xi}} + \sigma_L \frac{\partial \bar{H}_L(\bar{a}^*)}{\partial \bar{\xi}}. \end{array} \tag{2.81}$$

Note the RHS of the both lines, which is also $\frac{\partial \gamma}{\partial \bar{\xi}} = \frac{\partial}{\partial \bar{\xi}}\left[\sigma_L \bar{H}_L(\bar{a}^*)\right]$, equals

$$= \left[\frac{\sigma_L \bar{H}_L'(\bar{a}^*)}{-\bar{H}_S(\bar{a}^*) + (1+\sigma_S - \bar{a}^*)\bar{H}_S'(\bar{a}^*) - \sigma_L \bar{H}_L'(\bar{a}^*)} + 1\right] \sigma_L \frac{\partial \bar{H}_L(\bar{a}^*)}{\partial \bar{\xi}} \tag{2.82}$$

$$= \left[\frac{-\bar{H}_S(\bar{a}^*) + (1+\sigma_S - \bar{a}^*)\bar{H}_S'(\bar{a}^*)}{-\bar{H}_S(\bar{a}^*) + (1+\sigma_S - \bar{a}^*)\bar{H}_S'(\bar{a}^*) - \sigma_L \bar{H}_L'(\bar{a}^*)}\right] \sigma_L \frac{\partial \bar{H}_L(\bar{a}^*)}{\partial \bar{\xi}} > 0, \tag{2.83}$$

a positive term I call P. Hence from the second line in (2.81), $\frac{\partial a^*}{\partial \bar{\xi}} = -\frac{P}{H_S(a^*)} < 0$,and from the first,$\frac{\partial b^*}{\partial \bar{\xi}} = -[\frac{(1+\sigma_L - b^*)H_L'(a^*)}{H_S(a^*)} + 1]P/H_L(a^*) < 0$. Now suppose ξ increases in the lagging country only. Then

$$-\frac{\partial b^*}{\partial \xi} H_L(a^*) + (1+\sigma_L - b^*) \times \left[H_L'(a^*) \frac{\partial a^*}{\partial \xi} + \frac{\partial H_L(a^*)}{\partial \xi}\right] = 0, \tag{2.84}$$

$$\left[-H_S(a^*) + (1+\sigma_S - a^*) H_S'(a^*)\right] \frac{\partial a^*}{\partial \xi} = 0. \tag{2.85}$$

Since $H_S'(a^*) < 0$, it follows from the second equation that $\frac{\partial a^*}{\partial \xi} = 0$, and therefore from the first equation, $\frac{\partial b^*}{\partial \xi} = \frac{(1+\sigma_L - b^*)}{H_L(a^*)} \frac{\partial H_L(a^*)}{\partial \xi} > 0.$■

3.

Conclusions

The model constructed here allows evaluating the optimality properties of a market economy consisting of a mass production sector with market power and a competitive sector with small scale production. Aggregate net income is decreasing in the large scale sector market power, because higher prices for mass produced goods deviate resources from inputs to profits (Proposition 1). In addition (Theorem 1) aggregate profits and profits per sector are increasing in market power, while wages and aggregate wage participation are decreasing in market power. The aggregate wage to profit ratio is decreasing in market power and in the number of large-scale sectors. Employment intensity in the large scale sector is decreasing in market power, the opposite holding for the small scale sector. Next (Theorem 2), the impact on wages of an increase in the number of large scale sectors is decreasing in market power. When market power is high enough, wages can remain unaffected by the technological level of the large scale sector. An important result is that a government can reduce the inefficiencies due to market power by introducing a *market power tax* that encourages production rather than profit rates, thus reducing the diversion of resources to profits. Market power tax schedules exist that approximate the first best solution by establishing a maximum socially sanctioned profit *rate*. Moreover, in equilibrium the amount of tax levied is zero (Theorem 3).

Because market power is important as an innovation incentive, technological change is introduced in the model, including the analysis of both developed and underdeveloped countries. To this purpose I introduce, for simplicity, the concept of perfect myopic foresight into the infinitesimal future. The innovation inputs are the current state of knowledge, including installed technology; material inputs; and externalities from more advanced technologies. I assume the incumbent has a small innovation advantage. In each country the small scale sector is modeled as a recipient of innovation externalities from the large scale sector, and innovation externalities between countries occur between large scale sectors. First I establish the existence of technological steady states for both leading and lagging countries (Propositions 2 and 3). Leading countries determine the overall growth rate. Lagging countries may diverge in levels or in growth rates from leading countries. Next the efficiency properties of the two sector economy are analyzed for innovation (Theorem 4). The presence of market power makes innovation inefficient, and as market power decreases innovation tends to efficiency. Moreover, a market power tax, which can be defined about a socially sanctioned profit rate, makes the economy approach efficiency for small profit rates. As compared to a government innovation assignment maximizing the growth of income, the private assignment of innovation investment (also consistent with myopic perfect foresight) is deficient in both sectors, again because of the deviation of resources from mass produced innovation inputs to profits. Given fixed or endogenous levels of market power, the government can reach an optimum level of innovation with a balanced budget by using an appropriate combination of taxes on profits and subsidies on innovation, requiring in addition a lump sum tax on wages if the profit level is not high enough.

This will not, however optimize production. This policy for optimizing innovation can complement a market power tax, that can only approximate efficiency.

In addition to proving these results on the efficiency for a market economy with mass production for both developed and underdeveloped countries, the model gives some comparative results between developed and underdeveloped countries. For simplicity I keep to the case when a combination of productivity fixed effects pertaining to both sectors, including institutional effects, are the reason for underdevelopment (Proposition 4). In this case a lagging country's small scale sector is relatively more backward with respect to its large scale sector than a leading country's small scale sector. This gives an explanation for the prevalence in underdeveloped countries of excluded and informal sectors, which form part of the small scale sector (independently for example of a tax avoidance or other explanations). Improvements in the overall small sector innovation technology can improve lagging country's convergence in both sectors. The even further backwardness of lagging country's small scale sector is reflected in a relatively lower innovation investment allocation as compared to its technological levels (public or private, Proposition 5). Innovation is a strong determinant of the income steady state. When market power is constant, innovation levels in both sectors hold the same proportion to income (Proposition 6). Finally, if the number of large scale production sectors in the leading country increases, the growth rate of its technological level rises, but the relative technological levels $\bar{a}^*$, b^*, a^* in all other production sectors fall behind. If instead the number of large scale production sectors in the lagging country increases, then their relative technological level, as compared to the leading country, rises, while the relative level of the small scale sector remains constant (Proposition 7). Both of these statements refer to the impact of changes in material input availability for innovation, not to changes in externalities.

There are specific issues about technological change in the small scale sector that have not been addressed, such as the repetition of learning across firms in this sector, that can give rise to serious inefficiencies. These may warrant, for example, the existence of public knowledge systems supporting small sector innovation. What the model does make clear, though, is that wage levels and income in an economy with mass production depend on both of its sectors. The dependence on the small scale sector is stronger the stronger the market power of the large scale sector.

Pro-poor growth must place a special focus on promoting technologies in the small scale sector, as well on reducing market power in the large scale sector, so as to diminish the deviation of resources from mass produced inputs to profits.

Mass production appeared more than a Century after Adam Smith. Because mass production wields market power, its operation in a market economy leads to inefficiencies in production and innovation, and to lower wages. The model shows that *efficiency and equity, in production and innovation, are promoted together by reducing market power*. This is the essence of Adam Smith's insights on competition. However, when competition becomes a tool for market power, it is necessary for government to correct the distortions. Perhaps a market power tax, encouraging *production* rather than *profit rates*, can be useful. The challenge is to make mass production, the workhorse of modern wealth, equitable and truly responsive to pressing economic needs.

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