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JOSÉ CARLOS RAMÍREZ

Does Malthus's principle of population still have a practical purpose nowadays?

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Carretera México Toluca 3655, Col. Lomas de Santa Fe, 01210,
Álvaro Obregón, México DF, México.
www.cide.edu

Dirección de Publicaciones
publicaciones@cide.edu
Tel. 5081 4003

Abstract

By introducing a delay differential equation into a population model this paper attempts to show that an update on Malthus's principle of population requires using new mathematical analyses involving oscillations around a stable attractor. Acceptance of the fact that Malthusian dynamic equilibria are smooth and unique is a valid simplification for setting broad theoretical relationships between the population and means of subsistence, but it is not enough for understanding the potential relevance of the principle in dealing with current demographic problems. Results based on Mexico's demographic experience show that new dynamic approaches on the principle can be useful to describe recent aspects of the population's behavior in developing countries.

Resumen

El documento presenta una versión actualizada del principio de población de Malthus utilizando un modelo dinámico con rezagos capaz de simular las oscilaciones de las trayectorias en torno al atractor estable o estacionario. La idea con esta actualización es ampliar el horizonte de equilibrios posibles entre la población y los medios de subsistencia (ingreso o producto) de una sociedad con el fin de hacer útil el principio en el tratamiento de problemas demográficos modernos. Los resultados basados en la experiencia demográfica de México confirman, efectivamente, que el principio puede ser de mucha utilidad para explicar algunos aspectos de la conducta demográfica de algunos países de ingreso medio.

Introduction

In many ways, writing about Malthus is tantamount to raising the devil. The reason is that if one dares review any issue explored by the author's critics or apologists, then the effort will almost surely end in an ideological dispute. Malthus's writings, specifically his famous *Essay on the Principle of Population,*[1] are inseparable from scandal and political animosity, which is why nothing said about them, is ever entirely free of fanaticism. This has been the story of the *Essay* since its first publication in 1798 and given the expanding demographic complexity shared by countries with an unequal degree of development in an increasingly unified world, there are unlikely to be any changes in the immediate future. Today, the ideas in the *Essay* continue polarizing demographic opinions just as they did over 200 years ago, in other words, inspiring conservatives with dogmas of faith (right-wing politicians and international agency officials) or serving as an object of ridicule for liberals (leftists of every stripe).

But the difficulties of undertaking a new interpretation on Malthus's demographic works cannot merely be reduced to ideological differences. There are also theoretical reasons —two in particular— that have contributed to distorting the original ideas contained in the first two chapters of the *Essay* and therefore to creating false assumptions about Malthus's thought. The first of these is the clumsy systematization of the principle of population in any of the six editions of the *Essay,* revised by the author himself. According to Tudela (1998) and Davis (1998) the *principle* is explained so loosely in the *Essay* that it is impossible to rank the variables in order of importance within a theoretical body or to separate moral from scientific reasoning.[2]

The empirical proofs Malthus provides to support the omnipresence of the *principle* in many kinds of societies and which led him to write eight chapters (Chapter III to X) are superfluous and lack evidential validity, since in the absence of a theory, the examples neither confirm nor disprove anything (Davis, 1998: LIX). The explanation of the *principle* is rather enunciative, with a slightly prescriptive tone since, instead of creating a scientific structure that explains the existence of the *principle*, Malthus confines himself to declaring it as though it were a truth in itself and, in the final chapters of the

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[1] The full title of the 1st edition is *An Essay on the Principle of Population, as it affects the Future Improvement of Society with remarks on the Speculations of Mr. Godwin, M. Condorcet, and Other Writers.* Hereafter, we shall refer it simply as *Essay.*

[2] Malthus defines the *principle* as the constant tendency of all animated life to increase beyond its stock of means of subsistence. In doing so he sets three assumptions which are considered as fixed laws of the human nature: i) food is necessary to the existence of man ; ii) passion between sexes in inevitable and; iii) the power of population is indefinitely greater than the power in the earth to produce subsistence for man. "From these assumptions, Malthus came immediately to his famous ratios and his thesis that strong and constant forces must necessarily hold the superior power of population over subsistence in check" (Dooley, 1988: 200).

Essay, to showing the dangers of favoring social policies (such as the poor laws) that ignore its effects. As a result, Malthus's critics and followers alike have attempted to take advantage of the *Essay's* informal tone in order to freely interpret the meaning, functioning and consequences of the *principle.*

The second reason is the link between the *principle* and the equilibrium mechanisms, firstly of Ricardo's value theory and secondly of *steady state* economic models.[3] In his theory, Ricardo establishes the mediations whereby the population's trajectory and the rate of profit converge on the stationary state, using the *principle* as the basis of one of the most important laws in his conceptual scheme: the law of diminishing returns. Through this law, the principle acquired scientific status for the first time ever in the explanation of the progressive decline of the rate of profit into a stationary state which at the same time, "made it almost impervious to criticism" (Pasinetti, 1978: 111). But it also modified its original meaning. The insertion of Malthus's demographic ideas into Ricardian value theory gave the *principle* a stable vision that has no solid basis in the *Essay.* Ricardo's virtually automatic use of the *principle* as a mechanism for the long-term stabilization of the rate of profit is undoubtedly a result that is consistent with only a part of the *Essay*. And this is no small matter.

The subsequent destiny of the *principle* took a different path in aggregate demand and neo-classical models, since it went from being a determinant factor of supply (wage regulator) to one of demand (regulator of aggregate utility functions); from ruling in macroeconomic (general equilibrium) to microeconomic spheres (equilibrium in household economy) and from being measured by quantity (growth rates) to quality (different types of human capital).[4] In each case, the *principle* acquired a different status in economic analysis to such an extent that it is by no means easy nowadays to recognize its original meaning.

However, modifications have not altered the idea of convergence on a stable attractor bequeathed by Ricardo.[5] The inheritance of this idea by new economic growth models has perpetuated the half-truth that the *Essay* contains a stable, unique subsistence floor, with no differences in demographic patterns by social class or heterogeneous behaviors among the population.

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3 It is important to stress that stationary state and steady state are not exchangeable concepts. For an explanation on this point see Galor and Weil (2000).

4 For more information on these conceptual modifications of the *principle*, see Ramírez and Morelos (2002).

5 The most significant modifications include the causes of relative overpopulation and the sign of the relationship between the growth of means of subsistence (or income) and population growth. Whereas for Malthus and Ricardo relative overpopulation is mostly explained by macroeconomic factors that determine the difference in growth between the population and means of subsistence, for neo-classical models, it is nothing more than a negative externality derived from family decisions. Likewise, whereas for the former, the relationship between the two rates is positive, for the latter the relationship is negative since they assume that the higher (lower) the income levels, the more likely there are to be lower (higher) population growth levels (at least in developed economies).

This paper attempts to show that far from being the inevitable destination of all possible trajectories of the population and means of subsistence, stationary state is merely a limit concept in the long-term Malthusian analysis. The results discussed here show, as in Goodwin (1978) and Waterman (1987) for instance, that oscillations in those trajectories may delay their convergence on the stable attractor and therefore alter predicted equilibrium conditions. The importance of highlighting the causes that explain oscillations is crucial to having a more comprehensive idea of the demographic dynamics of Malthusian thought and as we shall see further on, of the practical implications of the *principle.*

The paper is divided into three sections. The first discusses the two main problems faced by a researcher when attempting to formalize Malthus's *principle* as the result of the different points of view on his dynamic interpretation. The second section begins with a dynamic model that includes a delay differential equation and continues with a simulation using data from Mexico to illustrate the oscillations suggested by Malthus. Lastly, the third section summarizes our conclusions by highlighting the importance of incorporating the study of oscillations into current demo-economic analyses with the aim of giving the *principle* a new and modern place in Economic Demography.

1. The problems in formulating a dynamic model on the principle

In the *Essay,* Malthus (1998) describes the race between food production and the number of humans that must be fed on the basis of a non-proportional numerical arrangement, in other words, whereas the former grows arithmetically, the latter does so geometrically, in the absence of checks. Thus any society, Malthus argues, is doomed to live at a subsistence minimum level in the long term.[6]

The means of representing this race over time is not unique or free of polemic. In fact, it is possible to distinguish two main issues over which authors disagree when it comes to interpreting the *principle* in dynamic environments: (a) modeling growth rates; and (b) attractors' conditions of stability and convergence.

[6] The population is kept at a subsistence minimum level by the action of preventive or positive checks which operate in any society to a greater or lesser extent. The former checks are related to birth control (forgoing early marriage or sexual abstinence) while the latter are related to various factors that reduce one's life expectancy (such as exposure to dangerous jobs, childhood malnutrition, epidemics, wars or famine). The effectiveness of both checks varies inversely to society's level of development. The higher this level, the lower the corrective power of the positive checks and the higher that of preventive checks. This situation is inverted in backward societies. The two types of checks can also be understood as the result of moral restriction, vice and misery (Malthus, 1998: 15).

1.1. Disagreements over growth rates equations (Malthus ratios)

In the first issue, differences begin with the way the *principle* is mathematically translated. On the one hand, there are authors that use one-dimensional equations to interpret its equilibrium analytically whereas on the other, there are authors that use differential equation systems to provide qualitative solutions for its dynamics. The former use linear models that try to keep as close as possible to the first two chapters of the *Essay* (Waterman, 1987, 1998; Samuelson, 1978); whereas the latter use non-linear models yet without following the canonical analyses too closely (Goodwin, 1978). [7] The result is a myriad of formalizations of the *principle* that is not necessarily homogeneous.

The most illustrative example of this diversity of formalizations is undoubtedly growth rate modeling. In order to clarify this point, we will consider the system of species reproduction originally proposed by Lotka (1973):

$$dm_i/dt = F(m_1, m_2, ..., Q, R), \ (i = 1,2,...,n) \quad (1)$$

Where m_i are the masses of the species and Q and R the parameters that denote, respectively, the nature of the various species and the set of their external conditions, such as the climate or the topographical features of their surroundings (Lotka, 1973).When $m = 1$, the system is reduced to a non-linear differential equation that expresses the components of the expansion of a species that grows without interference from other species:

$$\frac{dG(t)}{dt} = a_0 + a_1 G(t) + a_2 G^2(t) + ... + a_n G^n(t) \quad (2)$$

However, if we assume, like Malthus, that subsistence levels grow arithmetically, then (2) becomes:

$$\frac{dS(t)}{dt} = a_0 \quad (3)$$

$$S(t) = a_0 t + S_0 \quad (3.1)$$

[7] Between the two extremes are studies such as those by Pingle (2003) that use neoclassical categories to reinterpret the more classical analysis drawn up, for example, by Waterman (1987).

Where $G(t) \equiv S(t)$ are the means of subsistence measured in units of a certain foodstuff at the moment $t\,(t = 1,2,...,n)$; $a_0 = 1$ and $S(0) = S_0$.[8] Likewise, if we follow the geometric population series $P(t)$ presented in the *Essay,* then (2) must be arranged in such a way as to eliminate a_0 and all the terms higher than one in $G(t) \equiv P(t)$:[9]

$$\frac{dP(t)}{dt} = a_1 P(t) \tag{4}$$

$$P(t) = P_0 e^{a_1 t} \tag{4.1}$$

To obtain this way the famous *Malthus Law* (equation 4.1) in which a_1 is the reproductive rate.

Up to this point, the mathematical interpretation of rates does not pose a problem, since equations (3.1) and (4.1) accurately reflect the progressions described by Malthus for the population (2, 4, 16, 256...) and means of subsistence (1,2,3,4...). Problems arise when we incorporate checks or the law of diminishing returns into dynamic analysis. Is it possible to expect the same population growth pattern in the presence of checks? Are there changes in the progression of subsistences with the introduction of diminishing returns in production?

Waterman (1998: 575) replies that the only thing that can change are the values of growth equations but not their formal structure. Specifically, he states that if we define $gP(t) = a_1(w - s)$ as the population's growth rate depending on *per capita* wages (w) and *per capita* means of subsistence (s), then the scale of $gP(t)$ should be adjusted in such a way that it will grow more when $w > s$ than when $w = s$, since in the latter case, the population will be completely under the corrective action of checks.[10] Likewise, if we wish to adapt a (3.1) type equation to a situation in which the law of diminishing returns operates, all we have to do is to introduce a land-scarce Malthusian production function that includes a few doses of labor as capital to solve the problem. Thus, "if successive 'doses' of the composite factor are applied to

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8 All the other coefficients different from a_0 disappear because there is no other way of satisfying Malthus's numerical rule about the growth of subsistences.

9 In human and animal populations $a_0 = 0$ because there must be at least one female to create a population (Lotka, 1969: 54).

10 This equation is of the same nature as (4.1) since g is the operator of the first logarithmic derivative of the $P(t)$ function. The only difference is that in this version $a_1 > 0$ is no longer the *per capita* birth rate that increases at a constant rate but now expands or contracts depending on the difference existing between w and s. From now on we will use the concepts of means of subsistence, income or product as synonymous.

the vector or ‘profile’ of land at a geometrically increasing rate, then output will increase only arithmetically” (Waterman, 1987: 261).

According to the author, this explanation is due to the fact that Malthus formulates growth rates with such mathematical rigor that one is presented as a condition of the other, or simply put, the rate of subsistences is arithmetic if and only if that of the population is geometric (Waterman, 1987: 260). Hence there is no need to modify them or deal with them separately when assuming diminishing returns or admitting existence of checks: their interdependence is part of the same mathematical unity that Malthus grounds on the theorems of proportions.

Pingle (2003) and Hartwick (1988) disagree with this point of view, echoing Stigler’s doubts (1952: 190) about the validity of taking the two progressions literally. They consider that whereas Malthus assigns a geometric rate to population growth because it was a widespread practice in his time, he arbitrarily poses the arithmetic rate of subsistences with the sole purpose of giving the two progressions a scientific veneer. For this reason, the authors conclude, all that is required is for the population to grow at a faster rate than the means of subsistence in order to adequately justify the *principle*.

The most liberal attitude promoted by these authors has given rise to two new mathematical formulations on the link between rates. The first incorporates the quadratic component of (2), with $a_2 < 0$, with the aim of lending greater realism to (4.1) which increases without considering the carrying capacity of the environment (K). The resulting outcome is Verhulst's logistic equation:[11]

$$\frac{dP(t)}{dt} = a_1 P(t) - a_2 P(t)^2 \tag{5}$$

$$P(t) = \frac{a_1 P_0 / (a_1 - a_2 P_0)}{\left[a_2 P_0 / (a_1 - a_2 P_0)\right] + e^{-a_1 t}} \tag{5.1}$$

Where $P(0) = P_0$ is the initial population and a_1 y a_2 are constants representing average birth and death rates respectively.

The other formulation is a special case of (1) when $m = 2$, which generates Lotka-Volterra type models. An example of these models is system (6) in

[11] The (5.1) limits are set by $a_1/a_2 = K$ when $t \to +\infty$ and by 0 when $t \to -\infty$ giving the Verhulst curve its characteristic S-shape. The minimum number of individuals guaranteeing population replacement in this equation varies and depends on the species under consideration. Other studies, particularly those undertaken by ecologists and biologists, have developed models that include the term cubic in equation (2) to represent the effects of pollution and other environmental elements on the population growth of species (see Kuang, 1993).

which the population $P(t)$ is assumed as a predator and subsistences $S(t)$as prey:

$$
\begin{aligned}
dP(t)/dt &= rS(t)P(t) - RP(t) \\
dS(t)/dt &= \gamma S(t) - bS(t)P(t)
\end{aligned}
\qquad (6)
$$

According to (6), whereas the population grows at a r rate and decreases at another R due to the effect of the presence and absence of subsistences, respectively, $S(t)$expands at a spontaneous γ rate at the same time as it is exhausted as a b decline factor when $P(t)$ increases. The forms (6) can adopt are extremely varied and range from variants that distinguish between renewable and non-renewable resources to systems that differentiate between land uses (Goodwin, 1978; Bonneuil, 1994).

The introduction of both formulations has revived the discussion on the validity of Malthus's linear growth equations. Those that defend the use of (5) and (6) argue that their non-linear natures help provide a better explanation on: i) the observed fluctuations between demographic and economic variables; ii) the interdependent and changing nature of growth rates and iii) the different behaviors of a population exposed to both checks and the action of diminishing returns (see Ramírez and Juárez, 2009; Liz, 2006). Those that disagree with this position hold that, rather than constituting an alternative paradigm, models based on (5) and (6) are a new expression of ultra-determinism (Blanchet, 1998: 142).

1.2. Disagreements over the stability and convergence of attractors

The second major issue under discussion is the stability and convergence of Malthusian attractors. In general, on the one hand, we have studies that favor unique, stable equilibriums (Samuelson, 1978; Waterman, 1988; Pingle, 2003) with trajectories that experience temporary fluctuations (Peacock, 1952; Waterman, 1987; Eltis, 1984; Wrigley, 1986) and on the other, research based on (5) and (6) which, in addition to stable equilibriums, yield catastrophes (Goodwin, 1978), bifurcations and chaos (Day, 1983; Praskawetz and Feichteinger, 1995).

The first group bases its conclusions on the dynamic analysis of an aggregate production function involving land input and a dose of labor-*cum*-capital input as arguments. The structural linear form of its models permits well-behaved analyses of *transitional* dynamics that are occasionally interrupted by oscillations around the stationary state. These oscillations are seen as temporary imbalances represented by a zig-zag path of real wages (Waterman, 1987: 265).

Conversely, the second group assumes non-linear relations between the population and means of subsistence that produce more abrupt *transitional* dynamics. The use of systems such as (6), for example, in which means of subsistence are regarded as non-renewable resources reveals changes in stability simply by modifying the birth parameter (Goodwin, 1978: 193), or cyclical trajectories around stationary points such as $P^* = \gamma/b$ and $P^* = R/r$ (Blanchet, 1998: 140). Likewise, the application of a discrete version of (5) in models that link the size of the population to the total product can create chaos if a small change in the intensity of capital is capable of creating enormous leaps of over 60 % in the population's annual growth rate (Day, 1983).

The conclusions in either case are diametrically opposed. Whereas for the first group, convergence and smooth stability around the stationary state is the general, inevitable solution in the *Essay,* for the second, it is a special case that is achieved in idealized conditions since oscillations appear as a logic result of a highly unstable system predicted by Malthus.[12]

[12] In criticizing the second group's position, Blanchet (1998: 148) argues that the alleged instabilities attributed to Malthusian equilibriums are based on highly restrictive, unrealistic assumptions, such as: extremely high sensitivity of income to population size, and population growth's extreme sensitivity to changes in living standards. This is why, Blanchet continues, the demonstration of unstable equilibriums is an inevitable result that has more to do with the chaotic dynamics of discrete logistic equations (observed for certain ranges of growth rates) than with the authors' theoretical explanations.

2. The new interpretation of the principle: equilibriums with oscillations

2.1. The dynamics of the principle with $K(t)$ expressed as a function of P(t)

In order to achieve a comprehensive interpretation on the dynamics of the *principle* that includes the reservations indicated above, it is essential to have a model that presents the stationary state as an extreme case but one which, in the interim, includes the spectrum of changes described by Malthus as oscillations.[13]

The first step in this direction involves adopting an equation such as (7) in which: i) subsistences are *socially* established by a production function $K(t)$ that depends on the population or labor force $P(t)$, the exogenous technological parameter $A > 0$ and the coefficient of decreasing marginal returns $P(t)$, $0 < \alpha < 1$;[14] and ii) the population grows logistically as an inverse function of the reciprocal of the *per capita* product of the economy $K(t)/P(t)$, expanded by a constant s. [15] The resulting equation is an improved version of (5).[16]

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[13] In Chapter II of his *Essay*, Malthus refers to oscillations as retrograde, progressive movements experienced by the population's welfare around the "subsistence floor." The duration and amplitude of these oscillations varies in each society according to the economic conditions of the various social classes, the effectiveness of the causes that interrupt population growth (such as the introduction or failure of certain manufactures, the greater or lesser degree of initiative of agricultural firms, the years of abundance or shortage, wars and epidemics, among others) and in particular, the difference between the nominal and actual price of labor. These oscillations are obviously associated with various population growth rates.

[14] This is a standard function proposed by Stigler (1952) and subsequently used by authors such as Pingle (2003). If we additionally assume a closed economy without international trade, where there are no unemployed resources, the supply of land and capital is fixed, and the units of labor applied to the fixed stock of land and capital are homogeneous, then we will have a typical Malthusian model as defined by Peacock (1952: 15). However, we shall use instead the "Malthusian" term in the same way as Blanchet (1990: 39) as to define a representation of demo-economic relations in which: 1) there are decreasing returns to scale as population increases, for a given sate of technology; 2) technical progress is not systematically correlated with standard living or with demographic pressure; and 3) population size is partially dependent on the standard of living.

[15] The S constant can also be understood as the lower bound of the per capita product's growth rate.

[16] Equation (7) differs from (5) in the way the term $K(t)$ is considered. Whereas $K(t)$ is commonly treated as a fixed factor, we let it here to change according to a Malthusian production function. This way, the economic and demographic fundamental assumptions of the dynamic Malthusian analysis can be condensed in a single differential equation. The further introduction of delays into (7) makes it even more flexible and suitable for interpreting different behaviors of the population within a Malthusian world.

$$\frac{dP(t)}{dt} = r_0 P(t)\left[1 - s\frac{P(t)}{K(t)}\right] \tag{7}$$
$$where: K(t) = AP(t)^{\alpha}$$

The second stage involves expressing (7) in terms of the *per capita* product. For this purpose, we will consider that $x(t) = K(t)/P(t) = AP(t)^{\alpha-1}$ grows according to the differential equation:

$$x'(t) = A(\alpha-1)P(t)^{\alpha-2}P'(t) = (\alpha-1)x(t)\frac{P'(t)}{P(t)}. \tag{8}$$

After separating variables and replacing $x(t)$ and the first equation of (7) in (8), we have:

$$\frac{x'(t)}{(\alpha-1)x(t)} = r_0\left(1 - \frac{s}{x(t)}\right) \tag{9}$$
$$x'(t) - r_0(\alpha-1)x(t) = -r_0 s(\alpha-1) \tag{10}$$

Whose result eventually produces the trajectory of the *per capita* product:

$$x(t) = [x_0 - s]e^{r_0(\alpha-1)t} + s \tag{11}$$

Or expressed in terms of rates of birth a and mortality b, with $a = r_0$ and $b = r_0 s$

$$x(t) = \left[x_0 - \frac{b}{a}\right]e^{a(\alpha-1)t} + \left(\frac{b}{a}\right) \tag{12}$$

Thus if we introduce (11) or (12) into our definition of $x(t) = AP(t)^{\alpha-1}$ and solve $P(t)$ then we will eventually have the equation for the target population:

$$P(t) = \left(\frac{x(t)}{A}\right)^{1/(\alpha-1)} \tag{13}$$

The observation stopped at (13) shows that the population's trajectory will converge on a value defined by birth and mortality rates and the technological constant, in other words:

$$\lim_{t\to\infty} P(t) = P_e(t) = \left(\frac{s}{A}\right)^{1/(\alpha-1)} \tag{14}$$

since the $\lim_{t\to\infty} x(t) = s$ and $s = b/a$. Likewise, if we replace (13) in the production function and apply limits, we find that the $K(t)$ attractor is regulated by the same constants as $P_e(t)$:

$$\lim_{t\to\infty} K(t) = K_e(t) = A\left(\frac{s}{A}\right)^{\alpha/(\alpha-1)} \tag{15}$$

The convergence of attractors in (14) and (15) will be more pronounced the smaller the value of α as t goes to infinity. In the limit case, the instantaneous rates of all variables will be nil:[17]

$$\lim_{t\to\infty} P'(t)/P(t) = \lim_{t\to\infty} K'(t)/K(t) = \lim_{t\to\infty} x'(t)/x(t) = 0 \tag{16}$$

The influence of *per capita* wages in this process can be seen through the relationship between *s* and the mortality rate. If we assume, as tends to happen, that the mean mortality rate is an inverse function of *per capita w,* and that $b = r_o s$ then:

$$s = f(w) \tag{17}$$

Where $w = \alpha A P^{\alpha-1} = \alpha x(t)$ is the marginal product of labor.

Since $\frac{ds}{dw} < 0$ and $\frac{dP_e(t)}{dw} = \frac{dP_e(t)}{ds}\frac{ds}{dw}$, then we can see that there is a positive relation between $P(t)$ and w

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17 The relation between the limits of instantaneous rates is clearer if we express $K(t)$ first in logarithmic terms and then derive the equation. The economic reason why instantaneous rates tend to be nil as α approaches zero is because production will grow at a slower rate than the population in presence of diminishing returns, creating a permanent fall in the *per capita* product. The resulting loss of welfare will in turn reduce demographic growth and production to the point where $x(t)$ stops growing or converges on the stationary state.

$$\frac{dP_e(t)}{dw} = \frac{1}{\alpha - 1}\left(\frac{s}{A}\right)^{\frac{1}{\alpha-1}-1} \frac{ds}{dw} > 0 \tag{18}$$

The link between the two variables is not, however, proportional, since it is mediated by α, meaning that a reduction of s due to a fall in *w* may lead to an even greater fall in $P(t)$ due to the effect of the inhibiting term $-bP^2(t)$- the smaller the value of α. And since this is also hold for $K(t)$, then a variation in w accompanied by low α values will tend to have a more than proportional influence on the changes experienced by the per capita product and the population.

2.2. The logistic equation with delays and the existence of oscillations

So far, the dynamic analysis of the *principle* expressed by equations (7) – (18) uses different tools to reproduce the traditional Malthusian equilibrium, but without providing clues on the possible irregular behavior of some variables before they reach the stationary state. For this reason and as a last step, we will assume that population growth is not instantly affected by the birth rate but rather that there is a period of delay during which $P(t-\tau)$ influences $P(t)$ through mean birth and death rates. Thus the non-linear effect created by oscillations in the means of subsistence can be directly captured in the $P(t)$ variable through a τ delay in the inhibiting term of the logistic equation (see Kuang, 1993; Liz, 2006):

$$\frac{dP(t)}{dt} = r_0 P(t)\left[1 - s\frac{P(t-\tau)}{K(t)}\right] \tag{19}$$

$$where: K(t) = AP(t)^{\alpha}$$

Since setting the length of the τ delay may have different implications for each population, it is important not to suggest any τ value without previously determining the specific demographic situation of a society. In fact, the only thing we can safely state is that if $r_0\tau < \pi/2$ then (19) will converge on a stable limit value in which $\lim_{t\to\infty}\frac{P'(t)}{P(t)} = 0$ and $\lim_{t\to\infty} P(t) = \lim_{t\to\infty} P(t-\tau) = P_e(t)$ (Kuang, 1993). What is this limit value? In order to obtain it, let us consider the limit of the logistic equation once the $K(t)$ function has been included:

$$\lim_{t\to\infty}\frac{dP(t)}{dt}=\lim_{t\to\infty}\left(r_0P(t)\left[1-s\frac{P(t-\tau)}{AP(t)^{\alpha}}\right]\right)$$

Let us then evaluate it by taking the previous results into account

$$\lim_{t\to\infty}\frac{1}{P(t)}\frac{dP(t)}{dt}=\lim_{t\to\infty}\left(r_0\left[1-s\frac{P(t-\tau)}{AP(t)^{\alpha}}\right]\right).\Rightarrow 0=r_0-\frac{rs}{A}\lim_{t\to\infty}\left(\frac{P(t-\tau)}{P(t)^{\alpha}}\right)=\left(\frac{s}{A}\right)^{1/(\alpha-1)}$$

$$where:\lim_{t\to\infty}\left(\frac{P(t-\tau)}{P(t)^{\alpha}}\right)=\frac{A}{s}$$

The conclusion is that the stationary value of the logistic equation with delays coincides with the attractor of the system (7), in other words, if $r_0\tau<\pi/2$ delays may alter convergence on the stable attractor but not its limit value. The way this is produced is through changes in values of r_0 and τ which, as Gopalsamy states, (1992) are responsible for the quasi-periodic behaviors or fluctuations in the $P(t-\tau)$ term.[18] When the $r_0\tau<\pi/2$ condition is no meet, then oscillations produced by (19) will be not temporary any more and alter the overall asymptotic stability of trajectories in the neighborhood of the stable attractor.[19] In such a case the population's trajectories will not converge on any stable attractor and oscillations will be permanent (Kuang, 1993).

2.3. Application to the Case of Mexico

2.3.1. Specifications regarding the modified logistic equation

Before performing any statistical exercise, it is important to consider that: 1) equation (19) can be indistinctly adapted to describe Malthusian equilibriums with high and low wages and 2) the limit value of (19) is the same with variable or constant birth and death rates.

Regarding the first point, Dooley (1998) mentions that the subsistence minimum is set by the population's response to per capita wages and that since this response is variable, then it is feasible to expect different subsistence minimums. In particular, he points out that there are two polar situations in societies: the "high pressure" one characterizing poor countries in

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[18] The fluctuations produced by $P(t-\tau)$ are due to the effect of the changes in means of subsistence on the mean birth and death rates in the $(t-\tau,t)$. interval. It is therefore feasible to expect the population to oscillate around its point of equilibrium, depending on the variations in existing means of subsistence.

[19] The extent of the oscillations will depend on the absolute value of the expansion factor as well as the length of τ .

which equilibrium is linked to low wages and the "low pressure" one (with high wages) more related to the experience of developed countries. The first situation resembles the one in the *Essay* where growth rates of the population and *per capita* means of subsistence move in the same direction, whereas the second describes the modern version of Malthusian equilibrium in which a drop in *per capita* wages may even be accompanied by a positive population growth or vice versa (Dooley, 1998: 4). Another way of appreciating these relations is by analyzing the instantaneous growth rate of the *per capita* product which can be obtained by dividing (10) by *x(t) = K(t)/P(t)*:

$$\frac{x'(t)}{x(t)} = \frac{A'(t)}{A(t)} + (\alpha - 1)\frac{P'(t)}{P(t)} \tag{20}$$

According to this expression, if $\frac{A'(t)}{A(t)} \geq (1-\alpha)\frac{P'(t)}{P(t)}$ and $0 < \alpha < 1$, then the instantaneous *per capita* growth rate of the product will be positive, regardless of the population growth, although in the opposite case, the situation may be very different: the first rate will decrease in relation to the last.[20] Consequently, if we consider different time intervals for one or various countries and empirically estimate $x'(t)/x(t)$ and $P'(t)/P(t)$, an inverse relationship can be observed for some periods, and a positive relationship for others, without moving outside the Malthusian scheme (Blanchet, 1990).

The second point is related to the fact that the introduction of variable birth and death rates does not alter the limit value of (19) although it does modify its convergence conditions. In order to appreciate this, we consider, like Ordorica (1990) that the birth rate follows a logistic behavior and by construction of (19), similar to that of the death rate (since as we know *b = r (t)s(t)/k* is an inverse function of the *per capita* wage):

$$r_0(t) = k_1 + \frac{k_2}{1+e^{a_1+a_2 t}} \tag{21}$$

Where k_1 and $k_1 + k_2$ are, respectively, the lower and upper asymptotes of the birth rate; a_1 is the birth level and a_2 the speed of change of $r(t)$. The equation resulting from the new addition is:

$$\frac{1}{P(t)}\frac{dP(t)}{dt} = \left[k_1 + \frac{k_2}{1+e^{a_1+a_2 t}}\right]\left[1 - s(t)\frac{P(t-\tau)}{K(t)}\right] \tag{22}$$

[20] Particularly if there is no technological change, this behavior will always be observed.

Whose limit value is similar to (19) given that $\lim_{t\to\infty} r_0(t) = k_1$; $\lim_{t\to\infty} s(t) = s$; $\lim_{t\to\infty} P(t-\tau) = P$; $\lim_{t\to\infty} K(t) = K = AP^{\alpha}$; and $\lim_{t\to\infty} A(t) = A$; in other words:

$$\lim_{t\to\infty} \frac{1}{P(t)}\frac{dP(t)}{dt} = \lim_{t\to\infty}\left[k_1 + \frac{k_2}{1+e^{a_1+a_2 t}}\right]\left[1 - s(t)\frac{P(t-\tau)}{K(t)}\right]$$

$$k_1\left[1 - \lim_{t\to\infty} s(t)\frac{\lim_{t\to\infty} P(t-\tau)}{\lim_{t\to\infty} K(t)}\right] = 0$$

$$k_1\left[1 - s\frac{P}{AP^{\alpha}}\right] = 0 \Rightarrow P_e(t) = \left(\frac{A}{s}\right)^{\frac{1}{1-\alpha}}$$

The basic difference between (22) and (19) lies in the population's paths, since the disturbance produced by $r_0(t)$ further accentuates the oscillations produced by the τ delay in the relation established by $\frac{x'(t)}{x(t)}$ and $\frac{P'(t)}{P(t)}$ in (20).

2.3.2. Simulation

Results presented below consider the impact of delays on the relation between $P'(t)/P(t)$ and $K'(t)/K(t)$ and then on the stability of Mexico's projected population between 1930 and 2050. For the first case, equation (20) is estimated using the following regression model:

$$Y_t = \beta_0 + \beta_1 X_t + u_t \tag{23}$$

Where $X_t = \frac{\Delta P_t}{P_t}$ is the population growth rate, $Y_t = \frac{\Delta K_t}{K_t}$ is the *per capita* product growth rate and u_t the random error. For the second case, the following model is used:

$$Y_t = \beta_0 + \beta_1 P_t + u_t \tag{24}$$

To estimate a simplified version of (22) with $X_t = \frac{\Delta P_t}{P_t} = r_0\left[1 - \frac{P_t}{K}\right]$ and u_t the random error.

The data on the population and product (at constant prices) used in both models are taken from INEGI (several years) while the τ values are set taking the initial population $P_{1930} = 17063300$.[21] The estimator for the α parameter is assumed to be the same as the expected value of the random value of a uniform zero-one distribution ($\alpha = 0.5$).[22] The P_t time series is used to calculate the annual mean birth rate and the stationary population, assuming that there is no migration or changes in age structure and that (24) evolves according to the following values calculated by Ordorica (1990): $k_1 = 0.010$, $k_2 = 0.040$, $a_2 = -3.53927$ and $a_2 = 0.059483$. The numerical method routines are based on software codes available at:

http://www.runet.edu/~thompson/webddes/.

Tables 1.1 and 1.2 summarize the relationship described by (23) by considering two different τ values (25 and 40 years) and successive quinquennial periods in which the impact of the delay is measured.[23] There one can see that regression coefficients between the two growth rates tend to be negative as τ and the period of its impact increase. However this is not a definite pattern. Table 1.1 shows, specifically, that with short delays, relations are alternately positive and negative much alike to regression coefficients in Table 2 which seem to have a more definite negative trend.[24] This same irregular pattern has been also documented for several countries by Blanchet (1990) with the use of a Malthusian model.

What do these results mean in terms of Mexico's demographic experience'? The answer is linked to the time period under consideration. According to Partida (2006) Mexico is currently undergoing a demographic transition that started about 1930 and is expected to be essentially completed during the next fifty years.[25] Throughout the first stage (1930-1969), Mexico's growth rates of population and per capita product basically moved in the same direction by growing over 3 and 2 per cent annual, respectively.[26] During the second stage (1970-2000), the pace of population growth began gradually to decrease, as a consequence of a sharp decline in the rate of fertility, whereas the per capita product underwent an oscillating growth which amounted to 1.5 per cent annual on average. A more accentuated inverse relationship is expected to take place during the third stage (2001-2050) as the growth rate of population approaches to 1 percent around 2050. The most

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21 The Population Census data published by INEGI were taken from http://www.inegi.gob.mx

22 By using (23) and data from INEGI we originally estimated $\alpha = 0.68$, but for reasons of simplicity, we decided to take $\alpha = 0.5$ as in Blanchet (1990). In any case, results do not vary substantially and conclusions are basically the same.

23 The reason why these delays and time periods are chosen is explained below.

24 Sometimes, regression coefficients tend to be positive (negative) in one period after having recorded negative (positive) correlations in the previous period.

25 "Mexico's demographic transition has followed a typical profile … Following the pre-transitional phase that lasted until about 1930, the first stage saw a rapid decline in mortality wile birth rates remained fairly steady and even rose between 1945 and 1960. The second phase began around 1970 when the decline in fertility, which began during the 1960s, became more rapid. The third stage of the process, when the birth and mortality figures converge, will occur during the first half of the twenty-first century." (Partida, 2006: 285).

26 Rates of per capita product and population grew 2.03 per cent and 3.4 per annum, respectively.

relevant aspect of this process is that stages interrelate since "The inertia of the high demographic growth observed up to 1970 is still present in the age structure and will continue to be so for decades…In relative terms, the impulse from generations born up to 1969 will increase the population of Mexico by 16.2 per cent between 2000 and 2015, or by an average of 1.1 per cent per annum during this period." Partida (2006: 296)

The basic features of the first two stages and their interrelations are partially captured by tables 1.1 and 1.2.[27] In both tables, it is observed that the growth rate of population decreases at a faster pace as τ and the period of impact increase. It means that the legacy of the past population is more important when it is not far-distant. This fact along with the observed oscillating path of the per capita product during the two stages gives a particular Malthusian flavor to Mexico's demographic transition. With the exception of the 1961-1965 period in the first stage and the economic crises periods in the second stage (1981-1985 and 1991-1995), simulations with 25 and 40 year delays yield figures which are in accordance with the above description of Mexico's demographic transition The problem is that such figures are not a result of opposite movements between the two rates as happened in developed countries, but of a general trend which is characterized by downward movements of both rates, as stated by Malthus. In fact, regression coefficients for the whole period of the second stage are positive because of that particular trend. In Mexico, therefore, larger delays seems to diminish the likelihood that higher (lower) living standards - measured in the *per capita* product of the year projected for the delay- are reflected in more (less) modern demographic schemes.

The effects of these delays on the population's trajectory can be seen more clearly by using a forward-looking approach. To this end, we will make a comparison between our projections and those carried out by Mexico's National Council on Population (CONAPO, 2006), and by Ordorica (1990). The idea is to compare the estimated values of the population with methodologies that equally share similarities and differences, but do not include delays.[28]

TABLE 1.1. SIMULATION WITH A 25-YEAR DELAY			
PERIODS	MEAN GROWTH RATE OF	MEAN POPULATION	REGRESSION COEFFICIENT

[27] Tables are classified according to specific delays which account for the past contribution of demographic variables in $t-\tau$ to the potential for population growth in t (or *momentum*). Likewise, divisions in quinquennial periods are heuristic devices to compare delayed growth rates (population) with present rates (per capita product) in specific moments of the two stages of Mexico's demographic transition .

[28] The similarities include the adoption by all three of logistic functional forms (expologistic in the case of Ordorica) and variable hypotheses on mortality and fertility rates while differences include the eminently demographic nature of certain methodologies (CONAPO and Ordorica) as opposed to the economic-demographic version of others (such as ours).

	PRODUCT PER CAPITA	GROWTH RATE	
1956-1960	0.0308	0.0384	2.5697*
1961-1965	0.0386	0.0301	-18.329*
1966-1970	0.024	0.0294	5.7339*
1956-1970	**0.0231**	**0.0287**	**-1.0297***

TABLE 1.2 SIMULATION WITH A 40-YEAR DELAY

PERIODS	MEAN GROWTH RATE OF PRODUCT PER CAPITA	MEAN POPULATION GROWTH RATE	REGRESSION COEFFICIENT
1971-1975	0.0282	0.0336	-3.1371*
1976-1980	0.0399	0.0303	-23.5727*
1981-1985	-0.0064	0.0268	10.8589*
1986-1990	-0.0049	0.0233	-26.7466*
1991-1995	-0.0036	0.0198	27.7852*
1996-2000	0.0375	0.0165	-0.7436*
1971-2000	0.0205	0.0303	0.6230*

Note: Asterisks indicate that coefficients are statistically significant at 0.05 of the significance level

Specifically, CONAPO (2206) describes the trajectory of the country's population as a logistic type curve between 1930 and 2050 with a limit value in the last year (see Table 2 and Figure 1). The hypotheses for its projection - which is made by cohort demographic components- are based on declining rates of mortality and fertility and on a scenario where Mexican-born migration to U.S. and return migration are hold constant until 2050. The resulting forecasted data reach a peak in 1940 (112.9 million) and a stable value of 121.9 million ten years later.[29] For its part, Ordorica's study includes hypotheses of logistic behavior on mortality and fertility rates but without considering changes in migration or age structure. His results do not register any limit value because his mathematical specification on the P(t) trajectory is not convergent (See Table 2 and Figure 1).

Lastly, our projection is calculated on the basis of (24) assuming a 25-year delay first. According to Table 2 and Figure 1, the introduction of an economic ceiling and the "drag effect" of delayed fertility patterns in (24) yield projected population volumes higher than.

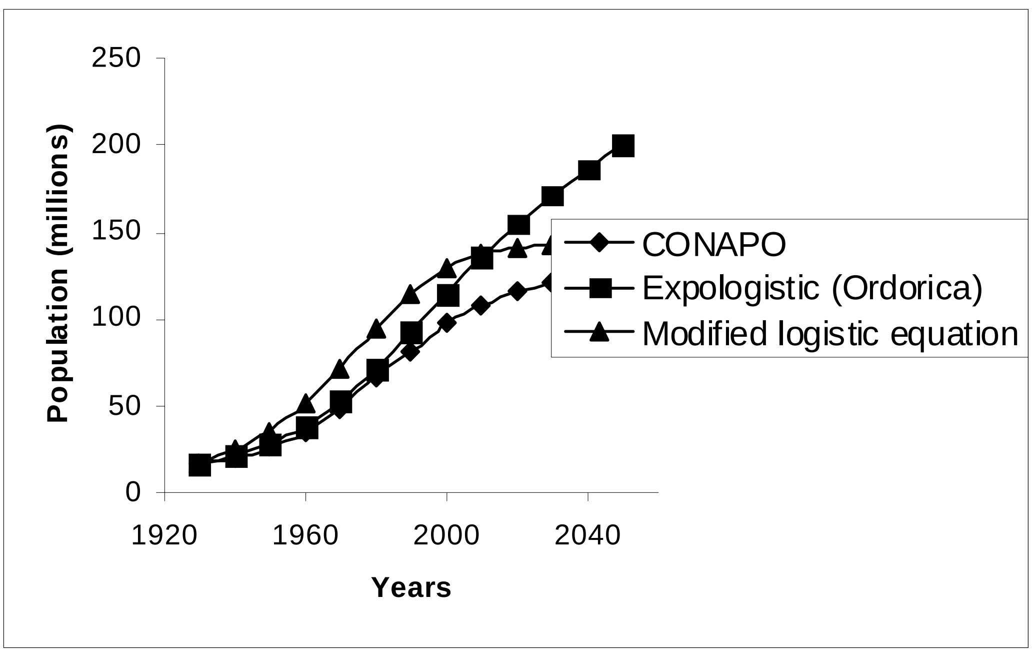
[29] When setting a maximum population growth ceiling CONAPO does not incorporate any economic hypothesis on the growth of the per capita product.

TABLE 2. PROJECTIONS FOR THE MEXICAN POPULATION ACCORDING TO VARIOUS SOURCES

	CONAPO	EXPOLOGISTIC (ORDORICA)	MODIFIED LOGISTIC EQUATION	
YEAR	POPULATION (MILLIONS)	POPULATION (MILLIONS)	PROJECTED POPULATION WITH A 25-YEAR DELAY (MILLIONS)	PROJECTED POPULATION WITH A 40-YEAR DELAY (MILLIONS)
1930	17.1	17.1	17.1	17.1
1940	19.7	21.4	24.9	24.9
1950	25.8	28.2	35.7	35.7
1960	34.9	38.3	51.1	48.7
1970	48.2	52.5	71.6	63.2
1980	66.8	70.9	94.1	86.6
1990	81.2	92.5	114.5	110.8
2000	97.5	114.9	129.4	132.7
2010	107.9	136.1	137.9	150.1
2020	115.4	154.8	141.4	161.8
2030	120.7	171.5	142.2	167.9
2040	122.9	186.6	141.8	169.8
2050	121.9	201.1	141.1	168.7

Source: INEGI (several years) and CONAPO (2006).

FIGURE 1. MEXICO: POPULATION PROJECTIONS BETWEEN 1930 AND 2050

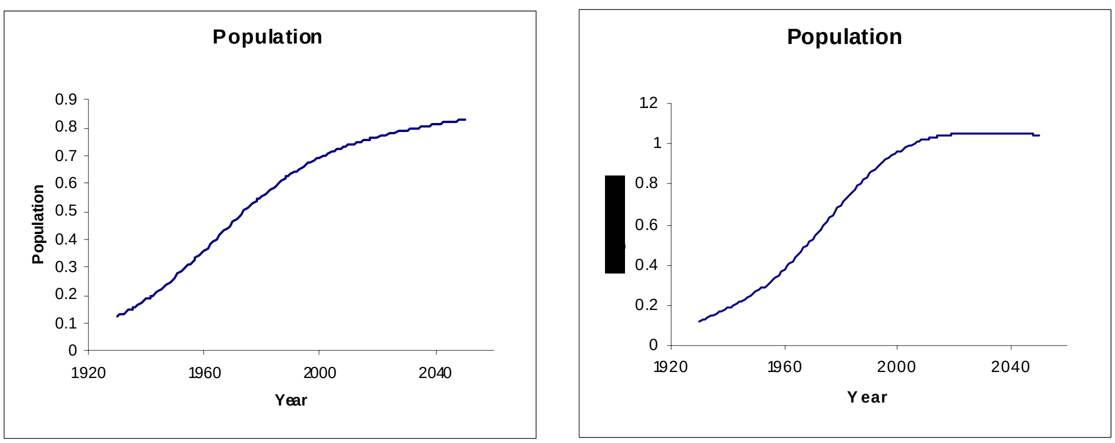


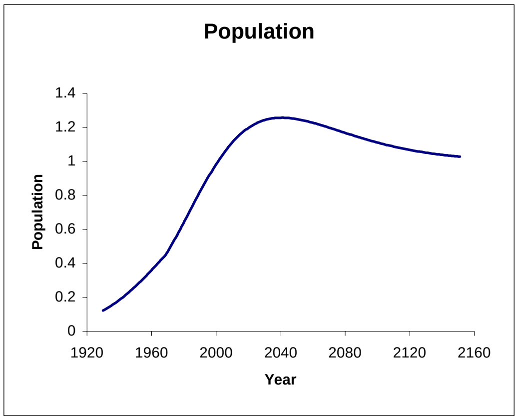
Source: INEGI (several years) and CONAPO (2006).

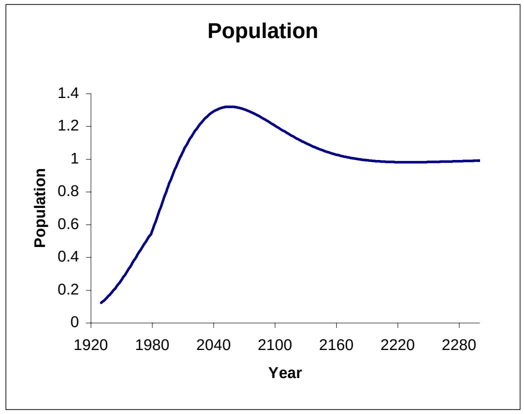
Those recorded by CONAPO (2006) yet lower than those by Ordorica, around 2050. In fact, the (24) data series achieve its limit value in a different year from 2050 since its convergence on $P_e(t) = 135\ \ 019\ \ 560$ depends on the value of τ . The four boxes in Figure 2 and the last column of table 2 show this aspect by demonstrating the fact that the greater the length of τ the longer it takes for the series to converge on $P_e(t)$ and the greater the presence of oscillations. Specifically, table 2 shows that projections with delays greater than 25 years tend to be quite different from CONAPO's figures and that a delay ranging from 0 to 25 years seems to be more appropriate to match the official projection.

In short, results from (24) support the idea that Mexico's growth rate of population will converge on a stable point around 2050, by virtue of the fact that $r_0 \tau = (0.031)(\tau) < \pi / 2$, $\tau \in (0{,}50]$. However, the future trend of the population does not

FIGURE 2. PROJECTIONS FOR THE MEXICAN POPULATION WITH VARIOUS DELAYS







Note. The boxes going from left to right and from top to bottom show the trajectory of the Mexican population with zero, 25, 40 and 50-year delays. Number 1 is equivalent to the value of the stationary population.

Seem to be quite smooth or explosive, as predicted by CONAPO and Ordorica, respectively. The presence of *momentums* –represented by delays- causes the population to oscillate because of the irregular movements of the $s\frac{P(t-\tau)}{K}$ component. As the size of the delay increases, the effect of a variation in the *per capita* product (reflected through $P(t-\tau)/K$) on the population is also accentuated. Particularly for the $25 \le \tau \le 50$ values, the oscillations are more pronounced in relation to the limit value.[30] These oscillations explain in turn the irregular pattern of interrelation between growth rates of the population and the per capita product in the two first stages of Mexico's demographic transition.

30The result is predictable and intuitive because in order to observe fluctuations in the size of the population, it is essential to consider sufficiently long periods of time in order for the mechanisms of the per capita product to have an effect on population dynamics.

Conclusions

Concluding remarks: Why should not one forget the principle of population?

This document holds that the up-to-date study of the principle of population requires using dynamic analyses with oscillations that can contribute, among other things, to a better understanding of the population's aggregate behavior in countries like Mexico. Though this conclusion is not revealing, it breaks away from the long-established idea that the *principle* is only useful to describe demographic situations of backward societies in which "the population rate is increasing in *per capita* consumption when living standards are low" (Hansen and Prescott, 2002: 1205). The model and its simulation above-developed are examples of the ways the *principle* can be returned to life as to grasp crucial aspects of current demographic issues in medium-income countries.

The results of the paper, based on (19) and more specifically (22) are interesting for two reasons. First, because the use of these equations can help describe different behaviors of the population by means of a combination of economic and demographic variables interacting with the presence of delays (which can account for *momentums*); the continuous, well-weighted adjustments to the values of the α , r_0 , A and τ parameters of (19) can help draw up general trends of the population in keeping with a country's economic situation. More important, however, is that introducing delays in a Malthusian model allows confirming that developing countries' demographic transition is neither a straight line of evolution nor a fixed pattern of relations between the growth rate of per capita product and the population. Tables 1.1 and 1.2 show, for instance, that though México is on the way of completing the demographic transition, its economic and demographic growth patterns differ from those observed in developed countries.

The second reason why both equations are worth noting is that they emphasize the fact that oscillations are a consequence of Malthus's different views on the prevailing demographic behaviors in a society. The cultural differences that affect the fertility decisions or material inequalities shaping mortality patterns are aspects that are present in the study of oscillations. For Malthus, the population reacts differently because in societies divided by social classes, not everyone has the same resources. Consequently, the instantaneous adjustments between growth rates and the homogenization of living standards in the stationary state are conceptual forms opposed to Malthusian thought that belong more to the realm of his neoclassical or Marxist interlocutors.

However, both equations can face serious criticisms due to the fact that they are based on a highly stylized model which supposedly involves restrictive assumptions. The model assumes that Mexico's production is ruled by decreasing returns to scale, that there is no migration and no changes in age structure, that the population's growth rate is the same as the one of the labor force, that the technological change is exogenous, and finally that the *momentum* is defined by the size of delays. In a word that Mexico lives in a Malthusian world with delays. As a result equations (19) and (22) seem to be doomed to draw a fake picture of Mexico's reality, or rather to fictionalize its current demographic phenomena with the use of an antediluvian principle of population. Though these criticisms are to some extent justified, in particular because exist other modern demo-economic models with less restrictive assumptions, this model can help enhance the idea that the *principle* is a powerful auxiliary in the explanation of Mexico's some demographic aspects.

Mexico is a country where demographic and economic conditions are heterogeneous and, consequently, the convergence of its population's paths on a stable attractor has no the same meaning as in developed countries. The marked diversity of demographic and living standard patterns within its regions, makes a little risky to ensure that the country's whole population is undergoing a smooth and homogeneous demographic transition. A quick review of some studies lets us to know that any kind of economic and welfare indices in Mexico is characterized by both diverging across regions and experiencing a constantly worsening over the time (Loayza *et al.,* 2004; Diaz, 2007; Núñez, 2006). Besides, it is well known that most of Mexico's rural and urban low-income inhabitants —a substantial part of the whole population— keep reproduction and death patterns much the same as those of the demographic transition's first stage.[31] For them, the demographic transition undergone for the "average citizen" in advanced societies does not make sense, particularly because their demographic rationality is not only based on modern economic criteria but also on ancient family traditions. In the world they live, children keep dying from "old fashion" diseases and fertility decisions still continue to be determined by long-standing religious beliefs, not by market rules. Hence it is an oversimplification to deal with different population strata by simply analyzing average rates, above all if it is done in a country where social and economic conditions can equally retard (retrograde movements) or foster (progressive movements) the passage from "old" to "modern" demographic behaviors. So, it is impossible to understand the driving forces behind a highly differentiated demographic transition if the proposed model is unable to give an explanation about what is meant by retrograde and progressive moments. In this aspect, a modern approach on Malthus´s principle with oscillations can be very helpful.

[31] Maybe because of that, the national average rates of mortality and fertility hide as much as they reveal.

Why do not we use a Boserupian or another tecnological endogenous growth type model insted of a Malthusian one to study these oscillations? There are some justifications for not doing that. Three of them have theoretical roots: 1) Blanchet (1990) shows that a typical Boserupian model does not add much different information to that supplied by a Malthusian model, when explaining the main features of a demographic transition in developing countries; 2) the introduction of tecnological progress in a production function does not guarantee that negative effects on productivity of persistent production growth can be overridden by the persistent growth of nonlabor factors and technological improvements. The reason is that "it remains unclear how new efforts to model technological innovation are likely to influence understanding on the relation between population growth and economic growth" (Mason, 2005: 561); and 3) a Malthusian model can be adapted to explain in a more realistic fashion some aspects of the theory of induced innovation in developing countries; its assumption of decreasing returns allows not only analyzing situations where demographic pressure creates relative scarcity, but also understanding how societies overturn such a scarcity. This last characteristic is not found in models that assume an $\alpha > 1$ (Blanchet, 1990). Other reasons have to do more with empirical facts that place Mexico as a country with definite Malthusian features in some economic sectors such as decreasing returns in production or progressive loss of importance of nonlabor factors in productivity.[32]

All of this makes it clear, then, that modeling Malthus's *principle* within this new perspective is not merely an exercise in exegesis. Instead, it is a means of using new tools to revitalize the object of study of the main economic and demographic problems which, in Goodwin's view (1978), was lost with the advent of the neoclassical theory of economic growth. The principle of population, with all its drawbacks and ideological burdens long pointed out by its detractors of every stripe (often with due reason), can be adapted to place again demography at the center of the discussion on economic problems (Ramírez and Morelos, 2002). A fruitful way to do that is following Mason's proposal consisting of studying the different relations between the rate of growth of per capita income ($\dot{y}$) and the growth rates of income per worker $\left(\dot{y}^{l}\right)$, labor force (l) and population (n) (Mason, 2005: 558). In doing so, it is important to include more realistic assumptions related to migration and age structure. Otherwise, it would be impossible to know,

[32] In Mexico, Factorial Total Productivity –a major productivity index- has been undergoing a serious declining between 1930- 2000. It went from an annual growth rate of 1.6 in the 1960-73 period to -0.8 in the 1973-1984 period. Likewise the growth rate of per worker product went from 3.8 per cent annual in the 1960-73 period to 0.7 and to 0.3 ten and twenty years later, respectively. Other indices such as labor productivity and per capita product have undergone the same growth pattern (Loayza *et al.,* 2004; Núñez, 2006).

among other things, the effects of $l-n$ on $\dot{y}$ or, rather the demographic dividend in a specific country.

The old demo-economic models and more recently, the models of overlapping generations have simplified the population's role to such an extent that nowadays it is virtually impossible to distinguish its function from other economic variables. And this can be dangerous, because the well-known technological solution to the Malthusian trap fails to guarantee the disappearance of the problems resulting from situations where demographic pressure creates relative scarcity. By this we mean the problems derived from deforestation, the exhaustion of the aquifers, pollution and social conflicts due over-urbanization, which need to be thought in the context of a general demographic framework, as Malthus suggests in the *Essay*, and not just in their individual (political or economic) terms. The shame or perhaps embarrassment about giving mechanical interpretations of the *principle* a new twist and accepting the currency of Malthus's ideas is one of the hardest demons to conquer. And that, given the overcrowded, impoverished conditions in which most of the world's population lives, sounds rather hypocritical.

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