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Abstract

In this paper we formulate an infinitely repeated Principal-Agent relationship as a Multi-Objective Optimization problem. We numerically approximate the solution of this model using a Multi-Objective Optimization Evolutionary Algorithm, named RankMOEA, for different values of the Agent's coefficient of relative risk aversion. Our numerical results indicate that as the Agent becomes more risk averse, the Pareto Frontier becomes less concave, the Principal-Agent relationship generates more value for the Agent, and the Principal appears to assume more of the risk sharing regardless of the contract we analyze along the Pareto Frontier.

Keywords: Asymmetric Information, Principal-Agent Model, Incentives, Pareto Frontier, Evolutionary Algorithms.

JEL Classification: C63, D61, D82, D86, L14.

Resumen

En este documento formulamos una relación Principal-Agente que se repite infinitamente como un problema de optimización multi-objetivo. Aproximamos la solución de este modelo usando un algoritmo evolutivo diseñado para trabajar con problemas de optimización multi-objetivo, denominado RankMOEA*, considerando distintos valores del coeficiente relativo de aversión al riesgo del agente. Nuestros resultados numéricos indican que cuando el agente se vuelve más averso al riesgo, la frontera de Pareto se vuelve menos cóncava, el agente obtiene un mayor valor de la relación Principal-Agente y el principal asume una mayor proporción en la compartición del riesgo, no importando cuál contrato a lo largo de la frontera de Pareto se analice.*

Palabras clave: información asimétrica, Modelo Principal-Agente, incentivos, frontera de Pareto, algoritmos evolutivos.

Clasificación JEL: C63, D61, D82, D86, L14.

Risk Aversion and the Pareto Frontier of a Dynamic Principal-Agent Model: An Evolutionary Approximation*

by

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Abstract

In this paper we formulate an infinitely repeated Principal-Agent relationship as a Multi-Objective Optimization problem. We numerically approximate the solution of this model using a Multi-Objective Optimization Evolutionary Algorithm, named RankMOEA, for different values of the Agent's coefficient of relative risk aversion. Our numerical results indicate that as the Agent becomes more risk averse, the Pareto Frontier becomes less concave, the Principal-Agent relationship generates more value for the Agent, and the Principal appears to assume more of the risk sharing regardless of the contract we analyze along the Pareto Frontier.

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1 Introduction

Principal-Agent models represent situations of information asymmetry with hidden actions, where a risk-neutral Principal, being the owner of a production technology, delegates the task of operating it to a risk-averse Agent. Uncertainty about the outcomes of the production process, and non-observability of the Agent's effort choices yield solutions that are not first-best Pareto Optimal. The static version of those models generates risk-sharing rules that depend on the stochastic relationship between the non-observable effort choice of the Agent and the productive outcome, and, consequently, the risk-averse Agent bears some of the risk associated with the production process, see Holmstrom (1979). The repeated Principal-Agent relationship has been modelled as an infinite-period dynamic problem with discounting, see Spear and Srivastava (1987) and Wang (1997). In this context, the Agent's compensation scheme becomes more complex and it includes two components, namely present and future compensation; but still risk-sharing rules emerge as the solution of the dynamic problem. Formally, the problem has been formulated as the maximization of the Principal's expected discounted utility subject to the participation constraint, which ensures that the Agent enters the contractual relationship, the incentive compatibility constraint, which ensures that the Agent, period by period, chooses the effort level that the Principal desires to implement, and some resource constraints, that involve the Principal's limited commitment, as in Wang (1997).

Many situations in the real world that involve optimizing conflicting objectives between two or more parts can be thought as Multi-Objective Optimization Problems (MOPs). Multi-Objective Optimization (MO) implies optimizing conflicting objectives subject to certain constraints, and most of the time it is impossible to find a unique solution to such problems. Thus, MOPs are characterized by a set of alternative and equivalent solutions because of the lack of information about the relevance of one objective with respect to the others. The set of optimal solutions is possibly of infinite dimensions by definition, and it is called the Pareto Optimal Frontier.

The structure of the Dynamic Principal-Agent problem makes it possible to envision it as a MOP, given the conflict of interest between the Principal and the Agent, see Di Giannatale *et al.* (2010). In this paper, we formulate the Dynamic Principal-Agent model as a MOP, and call it the M-Dynamic Principal-Agent model. This model allows us to analyze diverse contractual arrangements between the Principal and the Agent in which their discounted expected utilities have different levels of priority, and to obtain a better insight on how the creation of economic surplus is affected by such contractual arrangements.The formal problem is formulated as the maximization of both the discounted expected utility of the Principal and that of the Agent, subject to the Agent's participation constraint and some resource constraints. The Agent's effort remains unobservable to the Principal; but the M-Dynamic Principal-Agent model encompasses the Principal's effort implementation problem because of the structure of its objective functions, given the satisfaction of some conditions (Rogerson, 1985).

On the other hand, Evolutionary Algorithms (EAs) are stochastic methods of search, see Goldberg (1989), based on the evolutionary analogy of the "survival of the fittest." This analogy is inspired on the modern evolutionary synthesis, where natural selection can be seen as a learning process in which the fittest individuals, after a long time, survive in a defined environment. EAs are often used to provide numerical solutions to MOPs. We numerically approximate the Optimal Pareto Frontier that emerges from the M-Dynamic Principal-Agent model by using a recently proposed Multi-Objective EA (MOEA) named RankMOEA, see Herrera-Ortiz *et al.* (2011).

The remaining of this paper is organized as follows: in section 2 the M-Dynamic Principal-Agent model is presented. Section 3 explains the EA methodology that we use to numerically approximate the model's Pareto Frontier. The numerical results are discussed in section 4. Finally, our concluding remarks are presented in section 5.

2 The M-Dynamic Principal-Agent model

We assume that time is discrete and that it goes on until infinity: $t = 0, 1, 2, ...$ There are two individuals: a risk neutral Principal and a risk averse Agent, who are both discounted expected utility maximizers with a common discount rate $\beta \in (0, 1)$. Suppose that the Agent has a period utility function represented by: $v(w, a)$, which is assumed to be bounded and strictly increasing and strictly concave with respect to w, and strictly decreasing with respect to a . The variable w is the Agent's salary or present compensation at the end of every period. The variable a is the Agent's effort choice made at the beginning of every period, drawn from a compact set A, and it is unobservable to the Principal. We also assume that v is either additively or multiplicatively separable in its two arguments, w and a.

Every period a realization of the output y, drawn from the finite set Y, is observed by the Principal and the Agent. The stochastic relationship between the output realization and the Agent's effort choice is described by the distribution function $f(y; a) > 0$, $\forall y \in Y$ and $\forall a \in A$. We assume that the Monotone Likelihood Ratio Property and the Convexity of the Conditional Distribution hold in order to ensure that the First Order Approach to the Incentive Compatibility Constraint is valid, see Rogerson (1985).

Now, suppose that the history of output realizations is denoted by $h^t = \{y_s\}_{s=1}^t$, where every $y_s \in Y$. In the M-Dynamic Principal-Agent model, two conflicting objective functions are maximized: the *ex-ante* Principal's discounted expected utility, and the *ex-ante* Agent's discounted expected utility. A contract σ is defined by a history dependent Agent's effort recommendation $a(h^{t-1})$, and a history dependent Agent's compensation plan $w(h^t)$, such that $\sigma = \{a(h^{t-1}), w(h^t)\}$.

A contract σ is feasible if:

$$a(h^{t-1}) \in A, \qquad \forall t \geq 1, \qquad \forall h^{t-1} \in Y^{t-1}, \tag{1}$$

$$0 \leq w(h^t) \leq y_t, \quad \forall t \geq 1, \qquad \forall h^t \in Y^t. \tag{2}$$

Condition (2) requires that the Agent's salary be non-negative and not greater than the current ouput.

The continuation profile from time $t+1$ onwards for contract σ at any t, given h^t, is determined by $\sigma \mid h^t$, and this implies a continuation value from time $t+1$ onwards of $U(\sigma \mid h^t)$ for the Principal and of $W(\sigma \mid h^t)$ for the Agent.

Define $\mathcal{X} \in \mathbb{R}$ as the non-empty and compact set where the Agent's expected discounted utilities take values. Now, the set of the Agent's discounted expected utilities produced by a feasible contract is defined as follows:

$$\mathcal{W} = \{V \in \mathcal{X} \mid \sigma \ s.t. \ (1) \text{ and } (2)\}.$$

The optimization problem is, $\forall V \in \mathcal{W}$:

$$\max_{\sigma}\{U(\sigma \mid h^0), W(\sigma \mid h^0)\} \quad s.t. \ \ (1) \text{ and } (2).$$

A solution to this problem is an optimal contract that belongs to the Pareto Optimal Set, PS^*. An optimal contract σ belongs to the PS^* if there is no other contract φ in PS^* such that $F(\varphi)$ dominates $F(\sigma)$; that is, $F(\varphi) \prec F(\sigma)$, where F is the vector that contains the maximal expected discounted utilities of the Principal and of the Agent. We say that $F(\varphi)$ dominates $F(\sigma)$ if and only if contract φ improves the expected discounted utility of the Principal (or the Agent) with respect to contract σ, without inducing some simultaneous deterioration in the discounted expected utility of the Agent (or the Principal). On the other hand, the set of pairs of the discounted expected utilities of the Principal and the Agent that are generated by contracts that belong to PS^* form the Pareto Optimal Frontier, PF^*. Notice that in the M-Dynamic Principal Agent model we have not included the Incentive Compatibility Constraint in the constraint set because it is encompassed in the second objective function of this problem for one of the problem's decision variables is the Agent's effort choice, given that we have ensured that in our environment the First Order Approach to this constraint is valid.

Define $\mathcal{U}(V)$ as the set that contains pairs of the feasible discounted expected utilities of the Principal and the Agent, respectively, $\forall V \in \mathcal{W}$:

$$\mathcal{U}(V) = \{(U(\sigma \mid h^0), W(\sigma \mid h^0)) \mid \sigma \; s.t. \; \text{(1) and (2)}\}.$$

Proposition 1 *$\mathcal{U}(V)$ is compact, $\forall V \in \mathcal{W}$.*

Proof. Fix V. $\mathcal{U}(V)$ is bounded. We need to prove that $\mathcal{U}(V)$ is also closed. Let $\{U_n, W_n\} \in \mathcal{U}(V)$ such that $\lim_{n \longrightarrow \infty} \{U_n, W_n\} = \{U_\infty, W_\infty\}$. We have to show that $\{U_\infty, W_\infty\} \in \mathcal{U}(V)$, or that there exists a contract σ_∞ that satisfies (1), (2), $W(\sigma_\infty \mid h^0) = W_\infty$, and $U(\sigma_\infty \mid h^0) = U_\infty$. We construct this optimal contract σ_∞. The definition of $\mathcal{U}(V)$ allows us to say that there exists a sequence of contracts $\{\sigma_n\} = \{a_t^n(h^{t-1}), w_t^n(h^t)\}$ that satisfies (1) and (2), $\forall n$. Hence,

$$U_\infty = \lim_{n \longrightarrow \infty} \sum_{t=1}^{\infty} \beta^{t-1} \int_Y (y - w_t^n(h^t)) f(y; a_t^n(h^{t-1})) dh^t$$

$$W_\infty = \lim_{n \longrightarrow \infty} \sum_{t=1}^{\infty} \beta^{t-1} \int_Y (v(w_t^n(h^t), a_t^n(h^{t-1})) f(y; a_t^n(h^{t-1})) dh^t,$$

For $t = 1$, $\{a_1^n(h^0), w_1^n(h^1)\}$ is a finite collection of bounded sequences, so there exists a collection of subsequences $\{a_1^{n_q}(h^0), w_1^{n_q}(h^1)\}$ that satisfy:

$$\lim_{n_q \longrightarrow \infty} a_1^{n_q}(h^0) = a_1^\infty(h^0) \quad \text{and} \quad \lim_{n_q \longrightarrow \infty} w_1^{n_q}(h^1) = w_1^\infty(h^1)$$

Also, W_∞ must be equal to V; if $W_\infty < V$ the Agent would not enter the relationship, and if $W_\infty > V$ the Principal could lower the Agent's discounted expected utility and still the Agent would accept to enter the relationship, given that we have assumed that v is separable in w and a, see Grossman and Hart (1983).

We can repeat this procedure for $t = 2, ..., \infty$, and let $\sigma_\infty = \{a_t^\infty(h^{t-1}), w_t^\infty(h^t)\}$. The contract σ_∞ is the object we desire to obtain. ■

The maximal values of the discounted expected utilities that belong to $\mathcal{U}(V)$ are $(U^*(V), W^*(V))$. Now, we define Γ as an operator that maps from the space of continuous and bounded functions $U : \mathcal{W} \to \mathbb{R}$, because the Agent's compensations are bounded, and $W : \mathcal{W} \to \mathbb{R}$, because the Agent is risk-averse and his compensations are bounded, with the sup norm into itself; such that, $\forall\ V \in \mathcal{W}$:

$$\Gamma(U, W)(V) = \max_{a(V), w(y,V), \overline{V}(y,V)} \{U, W\} \tag{3}$$

$$U = \int_Y [y - w(y, V) + \beta U(\overline{V}(y, V))] f(y; a^*(V)) dy \tag{4}$$

$$W = \int_Y [v(w(y, V), a(V)) + \beta \overline{V}(y, V)] f(y; a(V)) dy \tag{5}$$

subject to

$$a(V) \in A \tag{6}$$

$$0 \leq w(y, V) \leq y \qquad \forall y \in Y \tag{7}$$

$$\overline{V}(y, V) \in \mathcal{W} \quad \forall y \in Y \tag{8}$$

where, equation (6) restricts actions to the space of feasible actions A; equation (7) indicates the Agent's temporary inability to borrow; and, equation (8) ensures that the Agent's future utility plan is feasible. We now demonstrate that $(U^*(V), W^*(V))$ is a fixed point of Γ.

Proposition 2 *$(U^*(V), W^*(V)) = \Gamma(U^*, W^*)(V)$, $\forall V \in \mathcal{W}$.*

Proof. Fix V. First, we show that $\Gamma(U^*, W^*)(V) \leq (U^*(V), W^*(V))$. This is true if $\exists \sigma$ that is feasible such that $(U(\sigma \mid h^0), W(\sigma \mid h^0)) = \Gamma(U^*, W^*)(V)$. We construct this contract σ by letting $a(V)$, $w(y, V)$, and $\overline{V}(y, V)$ be the solution to $\Gamma(U^*, W^*)(V)$, and:

$$a_1(h^0) = a(V), \text{ and } w_1(h^1) = w(y_1, V),\ \forall h^1.$$

For a given $y_1 \in Y$, $\exists \sigma_{y_1}$ such that the Agent receives $\overline{V}(y_1, V)$ and the Principal receives $U^*(\overline{V}(y_1, V))$. Let

$$\sigma \mid h^1 = \sigma_{y_1},\ \forall h^1.$$

Notice that σ_{y_1} belongs to the PS^*, because $\overline{V}(y_1, V) = W^*(\sigma_{y_1} \mid h^1)$. So, there is no other contract φ_{y_1} in the PS^* such that $F(\varphi_{y_1})$ dominates $F(\sigma_{y_1})$; that is, $F(\varphi_{y_1}) \prec F(\sigma_{y_1})$. So, σ_{y_1} is the contract we need, and $\Gamma(U^*, W^*)(V) \leq (U^*(V), W^*(V))$. The second part of the proof shows that $(U^*(V), W^*(V)) \leq \Gamma(U^*, W^*)(V)$. Let σ^* be an optimal contract. Hence,

$$U^*(V) = U(\sigma^* \mid h^0) = \int_Y [y_1 - w^*(y_1) + \beta U(\sigma^* \mid h^1)] f(y_1; a^*(h^0)) dy_1,$$

$$W^*(V) = W(\sigma^* \mid h^0) = \int_Y [v(w^*(y_1), a^*(h^0)) + \beta W(\sigma^* \mid h^1)] f(y; a^*(h^0)) dy_1 = V,$$

and

$$(U^*(V), W^*(V)) \leq \Gamma(U^*, W^*)(V)$$

if we set $a(V) = a^*(h^0)$, $w(y, V) = w^*(y_1)$, and $\overline{V}(y_1, V) = W^*(\sigma^* \mid y_1)$, for $y_1 \in Y$; for (1) and (2) are satisfied. It must be noted that $W^*(V) = V$ given the separability of v in w and a assumption, see Grossman and Hart (1983). Moreover, σ^* belongs to the PS^*, because $\overline{V}(y_1, V) = W^*(\sigma^* \mid y_1)$; and there is no other contract φ^* in the PS^* such that $F(\varphi^*)$ dominates $F(\sigma^*)$; that is, $F(\varphi^*) \prec F(\sigma^*)$. ■

The operator Γ satisfies Blackwell's sufficient conditions for a contraction, and the contraction mapping theorem ensures that the fixed point $(U^*(V), W^*(V))$ is unique, $\forall V \in \mathcal{W}$. This means that along the PF^* there exists only one maximal value of the Principal's discounted expected utility for every $V \in \mathcal{W}$. Now, PF^* must be non-increasing because otherwise either the Principal or the Agent can achieve a higher level of discounted expected utility and the other individual would be better off, see Spear and Srivastava (1987).

In the next section we propose a methodology to numerically approximate the Pareto Frontier derived from the M-Dynamic Principal-Agent Model.

3 Numerical Methodology

3.1 Functional Forms and Parameter Values

First, we make a set of assumptions regarding the relevant functional forms. In particular, the Principal's utility is: $u_t(y, w(y, V) = y - w(y, V)$; while the Agent's utility function is assumed to be: $v_t(a(V), w(y, V)) = \frac{(w(y,V))^h}{1-h} - a(V)$, where $1 > h > 0$. Notice that Agent's temporary utility function is of the CRRA type and that the coefficient of relative risk aversion is h, where higher degrees of risk aversion are observed with higher values of h. We assume that the Agent's feasible effort choices are continuous and that they belong to the compact set $A = [0; 10.0]$. The upper bound of this set has been chosen such that it will never be observed in the numerical solution.

Also, we suppose that there are two levels of output: low (L) or high (H), described by set $Y = \{y_L, y_H\}$. The probability function that stochastically associates effort and output is:

$$\begin{aligned} f(y_L; a) &= \exp(-a) \\ f(y_H; a) &= 1 - \exp(-a), \end{aligned}$$

these probabilities capture the idea that the higher the Agent's effort level choice is, the greater the likelihood of the realization of the high output level.

The parameter values we use for our numerical exercise are the following: $h = \{\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\}$, $\beta = 0.96$, $Y = \{y_L = 2, y_H = 4\}$, $A = [0; 10.0]$.

3.2 The Computational Approach

Multi-Objective Evolutionary Algorithms (MOEAs) constitute a reliable methodology to achieve the two ideal goals of MO: attaining a good convergence to the Optimal Pareto Frontier, and maintaining the distribution of the Pareto Frontier approximation as diverse as possible.

Evolutionary Algorithms (EAs) are stochastic methods of search often applied to optimization, see Goldberg (1989). EAs have shown to be a promising approach to deal with MOPs; however, they usually do not guarantee the identification of optimal trade-offs, only that they will find good assessments, *i.e.*, the set of solutions ($Pareto\ Frontier\ Approximation - PF^{*}_{known}$) whose objective vectors are not too far from the optimal objective vectors. In recent years, several MOEAs have been proposed, but most of them are unable to deal with incommensurable objectives. In this article, we use a recently proposed MOEA, named RankMOEA because of some advantages observed in numerical approximations of the solution of a dynamic model similar to the one proposed here, see Herrera *et al.* (2011).

Given a finite horizon, the chromosome of the individuals in the population is characterized by 2 substrings of length N, where N is the number of periods an individual lives, i.e. the length of each chromosome is $2N$. The first substring indicates the history of actions of the individual, the second and third one show the history of compensations conditional on a high or low output level respectively. Therefore, the phenotype of an individual is defined as:

$$\begin{bmatrix} w_1(y_H,V), w_2(y_H,V), ..., w_N(y_H,V); \\ w_1(y_L,V), w_2(y_L,V), ..., w_N(y_L,V) \end{bmatrix}$$

The Agent‘s effort levels follow the equation:

$$a(V) = \ln[\frac{(w(y_H,V))^h}{1-h} - \frac{(w(y_L,V))^h}{1-h} + \beta(\overline{V}(y_H,V) - \overline{V}(y_L,V))]$$

This equation is the result of maximizing the expected discounted expected utility of the Agent with respect to the Agent's effort choice. In order to compute U and W we use backward induction, for details see Herrera *et al.* (2011). The Agent's lifespan was set to $N = 70$.

4 Numerical Results

In this section we discuss our numerical results from using several values of the Agent's parameter of relative risk aversion, h. First, we present our results for $h = \frac{1}{2}$. In Figure 1 we show the results of the contract that gives priority to the Principal's expected discounted utility; that is, the contract that is located in the right extreme of the Pareto Frontier depicted in the upper-left panel. The Pareto Frontier is decreasing and strictly concave, as expected from other related articles (Spear and Srivastava, 1987; and Wang, 1997). The Agent's effort schedule is decreasing, meaning that the probability of the high productivity shock decreases as the life-span of the Agent approaches its maximal value of $N = 70$. Both the Agent´s expected discounted utilities for the low (L) and high (H) productivity levels are increasing and become closer (meaning that the spread between the two diminishes) as N approaches 70. It is interesting to notice that given that this contract is the most advantageous for the Principal, the incentives in future utility that the Principal offers to the Agent are very punitive (negative utilities); and that only when N approaches 70 and if there is a high productivity shock the Principal offers him a positive expected discounted utility. Negative Agent's discounted expected utilities are admissible under our optimization program, because they are a result of the Agent exerting feasible but high effort levels, and being paid positive but low salaries. The behavior of the Agent's current compensation schedules for the low and high productivity levels indicates that this incentive tool is used by the Principal when N approaches 70, since the spread between the two is almost zero for many periods and becomes larger as N approaches 70. So, in the initial periods of the Principal-Agent relationship, the incentive tool that the Principal favours is the future utility because it is cheaper in terms of the Principal's utility; while in the final periods of this relationship the Principal uses the Agent's current utility or salary as the preferred incentive tool.

[Insert Figure 1]

In Figure 2 we show the results of the most advantageous contract for the Agent; that is, the contract that is located in the left extreme of the same Pareto Frontier depicted in previous graph. The Agent's effort schedule has the same shape as in Figure 1; however, in the initial periods the Agent chooses higher effort levels and in the later periods the Agent chooses lower effort levels compared to those in Figure 1. The schedule of the Agent´s expected discounted utilities for the low (L) productivity shocks is increasing and starts at higher levels of utility (still negative) than that of Figure 1, and has postive values since period 25. The schedule of the Agent´s discounted expected utilities for the high (H) productivity shocks is decreasing and positive-valued. The spread between those two schedules lowers as N approaches 70, with a similar interpretation as in Figure 1. The behavior of the Agent's current compensation schedules for the low and high productivity levels indicates that this incentive tool is not being used in this contract since those schedules are flat for both the high and the low productivity shocks, even though the one for the high productivity shock is higher than the one for the low productivity shock. This means that the Agent is assuming very little risk inherent to the productive activity and only faces variability in the compensation scheme through future compensation.

[Insert Figure 2]

In Figure 3 we show the results of what we call the social contract; that is, the contract that is located exactly at the middle of the same Pareto Frontier depicted in previous two figures. The Agent's effort schedule has the same shape as in Figure 1and 2; however, in the initial periods the Agent chooses higher effort levels compared to those in Figure 2. The schedule of the Agent´s expected discounted utilities for the low (L) productivity shocks is increasing and starts at lower levels of utility (negative) than that of Figure 1. The schedule of the Agent´s discounted expected utilities for the high (H) productivity shocks is increasing in the first periods and then becomes non-ingreasing after period 16. It is always positive-valued. The spread between those two schedules lowers as N approaches 70, and it is higher than in the two previous contracts in the first periods. The behavior of the Agent's current compensation schedules for the low and high productivity levels is similar to that in Figure 1; however, the spread between the two is higher and with richer dynamics than that observed in Figure 1. This incentive tool is actively used by the Principal since the first periods, but it appears to be more intensively used in later periods.

[Insert Figure 3]

Secondly, we present our results for $h = \frac{1}{4}$. In Figure 4 we show the results of the contract that gives priority to the Principal's expected discounted utility; that is, the contract that is located in the right extreme of the Pareto Frontier depicted in the upper-left panel. The Pareto Frontier is decreasing and strictly concave, and the Agent achieves a lower maximal value in discounted expected utility with respect to Figure 1. The Agent's effort schedule, the Agent´s expected discounted utilities for the low (L) and high (H) productivity levels, and the Agent's current compensation schedules for the low and high productivity levels are very similar to those in Figure 1. Hence, the incentive provision mechanism works similarly; but given that the Agent is less risk-averse, his utility entitlement is lower.

[Insert Figure 4]

In Figure 5 we show the results of the most advantageous contract for the Agent; that is, the contract that is located in the left extreme of the same Pareto Frontier depicted in previous graph. The Agent's effort schedule, the Agent´s expected discounted utilities for the low (L) and high (H) productivity levels, and the Agent's current compensation schedules for the low and high productivity levels are very similar to those in Figure 2. Hence, the incentive provision mechanism works similarly; but, again, given that the value of h is lower, his utility entitlement is lower.

[Insert Figure 5]

In Figure 6 we show the results of the social contract; that is, the contract that is located exactly at the middle of the same Pareto Frontier depicted in previous two figures. The Agent's effort schedule, the Agent´s expected discounted utilities for the low (L) and high (H) productivity levels, and the Agent's current compensation schedules for the low and high productivity levels are very similar to those in Figure 3. Hence, the incentive provision mechanism works similarly; but, again, given that the Agent is less risk-averse, his utility entitlement is lower.

[Insert Figure 6]

Lastly, we present our results for $h = \frac{3}{4}$. In Figure 7 we show the results of the contract that gives priority to the Principal's expected discounted utility; that is, the contract that is located in the right extreme of the Pareto Frontier depicted in the upper-left panel. The Pareto Frontier is decreasing and concave, and the Agent achieves a higher maximal value in discounted expected utility with respect to Figure 1. It is interesting to point out that the Pareto Frontier is less concave than the one in Figure 1. That is, the Principal assumes more of the responsibility in risk sharing and the Agent achieves higher maximal levels of discounted expected utility than in the previous corresponding contracts. The Agent's effort schedule is similar to that in Figure 1, but the Agent chooses higher effort levels in most periods with respect to those in Figure 1. The Agent´s expected discounted utilities for the low (L) and high (H) productivity levels, and the Agent's current compensation schedules for the low and high productivity levels are very similar to those in Figure 1. Hence, the incentive provision mechanism works similarly; but given that the Agent is more risk-averse, his utility entitlement is higher.

[Insert Figure 7]

In Figure 8 we show the results of the most advantageous contract for the Agent; that is, the contract that is located in the left extreme of the same Pareto Frontier depicted in previous graph. The Agent's effort schedule is similar to that in Figure 2, but the Agent chooses higher effort levels in most periods with respect to those in Figure 2. The Agent´s expected discounted utilities for the low (L) and high (H) productivity levels, and the Agent's current compensation schedules for the low and high productivity levels are very similar to those in Figure 2. Hence, the incentive provision mechanism works similarly; but given that the Agent is more risk-averse, his utility entitlement is higher.

[Insert Figure 8]

In Figure 9 we show the results of the social contract; that is, the contract that is located exactly at the middle of the same Pareto Frontier depicted in previous two figures. The Agent's effort schedule is similar to that in Figure 3, but the Agent chooses higher effort levels in most periods with respect to those in Figure 3. The Agent´s expected discounted utilities for the low (L) and high (H) productivity levels are very similar to those in Figure 3. The Agent's current compensation schedules for the low and high productivity levels are similar to those in Figure 3 in the initial periods, but after period 50 those schedules look more as those corresponding to the Agent's most advantageous contract depicted in Figure 8. Hence, some of the incentive provision mechanism works similarly, but given that the Agent is more risk-averse, his utility entitlement is higher and the social contract becomes more similar to the the Agent's most advantageous contract in the last periods.

[Insert Figure 9]

5 Conclusions

In this paper we formulate an infinitely repeated Principal-Agent relationship as a Multi-Objective Optimization problem, and we call it the M-Dynamic Principal-Agent model. Given the inherent difficulties of obtaining a closed-form solution of this model, we numerically approximate it using a Multi-Objective Optimization Evolutionary Algorithm for several values of the Agent's coefficient of relative risk aversion.

Our numerical results indicate that the more risk averse the Agent becomes becomes, the higher the effort levels the Agent chooses, and the higher the values of the Agent's discounted expected utility. That is, as the Agent's coefficient of relative risk aversion increases, the probability of the high productivity shock increases, the Principal-Agent relationship generates more value to the Agent, and the Pareto Frontier becomes less concave. For the highest value that we consider for the Agent's coefficient of relative risk aversion ($h = \frac{3}{4}$), the Pareto Frontier is almost linear. That is, the Principal assumes more of the risk sharing, and the Agent achieves higher maximal levels of discounted expected utility than for the other two levels of the Agent's coefficient of relative risk aversion considered in this article ($h = \frac{1}{4}$ and $h = \frac{1}{2}$).

For every value that we consider for the Agent's coefficient of relative risk aversion, we obtain a Pareto Frontier. In each of those Pareto Frontiers, we identify three points that characterize three types of contracts. The first contract, which we label as the Principal's most advantageous contract, is obtained when the numerical algorithm solves the optimization program prioritizing the Principal's discounted expected utility over the Agent's discounted expected utility. The Agent's compensation schemes show a similar structure for the three values of the Agent's coefficient of relative risk aversion. In those contracts we observe that, in the initial periods of the Principal-Agent relationship, the incentive tool that the Principal favours is the Agent's future utility, while in the final periods of this relationship, the Principal uses the Agent's current utility or salary as the preferred incentive tool. The second contract, which we label as the Agent's most advantageous contract, is obtained when the numerical algorithm solves the optimization program giving higher priority to the Agent's discounted expected utility over the Principal's discounted expected utility. The schedules of the Agent's future expected discounted utility for the high and low productivity shock show a decreasing spread toward the final periods of the Principal-Agent relationship. The behavior of the Agent's current compensation schedules for the low and high productivity levels indicates that this incentive tool is not being used in this contract since those schedules are flat for both productivity shocks, even though the one for the high productivity shock is higher than the one for the low productivity shock. This means that the Agent is assuming very little risk inherent to the productive activity, and only faces variability in the compensation scheme through future compensation. The third contract, which we label as the social contract, is obtained when the numerical algorithm solves the optimization program giving equal priority to both the Principal's and the Agent's discounted expected utilities. The Agent chooses consistently higher effort levels under this contract than under the other two previously described contracts. The schedules of the Agent´s expected discounted utilities for the low and the high productivity shocks show a higher spread than that of the two previous contracts in the first periods of the Principal-Agent relationship, and, as in the previous two contracts, this spread lowers as the relationship approaches the final periods. The spread between the Agent's current compensation schedules for the low and high productivity levels is higher and with richer dynamics than that observed in the Principal's most advantageous contract. This incentive tool is actively used by the Principal since the first periods, but it appears to be more intensively used in later periods.

In future research, we intend to characterize the bargaining process between the Principal and the Agent when they have to make decisions on how to share the value that their relationship generates.

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Figure 1

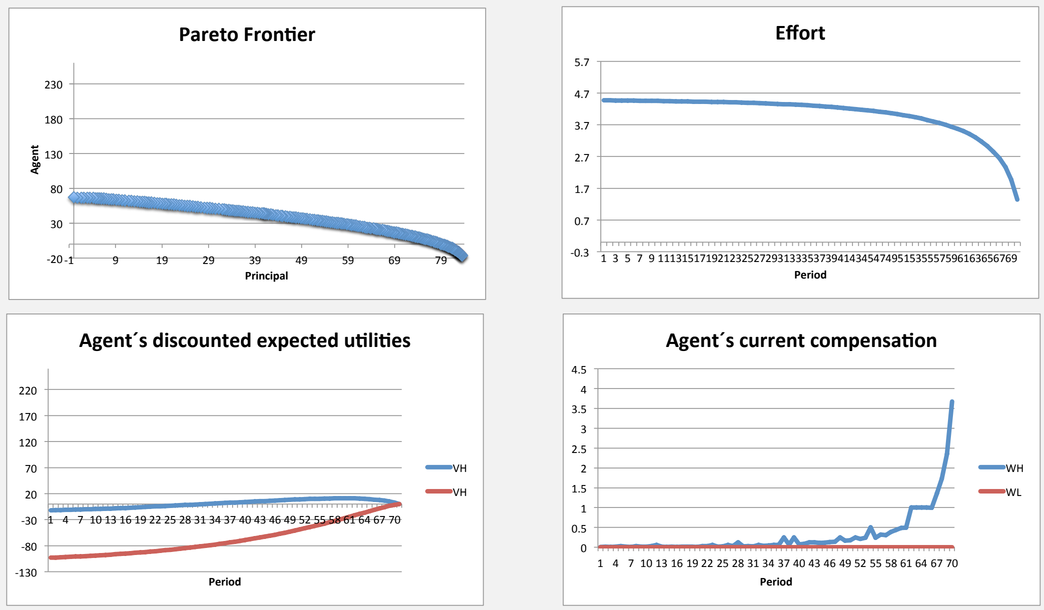

Pareto Frontier
Agent
Principal
Effort
Period
Agent´s discounted expected utilities
VH
VH
Period
Agent´s current compensation
WH
WL
Period

Figure 2

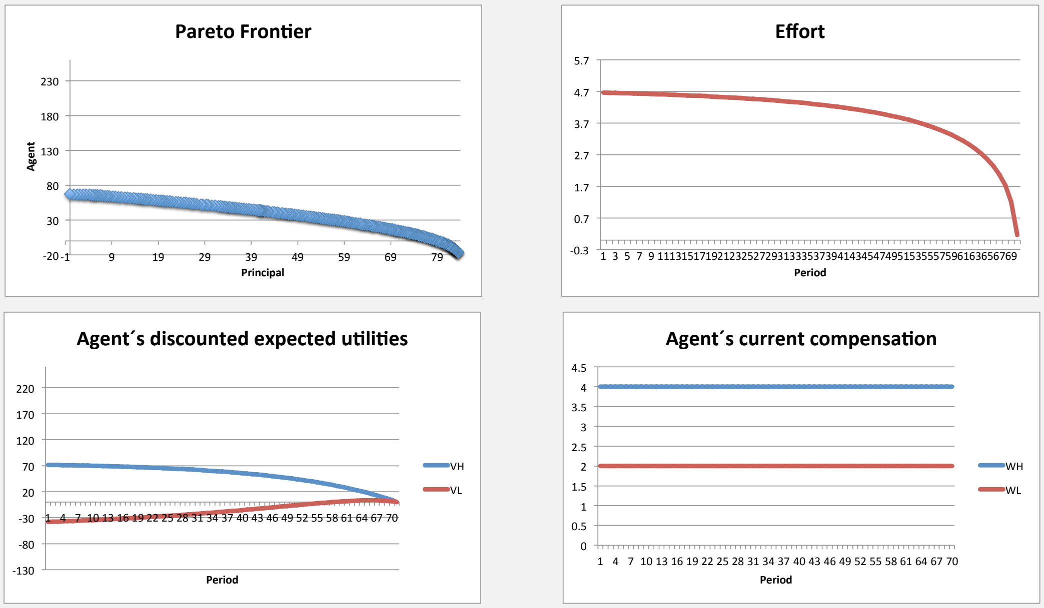

Pareto Frontier
Agent
Principal
Effort
Period
Agent´s discounted expected utilities
VH
VL
Period
Agent´s current compensation
WH
WL
Period

Figure 3

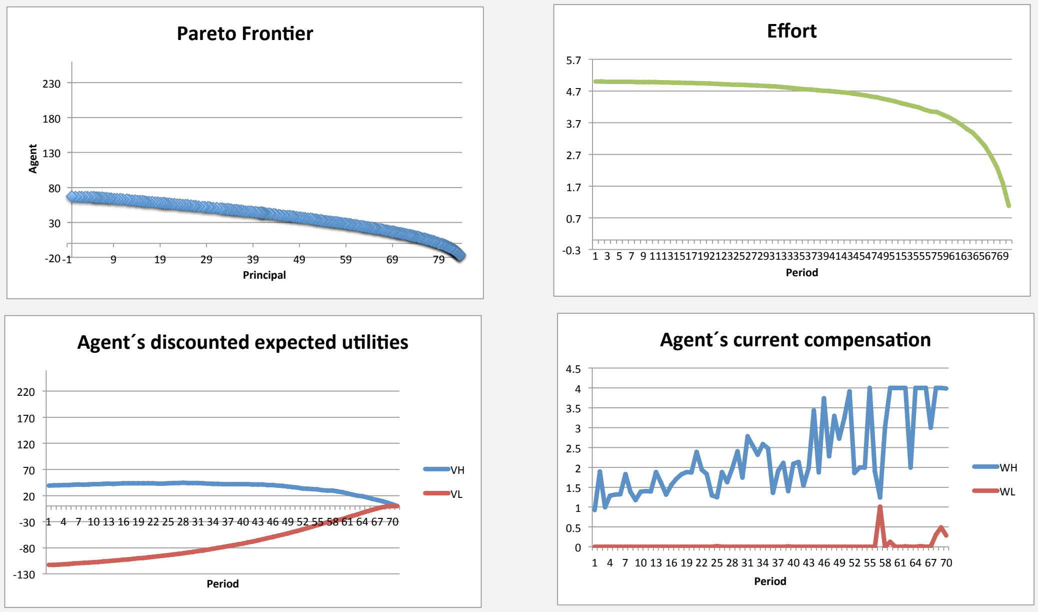
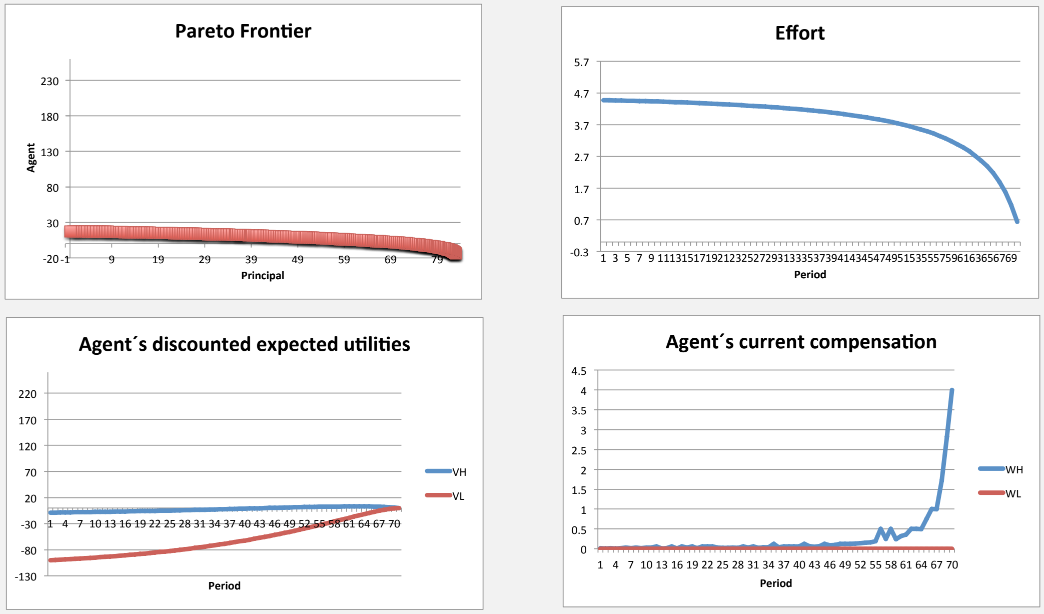

Pareto Frontier
Agent
Principal
230
180
130
80
30
-20
-1
9
19
29
39
49
59
69
79
Effort
Period
5.7
4.7
3.7
2.7
1.7
0.7
-0.3
Agent´s discounted expected utilities
220
170
120
70
20
-30
-80
-130
Period
VH
VL
Agent´s current compensation
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70
Period
WH
WL

Figure 4


Pareto Frontier
Agent
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5.7
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2.7
1.7
0.7
-0.3
Period
Agent´s discounted expected utilities
220
170
120
70
20
-30
-80
-130
VH
VL
Period
Agent´s current compensation
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
WH
WL
Period

Figure 5

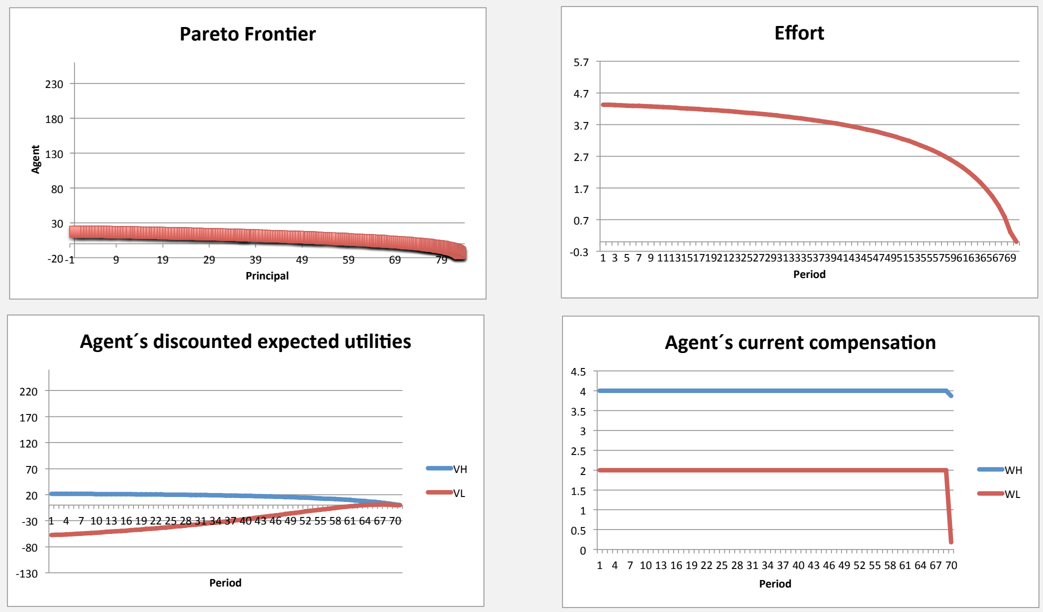
Pareto Frontier
Agent
Principal
Effort
Period
Agent´s discounted expected utilities
VH
VL
Period
Agent´s current compensation
WH
WL
Period

Figure 6

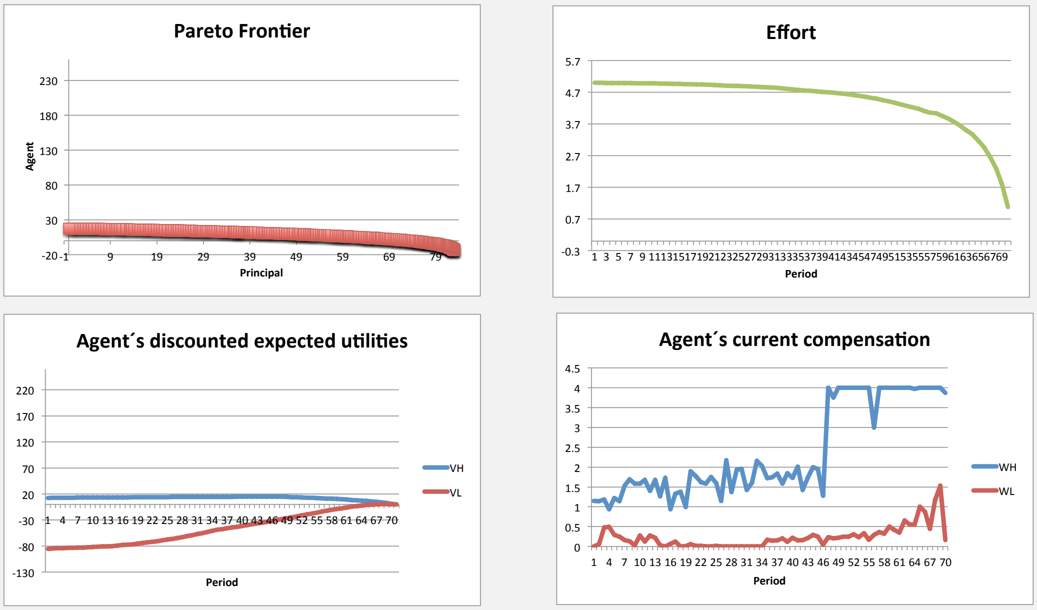
Pareto Frontier
Agent
Principal
Effort
Period
Agent´s discounted expected utilities
VH
VL
Period
Agent´s current compensation
WH
WL
Period

Figure 7


Pareto Frontier
230
180
130
80
30
-20
Agent
-1
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19
29
39
49
59
69
79
Pricipal
Effort
5.7
4.7
3.7
2.7
1.7
0.7
-0.3
Period
Agent´s discounted expected utilities
220
170
120
70
20
-30
-80
-130
1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70
Period
VH
VL
Agent´s current compensation
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
1 4 7 10 13 16 19 22 25 28 31 34 37 40 43 46 49 52 55 58 61 64 67 70
Period
WH
WL

Figure 8


Pareto Frontier
Agent
Principal
Effort
Period
Agent´s discounted expected utilities
VH
VL
Period
Agent´s current compensation
WH
WL
Period

Figure 9


Pareto Frontier
Agent
Principal
Effort
Period
Agent´s discounted expected utilities
VH
VL
Period
Agent´s current compensation
WH
WL
Period

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