

NÚMERO 490

LUCIANA MOSCOSO

Cooperation in the Prisoner's  
Dilemma With Short-Run Players

DICIEMBRE 2010



[www.cide.edu](http://www.cide.edu)

• Las colecciones de **Documentos de Trabajo** del **CIDE** representan un medio para difundir los avances de la labor de investigación, y para permitir que los autores reciban comentarios antes de su publicación definitiva. Se agradecerá que los comentarios se hagan llegar directamente al (los) autor(es).

• D.R. © 2010. Centro de Investigación y Docencia Económicas, carretera México-Toluca 3655 (km. 16.5), Lomas de Santa Fe, 01210, México, D.F.  
Fax: 5727•9800 ext. 6314  
Correo electrónico: [publicaciones@cide.edu](mailto:publicaciones@cide.edu)  
[www.cide.edu](http://www.cide.edu)

• Producción a cargo del (los) autor(es), por lo que tanto el contenido así como el estilo y la redacción son su responsabilidad.

## Abstract

---

*This paper studies cooperative behavior in communities with a subset of short-run players. This is done in the context of a repeated Prisoner's Dilemma game. The introduction of a short-run player in the population breaks any equilibrium supported by symmetric pure strategies. Despite this, I suggest a plausible information technology that ensures a cooperative outcome by identifying the cooperative members of a community. This technology resembles the informational institutions that allow firms to recognize clients and to make them recognizable for other firms.*

## Resumen

---

*Este trabajo analiza el comportamiento cooperativo en comunidades que contienen un subconjunto de jugadores de corto plazo. Esto se realiza en el contexto de un juego de Dilema del Prisionero repetido. La introducción de un jugador de corto plazo en la población destruye cualquier equilibrio sostenido por estrategias simétricas puras. A pesar de esto, el trabajo sugiere un mecanismo de información plausible que, al identificar a los miembros cooperativos de la comunidad, asegura la existencia de un equilibrio cooperativo. Este mecanismo se asemeja a las fuentes de información con que cuentan las empresas para identificar a sus clientes y hacerlos identificables para las otras firmas.*



# Cooperation in the Prisoner's Dilemma With Short-Run Players.

Luciana C. Moscoso Boedo\*

*CIDE*

September 2010

## **Abstract**

This paper studies cooperative behavior in communities with a subset of short-run players. This is done in the context of a repeated Prisoner's Dilemma game. The introduction of a short-run player in the population breaks any equilibrium supported by symmetric pure strategies. Despite this, I suggest a plausible information technology that ensures a cooperative outcome by identifying the cooperative members of a community. This technology resembles the informational institutions that allow firms to recognize clients and to make them recognizable for other firms.

---

\*Centro de Investigación y Docencia Económicas, Carretera México-Toluca 3655, Lomas de Santa Fe, 01210 Mexico City, Mexico. Email: luciana.moscoso@cide.edu

# 1 Introduction

Community enforcement is a well known mechanism for sustaining cooperation among agents in a society. When transactions among members are infrequent, there may be social norms that sustain cooperative outcomes. A key feature of such norms is the threat of sanctions by future partners to deter dishonest behavior. While such punishment structures depend crucially on the information available to agents, it has been shown that cooperation can be sustained even with very limited information when a large population of players is randomly matched. Examples of such results include Milgrom et al. (1990), Kandori (1992), Okuno-Fujiwara and Postlewaite (1995), Ellison (1994), Harrington (1995), and Ahn and Suominen (2001). In most of these models, defection is a dominant strategy of the stage game; cooperation hinges on the requirement that all players stay in the game indefinitely.

If, however, the transactions of some agents in the society are not only infrequent but also unique, then there is no reason to expect cooperation from those members. In certain settings, it may follow that disruption created by such agents also undermines the ability of the remaining long-run players to cooperate. This is the setting explored in this paper. In particular, I ask if and to what degree cooperation can be maintained when short-run players are introduced. I find that the answer depends crucially on the information available to agents. The ability of agents to distinguish between long and short-run players turns out to be critical. To simplify the analysis, I consider a repeated version of the Prisoner's Dilemma game with random matching.

Before continuing to the model, it is worth noting that the difficulty created by short-run players disappears in an environment with perfect information. For example, in small communities, where members know and observe each other's behavior, the presence of newcomers is easily detected, and cooperation can be sustained. In particular, equilibrium strategies allow agents to play a *cooperative* strategy against long-run players and a *myopic* strategy against short-run players. This paper focuses on the more interesting settings in which information is imperfect, such as those in which there is a large population.

In the first part of the paper, I consider the sustainability of cooperation in an environment without information technology. I build on Kandori's

arguments to show that the presence of a short-run player prevents cooperation in equilibrium when all players use a symmetric pure strategy. The restrictions imposed by this kind of strategy in this environment are mutually inconsistent. I show that for any discount factor there are no payoffs such that agents will follow the strategy. The presence of a non-cooperative player will always trigger the contagious process, destroying the incentives for long-run players to cooperate.

Next, I allow for a plausible information technology which may be used to sustain cooperation. I consider a mechanism that attaches labels to those who cooperate. This resembles the credit history mechanism used by credit card companies to recognize their clients. I analyze the sustainability of cooperation given this available information in two cases: one wherein long-run players enter at the beginning of the game and the other wherein there are new long-run players entering the game at every period. I show that the information institutions of this form play an important role. To reconcile cooperation in the presence of some opportunistic players, at least some information technology is necessary.

Finally, I show that in transactions where the option of a repeated partnership relationship is not available, information technology is the key to sustaining cooperation. This result is consistent with the cooperation result obtained by Ghosh and Ray (1996) in a context with heterogeneous agents. In their model players can identify their opponents and commit to a long-term partnership. This partnership opportunity constitutes an information technology that enables the support of cooperation. In contrast, my model assumes that each subsequent partner is anonymous and randomly chosen.

The paper is organized as follows. Section 2 introduces the specific example that will be used throughout the paper. Section 3 presents the case of perfect observability of types and actions. Section 4 proves why cooperation cannot be sustained if there is no information available. Section 5 shows how, with some information about cooperative players, i.e., good reports, cooperation can be sustained if players are sufficiently patient. This section also extends the result to a more sophisticated environment where long-run players enter every period. The last section provides my conclusion.

## 2 The Model

For the remainder of the paper, I analyze the model described below. There is a population of  $M$  players, where  $M$  is an even number.  $S$  of the players are short-run, while the remaining  $L = M - S$  are long-run. In each period, players are randomly matched into pairs to play a Prisoner's Dilemma game. The matching rule is uniform and independent across periods with:

$$\Pr \{ \mu(i, t) = j \mid h_{t-1} \} = \frac{1}{M-1}, \quad \forall j \neq i, \forall h_{t-1},$$

where the function  $\mu(i, t)$  represents the opponent of player  $i$  at time  $t$ .

In each period,  $S$  new short-run players enter the game to replace the last period short-run players who leave. Thus, the probability of a long-run facing a short-run player in a given period is  $\rho = \frac{S}{M-1}$ .

In each period, agents play a stage game in which they decide to cooperate ( $C$ ) or to defect ( $NC$ ). Letting  $l > 0$  denote the loss when cheated and  $g > 0$  the gain from defection, the payoff matrix is as depicted in the figure:

		<i>Player 2</i>	
		$C$	$NC$
<i>Player 1</i>	$C$	$1, 1$	$-l, 1 + g$
	$NC$	$1 + g, -l$	$0, 0$

Short-run players enter the game only for one period. Their discount factor is zero; they do not care about the future and play only the myopic best response:  $NC$ . In contrast, long-run players are concerned about the future and maximize the expected lifetime utility given their common discount factor  $\delta \in (0, 1)$ . In each period they have to decide whether to cooperate ( $C$ ) or defect ( $NC$ ).

Throughout, I focus on the sustainability of the cooperative outcome, in which long-run players cooperate with one another. I analyze the plausibility of a sequential equilibrium sustained by contagion strategies which induce cooperation among long-run players.



### 3 Perfect Information: Observable Types and Actions

If all agents' past actions and types are fully observable, then the cooperative outcome can be sustained in exactly the same way as in a population of only long-run players. The only complication resulting from the presence of short-run players is that players need to be more patient than in the homogeneous case, i.e., the discount factor required to sustain a cooperative equilibrium is higher. This model could be thought to describe a small community where members can readily know and observe one another.

To prove this, I first consider a contagious strategy where long-run players punish all players if one deviates. In this, the strategy for long-run player  $i$  is:

1. *In the first period, play (C) when facing a long-run player. After that, if all long-run players in the population played (C) against each other, play (C) whenever you face a long-run player.*
2. *Play (NC) otherwise.*

It follows that cooperation will be sustainable if:

$$\delta \geq \delta^* = \frac{g}{(g + 1 - \rho)},$$

which is the standard restriction for a two-player repeated Prisoner's Dilemma game.

The unappealing characteristic of this equilibrium is that a single deviator triggers the destruction of cooperation among the whole population. This calls for a more robust equilibrium. However, for this strategy the only information players need to make a decision is a general statement of the whole population: whether there is a non-cooperative long-run player or not. The identity of each player is not needed for this strategy.

In a setting where agents have full information about the identity of any deviator, they can adopt a strategy in which they stop cooperating only against deviators. If players follow the new strategy:

1. *In the first period, play (C) when facing a long-run player. After that, play (C) against long-run players that cooperated against another cooperative long-run player.*
2. *Play (NC) otherwise.*

Suppose there are  $K$  deviators. Define the probability of meeting a deviator as  $\kappa = \frac{K}{M-1}$ . In this setting,  $l > 0$  gives the incentives not to deviate from punishing. The incentives to cooperate when the strategy asks players to do so is given by the following condition:

$$1 + \frac{\delta}{1 - \delta}(1 - \rho - \kappa) \geq 1 + g$$

$$\Leftrightarrow \delta \geq \delta^{**} = \frac{g}{(g + 1 - \rho - \kappa)}$$

It follows immediately that  $\delta^{**} > \delta^*$  and that  $\delta^{**}$  is increasing in  $\kappa$  and  $\rho$ . In other words, as the number of deviators increases, one needs more patient players to sustain an equilibrium with the contagious strategy. If there are no deviators, the two conditions are the same, i.e.,  $\delta^* = \delta^{**}$ .

In this strategy players need much more information than before. They need to know the identity and the history of each opponent.

## 4 No Information: Unobservable Types and Actions

### 4.1 Non-Existence of Equilibrium With Cooperation: Symmetric Pure Strategy Case

In contrast to the game with perfect information, in a setting where interactions are anonymous and agents do not observe the history of other players, the cooperative result proved in the prior section breaks down. I show that in a population with one short-run player ( $S = 1$ ) there is no symmetric pure strategy supporting cooperation that is an equilibrium strategy. Long-run

players' behavior depends on their beliefs regarding the number of deviators in the population. I denote  $V_i^a(b_i(S|h_{it}))$  the value for player  $i$  of following stage  $a$  of the strategy given the belief that there are  $S$  players behaving as short-runs (not cooperating). As players have only two actions available ( $C$  or  $NC$ ), strategies can only induce players to take one of these actions in any period. The belief a player has regarding the amount of non-cooperative players in the society depends on his own private history  $h_{it}$ .

When I consider symmetric pure strategies, the following statements hold:

1. To sustain cooperation the strategy must require long-run players to cooperate in the first period. If that is not true, one can always disregard the periods in which the strategy asks agents to defect and restrict attention to the periods starting with cooperation, in which there is something at stake for the future.<sup>1</sup>
2. Any strategy that requires long-run players to cooperate regardless of history is not incentive compatible. If all long-run players are following one such strategy, player  $i$ 's payoffs are:

$$(1 - \delta)V_i^C(1) = \rho(-l) + (1 - \rho).$$

If  $i$  deviates and defects:

$$(1 - \delta)V_i^{NC}(1) = (1 - \rho)(1 + g).$$

Thus, always cooperating would be an equilibrium strategy if:

$$\rho(-l) \geq (1 - \rho)g,$$

which is impossible given  $l$  and  $g$  are both positive.

3. The punishment period is triggered by some history containing observations of defections. If that is not the case, i.e., if the strategy triggers defection after a cooperative history, then by cooperating players would have a present and a future loss. Here, defecting would be a profitable deviation from that strategy. Hence, there must exist some history  $G$  that triggers defection.

---

<sup>1</sup>Defecting is the stage-game dominant strategy for each player. A strategy that asks players to defect would implement the stage game Nash Equilibrium the first period. There are no incentives for players to deviate from that; nothing is at stake for the future. Those initial periods can be disregarded in the repeated game.

It is clear that any strategy that supports cooperation among long-run players will require them to cooperate in the first period of the game and will induce defection only after a history where they suffered defection. As players are anonymous, histories only contain their own and opponents past actions.

In order to support cooperation among long-run players, a symmetric pure strategy would consist of two stages:

- Stage I: Long-run players will have to cooperate in the first period, after the null history, given they know there is  $S = 1$  short-run player. Long-run players have to cooperate as long as history  $G$  did not occur or after the  $T$  periods of punishment took place. The value of being in this stage is denoted  $V_i^{SI}(b_i(S|h_{it}))$ .
- Stage II: Long-run players will have to defect for  $T \geq 1$  (possibly infinite) periods after history  $G$ ; in history  $G$  they experienced defection. This is denoted  $V_i^{SII}(b_i(S|h_{it}))$ .

Notice that history  $G$  is quite general. It includes any history in which defection has been observed.  $G$  can specify the total number of defections or only those observed in certain (even/odd) periods, the number of recurrent defections, etc. The length of the punishment period is also flexible. It can be finite or infinite, and it can be coordinated by a public randomization device or not.

I argue that the strategies described above are not equilibrium strategies when there is a short-run player. It is worth noting that in each period, before choosing an action (cooperate or defect), each long-run player forms a belief regarding the number of non-cooperative players in the society. Thus, different histories of play may induce different beliefs. We call  $S$  the number of non-cooperative opponents in the society and  $b_i(S|h_{it})$  the belief regarding the number of non-cooperative opponents in society after history  $h_{it}$ . Each private history implies a distribution of the non-cooperative players,  $b(S|h_{it}) = pmf(S_t | h_{it})$ , and a single distribution might be induced by different histories of play. Notice that:

$$b(S|\emptyset) = pmf(S | \emptyset) = \{S = 1, \Pr(S = 1) = 1\}.$$

The result is:

**Theorem 1** *In a uniform random matching model with an even number of players and one short-run player, the symmetric pure strategies described above are not equilibrium strategies.*

**Proof.** Given that the number of short-run players at the beginning of the game is known by all long-run players, the first period incentive constraint is:

$$V_i^{SI} (b_i (S|\emptyset)) \geq V_i^{SII} (b_i (S|\emptyset)). \quad (R_\emptyset)$$

History  $G$  triggers the punishment period, in which long-run players should defect.<sup>2</sup> For every  $G$  there exists a history  $\tilde{G}$  such that  $b(S|\tilde{G}) = b(S|\emptyset)$ , i.e. after  $\tilde{G}$  players believe there is only one non-cooperative player in the population. For instance,  $\tilde{G}$  can be the history in which player  $i$  has only met the short-run player. As the expected payoffs of a player are only affected by the distribution of non-cooperative players in the society, after observing  $\tilde{G}$ , player  $i$  will follow the stage II of the strategy if:

$$V_i^{SII} (b_i (S|\tilde{G})) \geq V_i^D (b_i (S|\tilde{G})). \quad (R_{\tilde{G}})$$

Thus,  $R_{\tilde{G}}$  implies that there is no profitable one shot deviation (denoted  $V_i^D$ ) to  $V_i^{SII} (b_i (S|\tilde{G}))$ . Given that  $b(\tilde{G}) = b(\emptyset)$ , by the null history's condition  $R_\emptyset$  we know that there is a profitable deviation to  $V(b(\tilde{G}), NC)$  and that is  $V(b(\emptyset), C)$ . The one-shot-deviation principle establishes that if there is a profitable deviation, then there is a one-shot profitable deviation:  $V_i^{SII} (b_i (S|\tilde{G})) \leq V_i^D (b_i (S|\tilde{G}))$ . Thus, both restrictions  $R_\emptyset$  and  $R_{\tilde{G}}$  can only hold with equality.

To see this, consider the value of deviating from Stage II by cooperating for  $K$  periods:  $V_i^{DK} (b_i (S|\emptyset))$ . We can rewrite  $R_{\tilde{G}}$  as:

$$V_i^{D0} (b_i (S|\emptyset)) \geq V_i^{D1} (b_i (S|\emptyset)).$$

By induction, given that I can always disregard the beginning periods of cooperation:

---

<sup>2</sup> $G$  describes the conditions under which (number of defections, frequency and periods in which they occurred) a player should start defection for  $N$  periods.

$$V_i^{D_K} (b_i (S|\emptyset)) \geq V_i^{D_{(K+1)}} (b_i (S|\emptyset))$$

If the restriction making  $R_{\tilde{G}}$  incentive compatible is satisfied, then it has to be the case that:

$$V_i^{S_{II}} (b_i (S|\emptyset)) = V_i^{D_0} (b_i (S|\emptyset)) \geq V_i^{D_1} (b_i (S|\emptyset)) \geq V_i^{D_\infty} (b_i (S|\emptyset)) = V_i^{S_I} (b_i (S|\emptyset))$$

Under the null history, the strategy is incentive compatible if and only if the restrictions are satisfied with equality. This implies that players have to be indifferent between cooperating and defecting at the beginning of the game.

Depending on  $G$ , there exist other private histories that induce the same belief regarding the number of deviators in the economy ( $K$ ). In some cases,  $G$  occurred and the strategy asks for Stage II, while in the other  $G$  did not occur and it requests cooperation (Stage I). Given this new belief, the two incentive compatibility constraints require equality again. Two possibilities need to be considered. First, if  $i$ 's defection does not affect future play of his opponents, this is not an equilibrium strategy. There is nothing at stake for the future, and  $(NC)$  is a best response in the one-shot game, breaking the indifference. Secondly, if  $i$ 's defection does affect the future, then it has to be exactly in the same way that it did under the starting belief. As a result, the marginal effect of an extra defection is constant for any number of defectors in the population. This is false. The effect of  $i$ 's behavior is decreasing in the number of deviators given that the probability of history  $G$  is greater for all non-defecting players when there are more defectors in the population.<sup>3</sup>

■

This general argument implies that any symmetric pure strategy that attempts to sustain cooperation among long-run players is not an equilibrium strategy because it requires different responses for the exact same beliefs. This argument can be extended to the case of asymmetric pure strategies, as we show in Appendix 1. A contagious strategy à la Kandori is a special example of the general strategy described above. In that strategy  $N = \infty$ ,  $G$  states “*defect after you experienced a defection or after you have defected,*”

---

<sup>3</sup>This proof follows Ellison’s 1994 reasoning closely, acknowledging that a defection triggers stage II only in the case of a contagious strategy. For more general strategies, it is in any way affecting the probability that the trigger ( $G$ ) occurs, and the same reasoning ensures that the marginal effect of a deviation decreases with the number of deviators.

and  $D$  implies cooperation for one period and defection ever after. A detailed proof for this example is presented in the following section.

## 4.2 The Contagious Strategy With a Short-Run Player

In this section I focus on the proof for the contagious strategy case. I show that this commonly used strategy is not an equilibrium strategy when there is a short-run player in the population.

It is straightforward to show that under assumptions of no information and anonymity a contagious process will spread with probability one. In other words, the probability that the short-run players are the only ones not cooperating in equilibrium approaches zero. The probability that the number of non-cooperating players does not increase in a period given long-run players following a contagious strategy and with  $S$  short-runs is given by:

$$P_0(S) = \begin{cases} \prod_{j=0}^{\frac{S}{2}-1} \frac{S-(1+2j)}{M-(1+2j)} & \text{if } S \text{ is even,} \\ 0 & \text{if } S \text{ is odd.} \end{cases}$$

Taking the limit as the number of independent interactions goes to infinity:

$$\lim_{t \rightarrow \infty} [P_0(S)]^t = 0 \quad \forall S.$$

Given this, the contagious process will start almost surely.

The following result shows that a contagious strategy cannot support cooperation among long-run players when there is a short-run player in the population. In other words:

**Proposition 2** *In a uniform random matching model with an even number of players and one short-run player, a contagious strategy is not an equilibrium strategy.*

Before proving the proposition, it is necessary to introduce notation and prove some preliminary results. Following Kandori's (1992) paper, I will

define the diffusion Markov matrix  $A$ , of dimension  $(M \times M)$  and define  $X_t$  as the number of non-cooperative players at time  $t$ . Each element of the matrix is defined by  $a_{ij} = \Pr(X_{t+1} = j \mid X_t = i)$ . Notice that  $a_{ij} = 0$  for all  $(j \leq i)$ , for all  $j$  odd and for all  $j \geq 2i$ . An example of matrix  $A$  when  $M = 6$  is provided in Appendix 4. Matrix  $A$  has a unique absorbent state, which occurs when all  $M$  players are non-cooperative. For later use, define  $\tau = \frac{1}{M-1}(M-1, M-2, M-3, \dots, 1, 0)^T$ , a column vector of dimension  $(M \times 1)$ . The  $i$ th element of  $\tau$  represents the conditional probability that a non-cooperative player meets a cooperative player given that there are  $[M-i]$  cooperative players in the economy. In addition,  $e_i$  is defined as to be the  $[1 \times M]$  row vector with  $i$ th element 1 and zeros everywhere else.

In this setup, the contagious strategy has two stages:

- *Stage I: Start the game cooperating, then play (C) if you played (C) before and nobody played (NC) against you.*
- *Stage II: Play (NC) if you played (NC) before or someone played (NC) against you. That is, after these histories the strategy asks you to behave as if you were a short-run player.*

Each stage imposes a constraint on the parameters for which the strategy can be sustained in equilibrium. Restriction I, the one imposed in Stage I, asks players to cooperate when they have experienced a history of all cooperative encounters. To keep track of player's histories, I define  $\lambda = 1$  when all prior interactions are cooperative and  $\lambda = 0$  otherwise. Restriction II, on the other hand, asks players to defect after any non-cooperative encounter. It is worth noting that Restriction II in Kandori's framework refers to out-of-equilibrium behavior. In contrast, in this model this restriction occurs on the equilibrium path, imposing an additional restriction on players' beliefs regarding the number of non-cooperative players.

To check whether the contagious strategy is an equilibrium strategy, suppose first that all but one long-run player follow this strategy, then consider the incentive to deviate for the remaining long-run agent. For the agent not to deviate, there must exist a set of parameters  $(l, g, \delta)$  such that the following two conditions hold:



1. First, for all  $t$  such that all previous encounters have been cooperative,

$$E_t [V^{S_I}(S_t) | S_o = 1] - E_t [V^{S_{II}}(S_t) | S_o = 1] \geq 0. \quad (R_I)$$

2. Second, for all  $t$  and all non-cooperative histories,

$$E_t [V^{S_{II}}(S_t) | S_o = 1] - E_t [V^D(S_t) | S_o = 1] \geq 0. \quad (R_{II})$$

Here,  $S_t^4$  refers to the number of players, excluding the decision maker, who behave myopically in period  $t$ .  $D$  refers to the behavior in which an agent cooperates for one period and does not cooperate ever after (a one-shot deviation from stage II).

In this game players form beliefs about the number of short-run-behaving players in the society following each cooperative history. Given those beliefs, the restriction  $R_I$  says that a player should prefer to continue cooperating rather than to deviate and defect forever. In each case, agents know the initial number of short-run players and the contagious process as represented by matrix  $A$ .

Before moving on to the proof, I present a series of preliminary results to ease exposition.

1. After the first period, the number of non-cooperating players is uncertain. For any uncertain  $S_t$ , I define the value in  $t$  of following each stage of the strategy as,

$$E_t [V^{S_I}(S_t) | S_o = 1] = \sum_{x=1}^{M-1} \Pr(S_t = x | S_o = 1 \text{ and } \lambda = 1) \times V^{S_I}(x),$$

$$E_t [V^{S_{II}}(S_t) | S_o = 1] = \sum_{x=1}^{M-1} \Pr(S_t = x | S_o = 1 \text{ and } \lambda = 0) \times V^{S_{II}}(x).$$

---

<sup>4</sup>Notice that  $X_t$  includes all players while  $S_t$  excludes the decision maker. When the decision maker defects  $X_t = S_t + 1$ .

2. The value of not cooperating when all opponents are defecting is zero. In other words,

$$V^{S_{II}}(M - 1) = 0.$$

3. After observing a cooperative outcome, a player knows with certainty that it is not the case that all prospective opponents are defecting. Thus, given the information about the diffusion process contained in matrix  $A$ , players update their beliefs regarding the number of non-cooperative players in society by shifting the weight assigned to the case  $S = (M - 1)$  to the remaining alternatives. In particular,

$$\Pr(S_t = M - 1 \mid S_o = 1 \text{ and } \lambda = 1) = 0.$$

4. After a cooperative history of  $t$  periods the only consistent belief will put high probability on the event that all but one of the prospective opponents are defecting. That is,

$$\lim_{t \rightarrow \infty} [\Pr(S_t = M - 2 \mid S_o = 1 \ \& \ \lambda = 1)] = 1.$$

The intuition is as follows. Matrix  $A$  involves a diffusion process that is shifting weight to a larger  $S$ , since it is an upper-triangular matrix with only one absorbent state when  $S = M$ . However, when a player has experienced a  $t$  period history of cooperation he knows for sure that not all players are defecting (see point (3) above). Thus, after  $t$  periods he knows that all events where  $S \in \{2, (M - 2)\}$ ,  $S$  even, have positive probability. Given that  $A$  is upper triangular and that the probability of staying at any given state  $S$  is decreasing in  $S$ , the event probability is accumulating at the largest  $S_t$  allowed by the observed history. This event is  $S = (M - 2)$ .

5. In stage II of the game, the value of not cooperating is decreasing in the number of players already not cooperating in the economy. That is,

$$V^{S_{II}}(S) \geq V^{S_{II}}(S + 1).$$

Notice that for any  $S$ , we can rewrite the value function

$$V^{S_{II}}(S) = \sum_{t=0}^{\infty} \delta^t e_{S+1} A^t \tau,$$

as in Kandori (1992). In this alternate expression,  $e_{S+1}A^t$  first order stochastically dominates  $e_S A^t$ . Given that  $\tau$  is a decreasing function, the result follows.

With these pieces in place, I am now ready to prove Proposition 1.

**Proof.** I want to show that

$$\{(l, g) : R_I \geq 0\} \cap \{(l, g) : R_{II} \geq 0\} = \emptyset. \quad (1)$$

I first consider  $R_I$ , i.e.,  $\{(l, g) : R_I \geq 0\}$ . By points (1) and (4) above, it suffices to analyze the restriction when  $S = (M - 2)$ . That is, after any history of  $t$  cooperative encounters, a player is almost sure that there are  $(M - 2)$  non-cooperative players and the strategy asks him to prefer to cooperate the next period rather than deviate and not cooperate. Thus  $R_I$  can be replaced by the restriction

$$V^{S_I}(M - 2) - V^{S_{II}}(M - 2) \geq 0. \quad (2)$$

Moreover, since

$$V^{S_I}(M - 2) = \left[ \frac{(M - 2)}{(M - 1)}(-l) + \left(1 - \frac{(M - 2)}{(M - 1)}\right) \right] \frac{1}{1 - \delta \left(1 - \frac{(M - 2)}{(M - 1)}\right)}$$

and

$$V^{S_{II}}(M - 2) = \left(1 - \frac{(M - 2)}{(M - 1)}\right) (1 + g).$$

Condition (2) can be rewritten

$$(1 + g) \leq [1 - l(M - 2)] \frac{1}{1 - \frac{\delta}{(M - 1)}}. \quad (R_{I(M-2)})$$

Next, consider the second bracketed term on the left hand side of (1). It is necessary to show that for any number of expected non-cooperative players, a player in Stage II prefers to defect rather than to deviate by cooperating one period and defecting thereafter. In particular, if a player meets the short-run player in Period 1, he knows there is only one non-cooperative opponent, and Stage II of the strategy asks him to defect. Thus, in that particular case, the restriction is

$$R_{II(1)} \equiv V^{S_{II}}(1) - V^D(1) \geq 0,$$

where

$$R_{II(1)} = \frac{1}{(M-1)}l + \frac{(M-2)}{(M-1)}g - \frac{(M-2)}{(M-1)}\delta [V^{S_{II}}(2) - V^{S_{II}}(3)].$$

With  $S = (M-3)$ , the restriction is

$$R_{II(M-3)} \equiv V^{S_{II}}(M-3) - V^D(M-3) \geq 0,$$

where

$$R_{II(M-3)} = \frac{(M-3)}{(M-1)}l + \frac{2}{(M-1)}g - \frac{2}{(M-1)^2}\delta(1+g).$$

This restriction implies

$$(1+g) \geq \left[1 - l \frac{(M-3)}{2}\right] \frac{1}{1 - \frac{\delta}{(M-1)}}. \quad (R_{II(M-3)})$$

From this, it is straightforward to show that<sup>5</sup>

$$\{(l, g) : R_I \geq 0\} \cap \{(l, g) : R_{II(M-3)} \geq 0\} = \emptyset, \quad \forall l, g > 0, \quad \forall \delta \quad (3)$$

It remains to show that for all  $(l, g)$ ,  $l > 0$ ,  $g > 0$ , for which  $R_{II(M-3)} \geq 0$  is true then  $R_{II(1)} \geq 0$  is also true. It will then follow that  $R_{II(1)} \geq 0 \Rightarrow R_{II(M-3)} \geq 0$ . Given that both equations are linear in  $l$  and  $g$ , it is sufficient to look at the restrictions when  $l = 0$  and  $g = 0$  and show that the  $\delta$  intercepts are larger when the restriction  $R_{II(1)}$  holds than when the restriction  $R_{II(M-3)}$  holds. Thus, with two linear restrictions, four intercepts are considered: two when  $g = 0$  and two when  $l = 0$ .

When  $g = 0$  and restriction  $R_{II(1)}$  holds,

$$l_{R_{II(1)}(g=0)} = \frac{(M-2)}{(M-1)(M-3)}\delta \{M-3 + (M-4)\delta\Lambda\},$$

---

<sup>5</sup>If we picture the restriction on the  $(l, 1+g)$  space, the y-intercept is the same for both equations  $(\frac{1}{1-\frac{\delta}{M-1}})$  and the x-intercept is larger in  $R_{II(1)}$ .  $(\frac{2}{M-3} > \frac{1}{M-2})$  (See Figure 2 below).

where  $\Lambda = 3V(3, NC) + (M - 8)V(5, NC) - (M - 5)V(7, NC) \geq 0$ , by point (5) above. Alternatively, when we consider  $R_{II(M-3)}$ ,

$$l_{R_{II(M-3)}(g=0)} = \frac{2\delta}{(M-1)(M-3)}.$$

Thus, for any  $M \geq 4$ ;  $l_{R_{II(1)}(g=0)} \geq l_{R_{II(M-3)}(g=0)}$ .

When  $l = 0$  and restriction  $R_{II(1)}$  holds,

$$g_{R_{II(1)}(l=0)} = \frac{\frac{\delta}{M-1}}{1 - \frac{\delta}{M-1}} + \frac{1}{1 - \frac{\delta}{M-1}} \frac{\delta^2 (M-4)}{(M-1)(M-3)} \Lambda,$$

and when  $R_{II(M-3)}$  holds,

$$g_{R_{II(M-3)}(l=0)} = \frac{\frac{\delta}{M-1}}{1 - \frac{\delta}{M-1}},$$

since  $\Lambda \geq 0$ ,  $g_{R_{II(1)}(l=0)} \geq g_{R_{II(M-3)}(l=0)}$ .

It follows that,  $R_{II(1)} \geq 0 \Rightarrow R_{II(M-3)} \geq 0$ .

Finally, this implies:

$$\begin{aligned} \{(l, g) : R_I \geq 0\} \cap \{(l, g) : R_{II(M-3)} \geq 0\} &= \emptyset \quad \forall l, g > 0, \forall \delta \Rightarrow \\ \{(l, g) : R_I \geq 0\} \cap \{(l, g) : R_{II(1)} \geq 0\} &= \emptyset \quad \forall l, g > 0, \forall \delta. \end{aligned}$$

By Condition 3 above, the former condition holds, establishing the latter. There is no set of parameters  $(l, g, \delta)$  such that both restrictions hold, proving the theorem.

A graphical display of the analysis is presented in Figure 1.<sup>6</sup> The intersection where both restrictions are satisfied is outside the relevant payoffs range. ■

To sum up, I have shown that when there is a short-run player, the contagious strategy is not an equilibrium strategy. The contagion will eventually affect the whole population. The threat of a faster contagious process after a deviation does not prevent agents from deviating to get  $(1 + g)$  and avoid the loss of  $l$  today.

---

<sup>6</sup>Payoffs  $g < 0$  ( $1 + g < 1$ ) are not the ones described in this paper. The axis in the graph are just convenient for the graphical representation. The relevant range is  $(1 + g) \geq 1$ .

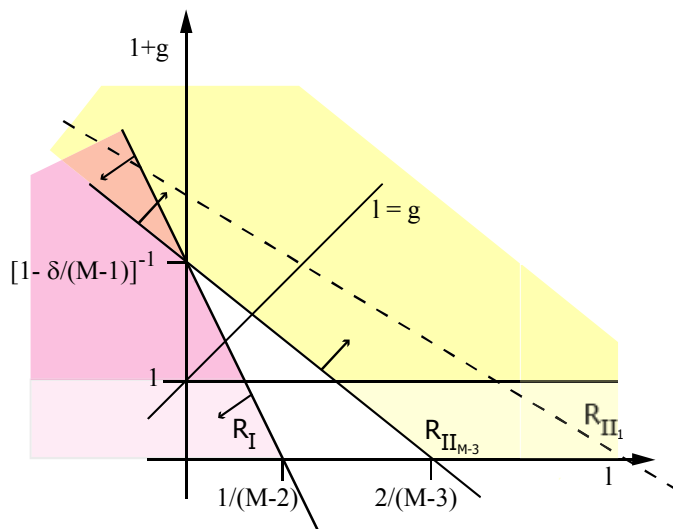


Figure 1: Non-existence of equilibrium

## 5 Identifying Cooperative Behavior

Given the previous section's result, when players use contagious strategies some information regarding opponents is necessary to get cooperative behavior. I now consider a plausible information technology that resembles the informational institutions that allow firms to recognize clients and to make them recognizable for other firms. Some real life examples of these information systems are airlines' frequent flyer programs, credit history and credit cards' holders.

The following information technology is considered: *Label all players who cooperated in the first period. Thereafter, erase labels from those who defect against labeled opponents.* After the first period, the technology monitors only transactions among labeled agents. All players enter the game unlabeled and at the same moment. This technology enables the identification of cooperative players, separating them from the  $S$  short-run players and the deviators.

Players observe their opponents labels after the first period. I consider the following strategy: *Cooperate the first period. Cooperate with labeled*

opponents if you are also labeled. Otherwise defect.

To verify that this is a sequential equilibrium strategy, I need to check :

1.  $(NC)$  once you are unlabeled is better than  $(C)$
2.  $(NC)$  against unlabeled people is better than  $(C)$
3.  $(C)$  against labeled people is better than  $(NC)$
4.  $(C)$  at the beginning is better than  $(NC)$ .

Player  $i$  makes his decision taking into account: whether  $i$  is labeled ( $L_i$ ) or unlabeled ( $U_i$ ); whether his opponent  $j$  is labeled ( $L_j$ ) or unlabeled ( $U_j$ ); whether  $i$  has seen a defecting labeled opponent or not ( $D_i$  or  $C_i$ ).

**Proof.**  $l > 0$  ensures that  $(NC)$  once unlabeled is better than  $(C)$  with all opponents and that  $(NC)$  against unlabeled people is better than  $(C)$  when labeled. Notice that the value to a player  $i$  of not having a label after period one is:

$$V_i^{NC}(U_i, j) = 0 \quad \forall j.$$

$V_i^{NC}(L_i, U_j) \geq V_i^C(L_i, U_j)$  because there is nothing at stake for the future. The information technology is not monitoring this transaction; no matter what  $i$  does;  $i$  will have a label next period, and  $j$ 's beliefs do not change with  $i$ 's action.

I need to show that  $(C)$  with labeled people is better than  $(NC)$ . Each labeled player  $i$  has to cooperate  $(C)$  with labeled opponents given his beliefs regarding the amount of unlabeled long-run players in the economy  $K$  ( $0 \leq K \leq L - 2$ ), who represent a proportion  $\kappa$  of the population. Once player  $i$  meets labeled player  $j$ , it is required that  $V_i^C(L_i, L_j, K) \geq V_i^{NC}(L_i, L_j, K)$ , so that:

$$1 + \frac{\delta}{1 - \delta} (1 - \rho - \kappa) \geq 1 + g$$

$$\delta \geq \delta^* = \frac{g}{g + (1 - \rho - \kappa)}.$$

Notice that  $\delta^* \in (0, 1)$  and is increasing in  $\kappa$ . The larger the proportion of the population that has already deviated, the more appealing it is for another long-run player to deviate. For any history in which a player met a labeled opponent,  $\kappa$  is lower than  $\frac{L-2}{M-1}$  which implies that  $(1 - \rho - \kappa)$  is at least  $\frac{1}{M-1}$  so  $\delta^*$  is less than 1. Finally, it has to be the case that cooperating  $(C)$  at the

beginning of the game is better than defecting ( $NC$ ),

$$\begin{aligned} V_i^C(\emptyset) &\geq V_i^{NC}(\emptyset) \\ \rho(-l) + (1 - \rho) + \frac{\delta}{1 - \delta} (1 - \rho) &\geq (1 - \rho)(1 + g) \\ \delta &\geq \hat{\delta} = \frac{\rho l + (1 - \rho)g}{\rho l + (1 - \rho)(1 + g)}. \end{aligned}$$

Notice that  $\hat{\delta} \in (0, 1)$ . ■

When  $\kappa = 0$ , the difference between the two discount factors depends on the difference between the payoffs, in other words:

$$\text{sgn}(\delta^* - \hat{\delta}) = \text{sgn}(g - l).$$

If  $g > l$ ,  $\delta^*$  is the binding condition given that the temptation to cheat against a labeled player is high. If  $g < l$ ,  $\hat{\delta}$  is the binding condition, because there are high incentives to avoid the possible loss when cheated in the first period. Naturally, for large  $\kappa$ ,  $\delta^*$  is the binding condition.

This technology involves the payment of a fee in order to belong to the ‘labeled club’. Players pay at the beginning of the game for the monitoring of the first transaction.

The payoff for following the strategy in this setting is given by:

$$V_{GR}^C = \rho(-l) + \frac{1}{1 - \delta} (1 - \rho).$$

Thus, this system involves a present loss given by the probability of meeting the short-run player. This loss enables the system to identify cooperative long-runs forever.

## 5.1 Sequential Entrance of Long-run Players

As an extension of the previous model I allow long-run players to enter the game at any time. A realistic information technology would need to include this possibility.



In each period a proportion  $N$  of the long-run players is replaced with new long-run players. Thus, in any period, the opponent is an unlabeled short-run player with probability  $\rho = \frac{S}{M-1}$  and a new long-run with probability  $\eta = \frac{N}{M-1}$ . Notice that  $\eta$  affects the survival rate of the long-run players. In particular, if in every period there are  $M - S$  long run players, then the probability of being replaced is given by  $\pi = \frac{N}{M-S}$ .<sup>7</sup> The information mechanism monitors all interactions involving labeled and new players in all periods. This enables it to enforce behavior of labeled opponents and to evaluate new players. We assume that the mechanism recognizes new players.<sup>8</sup>

Players observe their opponents labels every period and are aware of the replacement rate of short and long-run players. I now consider the following strategy: *Cooperate your first period. Cooperate with labeled opponents if you are also labeled, otherwise defect.*

To verify this strategy conforms a sequential equilibrium, we need to check:

1.  $(NC)$  once you are unlabeled is better than  $(C)$
2.  $(NC)$  against unlabeled people is better than  $(C)$
3.  $(C)$  against labeled people is better than  $(NC)$
4.  $(C)$  in your first period is better than  $(NC)$ .

**Proof.** The value to a player  $i$  of not having a label is:

$$V_i^{NC}(U_i, j) = \frac{\eta}{1 - \delta(1 - \pi)}(1 + g) \quad \forall j.$$

$V_i^{NC}(U_i, j) \geq V_i^C(U_i, j)$  because there is nothing at stake for the future. The information technology will not change  $i$ 's status; no matter what  $i$  does,  $i$  will have a label next period. Beliefs of  $j$  do not change with  $i$ 's action. The same is true for a labeled player  $i$  facing an unlabeled opponent. As the only difference is the present behavior, these restrictions are satisfied because  $NC$  is a dominant strategy of the stage game.

---

<sup>7</sup> $\pi < 1$  implies that  $N < M - S$ , thus the proportion of long-runs that are replaced cannot be larger than the proportion of long-runs. Adding a survival rate is the same as reducing the discount factor to  $\tilde{\delta} = \delta(1 - \pi)$ .

<sup>8</sup>And new players are essentially different from unlabeled old players. This is a non-forgiving mechanism: once a player lost his label, he can't recover it.

I also need to show that  $(C)$  with labeled people is better than  $(NC)$ . Each labeled player  $i$  has to cooperate  $(C)$  with labeled opponents given his beliefs regarding the amount of unlabeled long-run players in the economy  $K$  ( $0 \leq K \leq L-2$ ), who represent a proportion  $\kappa$  of the population. Once player  $i$  meets labeled player  $j$ , it is required that  $V_i^C(L_i, L_j, K) \geq V_i^{NC}(L_i, L_j, K)$ , so that:

$$\frac{\delta(1-\pi)}{1-\delta(1-\pi)}(1-\rho-\kappa-\eta) \geq g$$

$$\delta(1-\pi) \geq [\delta(1-\pi)]^* = \frac{g}{g+(1-\rho-\kappa-\eta)}.$$

Notice that  $[\delta(1-\pi)]^* \in (0, 1)$  and is increasing in  $\kappa$ . The larger the proportion of the population that has already deviated, the more appealing it is for another long-run player to deviate. Given a player met a labeled opponent,  $\kappa$  is lower than  $\frac{L-2}{M-1}$  which implies that  $(1-\rho-\kappa-\eta)$  is at least  $\frac{1}{M-1}$  so  $[\delta(1-\pi)]^*$  is less than 1.

Finally, it has to be the case that cooperating  $(C)$  when entering the game is better than defecting  $(NC)$ , so that:

$$V_i^C(\emptyset) \geq V_i^{NC}(\emptyset)$$

$$\frac{\delta(1-\pi)}{1-\delta(1-\pi)}(1-\rho-\eta) \geq \rho l + (1-\rho)g$$

$$\delta(1-\pi) \geq \overline{\delta(1-\pi)} = \frac{\rho l + (1-\rho)g}{\rho l + (1-\rho)(1+g) - \eta}.$$

Notice that  $\overline{\delta(1-\pi)} \in (0, 1)$ . ■

All restrictions are similar to the case with common entrance except that they are on the effective discount factor, which is modified by the survival rate. With the presence of new long-run players, every period requires more patient players. Now the loss from deviating is lower than before since the payoff of being unlabeled is now positive because some opponents will be trying to earn a label. Players need to be much more patient when there is entrance every period.

The payoff for following the strategy in this setting is given by:

$$V_{GR}^C = \rho(-l) + (1-\rho) + \frac{\delta(1-\pi)}{1-\delta(1-\pi)}(1-\rho+\eta g).$$

This system involves a present loss given by the probability of meeting the short-run player that enables the system to identify them forever.

Due to the survival probability, the value of cooperating in this setting is lowered. Nevertheless, if the payoff derived when defecting against a cooperative player is large enough (large  $g$ ), then this sequential entry system might involve a higher payoff than that with only one entry. The new players not only forgo some income at entry but also generate a positive payment to the existing long run players.

Comparing the two equilibrium outcomes for a  $\delta$  that supports cooperation under both settings; the gain from sequential entrance has to be larger than the loss of it, as:

$$\begin{aligned} \frac{\delta(1-\pi)}{1-\delta(1-\pi)}\eta g &\geq \frac{\delta\pi(1-\rho)}{(1-\delta)[1-\delta(1-\pi)]} \\ \eta g &\geq \frac{\pi(1-\rho)}{(1-\pi)(1-\delta)} \\ g &\geq \frac{1}{(1-\delta)}\frac{M-S-1}{M-S-N}. \end{aligned}$$

The gain from defecting has to be larger than the present value of the proportion of long-run opponents to old long-run opponents. More new players involves a lower probability of surviving and thus players have to be compensated by a higher payoff when defecting.

## 6 Conclusion

In this paper, I have presented a model of random pairwise interactions in a large population of agents who play a Prisoner's Dilemma stage game. I have shown how the inclusion of a short-run player makes the sustainability of a cooperative outcome more complex. The short-run player will trigger the diffusion of defection with probability one. Thus, when no information is available, any cooperative equilibrium sustained by a symmetric pure

strategy collapses. In addition, I have shown how plausible information institutions can sustain cooperation in equilibrium by imposing a present cost to the players.

My model is an application of Kandori's (1992) model. In that paper, a Folk theorem for a matching game with homogenous agents was proven. The generalization of Folk theorems to populations was initiated by Milgrom et al. (1990) and Okuno-Fujiwara and Postlewaite (1995) who restricted attention to games with an infinite number of players. Ellison (1994) extended Kandori's result by allowing for a public randomization device. In the setting presented in this paper, a short-run player will disrupt cooperation even when a public randomization device is available. Harrington (1995) allowed for non-uniform matching and non-anonymous players, and Ahn-Suominen (2001) analyzed the possibility of local communication. None of these results allowed for heterogeneous agents. Gosh and Ray's (1996) paper sustained cooperation in a model with some myopic players. Their result does not contradict the one presented in this paper because they departed from the random matching framework and allowed agents to choose frequent interaction with the same partner. I have shown in this paper that allowing for heterogeneity in a random matching framework results in the breakdown of cooperation sustained by a symmetric pure strategy.

While this paper is able to explain the crucial role of informational technology in an economy with heterogeneous players, some interesting questions are still unanswered. First, a *least costly information technology* remains to be defined. Second, this could be extended to situations where the labeling mechanism need not work perfectly, at each stage agents getting labels with only a certain probability. Ahn and Suominen's witnesses game, for example, could be extended to this framework.

## 7 Appendix

### 7.1 Asymmetric Pure Strategies

The argument presented in Section 4.1 can be extended to asymmetric pure strategies. In that case, each player or group of players follows a different

strategy. In each of them, after a history  $G_j$ , they are required to defect. As history  $G$  in the symmetric case, each  $G_j$  involves a certain number of non-cooperative experiences. Thus, for each  $G_j$  there exists a  $\widetilde{G}_j$  in which player  $j$  believes he is the first player to be asked not to cooperate. Thus,  $b(\widetilde{G}_j) = pmf(S | \widetilde{G}_j) = \{S = 1, \Pr(S = 1) = 1\} = b(\emptyset)$ . As before, this strategy is not an equilibrium strategy because it requires each player to prefer different actions for the same belief.

## 7.2 The example for $M = 4$ and $S = 1$

As an example of the non-existence of the equilibrium it is worth analyzing the case  $M = 4$  and  $S = 1$ . This is a special case because after a history of cooperative encounters a player knows for sure the number of non-cooperative players in the population. As in the general case, in this setup Restriction I requires players to cooperate after a cooperative history instead of defecting forever. With only four players, the strategy asks long-run players to cooperate at the beginning of the game (when  $S = 1$ ) and after any cooperative history (when  $S = 2$ ).

When  $S = 2$  Restriction I implies

$$V(2, C) - V(2, NC) \geq 0$$

$$(1 - 2l) \left( \frac{1}{1 - \frac{1}{3}\delta} \right) \geq (1 + g)$$

and when  $S = 1$

$$V(1, C) - V(1, NC) \geq 0$$

$$\left( 1 - \frac{(1 + \delta)l}{2} \right) \left( \frac{1}{1 - \frac{1}{3}\delta} \right) \geq (1 + g).$$

It is sufficient to show that there is no equilibrium when the second condition is satisfied. For any history  $t \geq 2$  of good encounters a player knows for sure that there are  $S = 2$  non-cooperative players in the population. It is necessary and sufficient to check that there is no equilibrium when the first restriction holds.

Stage II of the game imposes Restriction II, i.e., a player has to defect after he has seen a defection. In this setup, when a player sees a defection in Period 1 he knows that there are  $S = 1$  non-cooperative opponents in the population, and when he sees a defection in any future period, he knows that there are  $S = 3$  non-cooperative players. As long as  $l > 0$ , the restriction for  $S = 3$  holds:

$$V(3, NC) = 0 \geq V(3, C_1) = -l.$$

The strategy also requires the player to defect forever after suffering a defection in the first period. This restriction is given by:

$$\begin{aligned} V(1, NC) &\geq V(1, C_1) \\ (1 + g) &\geq \left(1 - \frac{1}{2}l\right) \frac{1}{\left(1 - \frac{1}{3}\delta\right)}. \end{aligned}$$

Notice that when  $l = 0$ , all restrictions reach equality at the same point. When  $(1 + g) = 0$ ,  $l = 2$  according to the last restriction. That is, this intercept<sup>9</sup> is higher than  $\frac{2}{(1+\delta)} = l$  given  $\delta > 0$ . Thus, there is no equilibrium where cooperation can be sustained with a contagious strategy in the case  $M = 4$  and  $S = 1$ , as Figure 2 shows.

### 7.3 Kandori with $M = 4$

For the sake of comparison, I introduce a graphical analysis of Kandori's folk theorem result when  $M = 4$ . On the equilibrium path, Kandori's condition is:

$$\begin{aligned} V(0, C) &\geq V(0, NC) \\ \frac{(3 + \delta)}{(1 - \delta)(3 - \delta)} &\geq (1 + g). \end{aligned}$$

Notice that the number  $\frac{(3+\delta)}{(1-\delta)(3-\delta)} > 1 \forall \delta \in (0, 1)$ . Thus, for each  $g > 0$  there exists a  $\delta \in (0, 1)$  such that this restriction holds. Kandori's off-equilibrium path condition has to hold for any  $S \geq 1$ .

---

<sup>9</sup>If we consider the space  $(l, (1 + g))$  as in the figure. All restrictions have the same y-intercept, but the x-intercept is higher in restriction II.

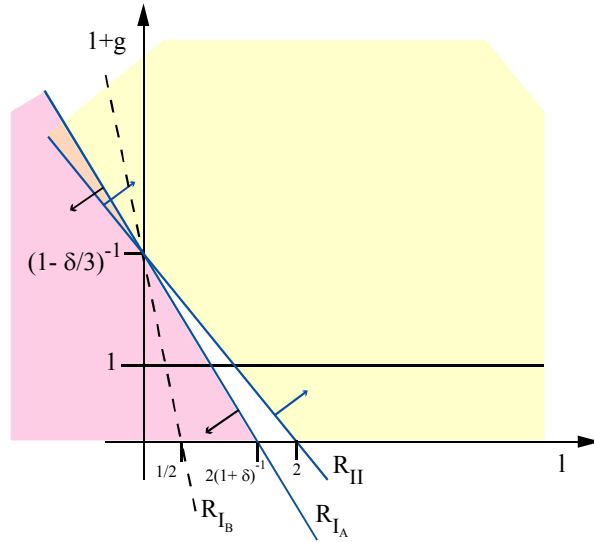


Figure 2:  $M = 4, S = 1$

For  $S = 3$ ,

$$\begin{aligned} V(3, NC) &\geq V(3, C_1) \\ 0 &\geq -l. \end{aligned}$$

For  $S = 2$ , the restriction is

$$\begin{aligned} V(2, NC) &\geq V(2, C_1) \\ (1 + g) &\geq (1 - 2l) \frac{1}{(1 - \frac{1}{3}\delta)} \end{aligned}$$

and for  $S = 1$ , it is

$$\begin{aligned} V(1, NC) &\geq V(1, C_1) \\ (1 + g) &\geq \left(1 - \frac{l}{2}\right) \frac{1}{(1 - \frac{1}{3}\delta)}. \end{aligned}$$

It is worth noticing that whenever the restriction for  $S = 1$  is satisfied, then the other ones are also satisfied (given  $l > 0$ ), as:

$$(1 + g) \geq \left(1 - \frac{l}{2}\right) \frac{1}{\left(1 - \frac{1}{3}\delta\right)} \geq (1 - 2l) \frac{1}{\left(1 - \frac{1}{3}\delta\right)}.$$

Thus, the two restrictions in Kandori's  $M = 4$  model are:

$$(1 + g) \leq \frac{(3 + \delta)}{(1 - \delta)(3 - \delta)}$$

$$(1 + g) \geq \left(1 - \frac{l}{2}\right) \frac{1}{\left(1 - \frac{1}{3}\delta\right)}.$$

As Figure 3 shows, when  $\delta$  and  $l$  are sufficiently large, there exists an equilibrium supported by this strategy. In particular, the equilibrium involves cooperation along the equilibrium path.

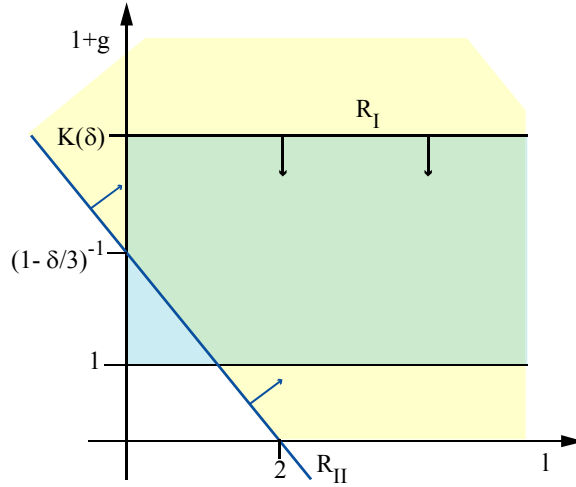


Figure 3: Kandori.  $M = 4$

#### 7.4 Diffusion Matrix $\mathbf{A}$ when $M = 6$

$$A_6 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{5} & 0 & \frac{4}{5} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{5} & 0 & \frac{2}{5} \\ 0 & 0 & 0 & \frac{1}{5} & 0 & \frac{4}{5} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$



## References

- [1] Ahn, I., and M. Suominen (2001), “Word-of-Mouth Communication and Community Enforcement,” *International Economic Review*, 42: 399-415.
- [2] Araujo, L. (2004), “Social Norms and Money,” *Journal of Monetary Economics*, 51(2): 241-256.
- [3] Dal Bo, P. (2007), “Social Norms, Cooperation and Inequality,” *Economic Theory*, January.
- [4] Ellison, G. (1994), “Cooperation in the Prisoner’s Dilemma With Anonymous Random Matching,” *Review of Economic Studies*, 61: 567-88.
- [5] Ely, J. and Valimaki J. (2002), “A Robust Folk Theorem for the Prisoner’s Dilemma,” *Journal of Economic Theory*, 102: 84-105.
- [6] Fudenberg, D. and Maskin E. (1986), “The Folk Theorem in Repeated Games With Discounting or With Incomplete Information,” *Econometrica*, 54: 533-56.
- [7] Fudenberg, D. Kreps, D. and Maskin E. (1990), “Repeated Games With Long-run and Short-run players,” *Review of Economic Studies*, 57: 555-73.
- [8] Ghosh P, Ray D, (1996), “Cooperation in Community Interaction Without Information Flows,” *Review of Economic Studies* 63 (3): 491-519.
- [9] Haag, M. and Lagunoff R. (2004), “Social Norms, Local Interaction, and Neighborhood Planning,” *International Economic Review*, forthcoming.
- [10] Harrington, J. E. (1995), “Cooperation in a One-Shot Prisoners’ Dilemma,” *Games and Economic Behavior*, 8: 364-377.
- [11] Kandori, M. (1992), “Social Norms and Community Enforcement,” *Review of Economic Studies*, 59: 63-80.
- [12] Kandori, M.(2002), “Introduction to Repeated Games With Private Monitoring,” *Journal of Economic Theory*,102 (1): 1-15.
- [13] Milgrom, P., North, D. and Weingast, B. (1990), “The Role of Institutions in The Revival of Trade: The Law Merchant, Private Judges, and the Champagne Fairs,” *Economics and Politics* , 2: 1-23.

- [14] Okuno-Fujiwara, M. and Postlewaite A. (1995), "Social Norms in Random Matching Games," *Games and Economic Behavior*, 9: 79-109.
- [15] Palomino F, Vega-Redondo F. (1999), "Convergence of Aspirations and (Partial) Cooperation in the Prisoner's Dilemma," *International Journal of Game Theory*, 28 (4): 465-488. The Chicago Bibliography System.
- [16] Robbins E. H., Sarath B. (1998), "Ranking Agencies Under Moral Hazard," *Economic Theory*, 11: 129-155.
- [17] Takahashi, S. (2008), "Community Enforcement When Players Observe Partners' Past Play," Working Paper.

## Novedades

---

### DIVISIÓN DE ADMINISTRACIÓN PÚBLICA

- María del Carmen Pardo, *Los mecanismos de rendición de cuentas en el ámbito ejecutivo de gobierno*, DTAP-245
- Sergio Cárdenas, *Separados y desiguales: Las escuelas de doble turno en México*, DTAP-244
- Sergio Cárdenas, *Obstáculos para la calidad y la equidad: La corrupción en los sistemas educativos*, DTAP-243
- Sergio Cárdenas, Ignacio Lozano, Miguel Torres y Katsumi Yamaguchi, *Identificando beneficiarios de programas gubernamentales*, DTAP-242
- Ma. Amparo Casar, Ignacio Marván y Khemvirg Puente, *La rendición de cuentas y el poder legislativo*, DTAP-241
- Lizbeth Herrera y José Ramón Gil García, *Implementación del e-gobierno en México*, DTAP-240
- Laura Sour, *Gender Equity, Enforcement Spending and Tax Compliance in Mexico*, DTAP-239
- Laura Sour y Fredy Girón, *Electoral Competition and the Flypaper Effect in Mexican Local Governments*, DTAP-238
- Ma. Amparo Casar, *La otra reforma*, DTAP-237
- Judith Mariscal y Federico Kuhlmann, *Effective Regulation in Latin American Countries. The cases of Chile, Mexico and Peru*, DTAP-236

### DIVISIÓN DE ECONOMÍA

- Alejandro López, *Poverty and Commercialization of Non-timber Forest Products*, DTE-486
- Alejandro López et al., *Natural Resource Dependence in Rural Mexico*, DTE-485
- Fausto Hernández, *Obstáculos al desarrollo del sistema financiero en México*, DTE-484
- Rodolfo Cermeño y Benjamín Oliva, *Incertidumbre, crecimiento del producto, inflación y depreciación cambiaria en México*, DTE-483
- Kurt Unger, *Mercado y autoconsumo. Vocación agropecuaria de los municipios de Guanajuato*, DTE-482
- David Mayer, *Divergences and Convergences in Human Development*, DTE-481
- Arturo Antón y Fausto Hernández, *VAT Collection and Social Security Contributions under Tax Evasion: Is There a Link?*, DTE-480
- Eric Zenón y Juan Rosellón, *Expansión de las redes de transmisión eléctrica en Norteamérica: Teoría y aplicaciones*, DTE-479
- María José Roa, *Racionalidad, uso de información y decisiones financieras*, DTE-478
- Alexander Elbittar y Sonia Di Giannatale, *King Solomon's Dilemma: An Experimental Study on Implementation*, DTE-477

## DIVISIÓN DE ESTUDIOS INTERNACIONALES

- Irina Alberro and J. Schiavon, *Shaping or Constraining Foreign Policy?*, DTEI-202
- Jorge Schiavon, *La diplomacia local de los gobiernos estatales en México (2000-2010)*, DTEI-201
- Luis Fernández y J. Schiavon, *La coordinación en la política exterior de Brasil y México*, DTEI-200
- Alejandro Anaya, *Internalización de las normas internacionales de derechos humanos en México*, DTEI-199
- Rafael Velázquez y Karen Marín, *Política exterior y diplomacia parlamentaria: El caso de los puntos de acuerdo durante la LX Legislatura*, DTEI-198
- Jorge Schiavon y Rafael Velázquez, *La creciente incidencia de la opinión pública en la política exterior de México: Teoría y realidad*, DTEI-197
- Jorge Chabat, *La respuesta del gobierno de Calderón al desafío del narcotráfico: Entre lo malo y lo peor*, DTEI-196
- Jorge Chabat, *La Iniciativa Mérida y la relación México-Estados Unidos*, DTEI-195
- Farid Kahhat y Carlos E. Pérez, *El Perú, Las Américas y el Mundo*, DTEI-194
- Jorge Chabat, *El narcotráfico en las relaciones México-Estados Unidos*, DTEI-193
- Jorge Schiavon y Rafael Velázquez, *La creciente incidencia de la opinión pública en la política exterior de México: Teoría y realidad*, DTEI-197
- Rafael Velázquez y Karen Marín, *Política exterior y diplomacia parlamentaria: El caso de los puntos de acuerdo durante la LX Legislatura*, DTEI-198
- Alejandro Anaya, *Internalización de las normas internacionales de derechos humanos en México*, DTEI-199

## DIVISIÓN DE ESTUDIOS JURÍDICOS

- Gustavo Fondevila, *Estudio de percepción de magistrados del servicio de administración de justicia familiar en el Distrito Federal*, DTEJ-47
- Jimena Moreno, Xiao Recio Blanco y Cynthia Michel, *La conservación del acuario del mundo*, DTEJ-46
- Gustavo Fondevila, *"Madrinas" en el cine. Informantes y parapolicías en México*, DTEJ-45
- María Mercedes Albornoz, *Utilidad y problemas actuales del crédito documentario*, DTEJ-44
- Carlos Elizondo y Ana Laura Magaloni, *La forma es fondo. Cómo se nombran y cómo deciden los ministros de la Suprema Corte de Justicia de la Nación*, DTEJ-43
- Ana Laura Magaloni, *El ministerio público desde adentro: Rutinas y métodos de trabajo en las agencias del MP*, DTEJ-42
- José Antonio Caballero, *La estructura de la rendición de cuentas en México: Los poderes judiciales*, DTEJ-41
- Marcelo Bergman, *Procuración de justicia en las entidades federativas. La eficacia del gasto fiscal de las Procuradurías Estatales*, DTEJ-40
- Ana Elena Fierro, *Transparencia: Herramienta de la justicia*, DTEJ-39
- Ana Elena Fierro y Adriana García, *¿Cómo sancionar a un servidor público del Distrito Federal y no morir en el intento?*, DTEJ-38

## DIVISIÓN DE ESTUDIOS POLÍTICOS

- Andreas Schedler, *The Limits to Bureaucratic Measurement. Observation and Judgment in Comparative Political Data Development*, DTEP-224
- Andrea Pozas and Julio Ríos, *Constituted Powers in Constitution-Making Processes. Supreme Court Judges, Constitutional Reform and the Design of Judicial Councils*, DTEP-223
- Andreas Schedler, *Transitions from Electoral Authoritarianism*, DTEP-222
- María de la Luz Inclán, *A Preliminary Study on Pro and Counter Zapatista Protests*, DTEP-221
- José Antonio Crespo, *México 2009: Abstención, voto nulo y triunfo del PRI*, DTEP-220
- Andreas Schedler, *Concept Formation in Political Science*, DTEP-219
- Ignacio Marván, *La revolución mexicana y la organización política de México. La cuestión del equilibrio de poderes, 1908-1932*, DTEP-218
- Francisco Javier Aparicio y Joy Langston, *Committee Leadership Selection without Seniority: The Mexican Case*, DTEP-217
- Julio Ríos Figueroa, *Institutions for Constitutional Justice in Latin America*, DTEP-216
- Andreas Schedler, *The New Institutionalism in the Study of Authoritarian Regimes*, DTEP-215

## DIVISIÓN DE HISTORIA

- Sergio Visacovsky, *"Hasta la próxima crisis". Historia cíclica, virtudes genealógicas y la identidad de clase media entre los afectados por la debacle financiera en la Argentina (2001-2002)*, DTH-68
- Rafael Rojas, *El debate de la Independencia. Opinión pública y guerra civil en México (1808-1830)*, DTH-67
- Michael Sauter, *The Liminality of Man: Astronomy and the Birth of Anthropology in the Eighteenth Century*, DTH-66
- Ugo Pipitone, *Criminalidad organizada e instituciones. El caso siciliano*, DTH-65
- Ugo Pipitone, *Kerala, desarrollo y descentralización*, DTH-64
- Jean Meyer, *Historia y ficción, hechos y quimeras*, DTH-63
- Luis Medina, *La Comanchería*, DTH-62
- Luis Medina, *La organización de la Guardia Nacional en Nuevo León*, DTH-61
- Luis Medina, *El Plan de Monterrey de 1855: un pronunciamiento regionalista en México*, DTH-60
- Mónica Judith Sánchez, *Liberal Multiculturalism and the Problems of Difference in the Canadian Experience*, DTH-59

## Ventas

---

El CIDE es una institución de educación superior especializada particularmente en las disciplinas de Economía, Administración Pública, Estudios Internacionales, Estudios Políticos, Historia y Estudios Jurídicos. El Centro publica, como producto del ejercicio intelectual de sus investigadores, libros, documentos de trabajo, y cuatro revistas especializadas: *Gestión y Política Pública*, *Política y Gobierno*, *Economía Mexicana Nueva Época* e *Istor*.

Para adquirir cualquiera de estas publicaciones, le ofrecemos las siguientes opciones:

VENTAS DIRECTAS:	VENTAS EN LÍNEA:
Tel. Directo: 5081-4003 Tel: 5727-9800 Ext. 6094 y 6091 Fax: 5727 9800 Ext. 6314  Av. Constituyentes 1046, 1er piso, Col. Lomas Altas, Del. Álvaro Obregón, 11950, México, D.F.	Librería virtual: <a href="http://www.e-cide.com">www.e-cide.com</a>  Dudas y comentarios: <a href="mailto:publicaciones@cide.edu">publicaciones@cide.edu</a>

## ¡¡Colecciones completas!!

Adquiere los CDs de las colecciones completas de los documentos de trabajo de todas las divisiones académicas del CIDE: Economía, Administración Pública, Estudios Internacionales, Estudios Políticos, Historia y Estudios Jurídicos.



## ¡Nuevo! ¡¡Arma tu CD!!



Visita nuestra Librería Virtual [www.e-cide.com](http://www.e-cide.com) y selecciona entre 10 y 20 documentos de trabajo. A partir de tu lista te enviaremos un CD con los documentos que elegiste.