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A General Equilibrium Analysis of the Credit Market

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Abstract

I analyse a model of incentive contracts where principals who each possesses the same monitoring technologies, contract with agents from a pool of individuals differing in their wealth endowments. Principals and agents are matched to form partnerships, and the matches are subject to a double-sided moral hazard problems. Agents need to borrow from the principals to finance their projects. In equilibrium, the payoffs to the principals and agents are determined endogenously. The wealthier agents consume higher payoffs, whereas all principals get the same payoff. I further analyse the effects of changes in the monitoring cost and the risk-free interest rate on the optimal monitoring and stock prices.

JEL Codes: D82, J33, J41.
Keywords: two-sided matching, stability, optimal contracts.

Resumen

En este trabajo se analiza un modelo de contratos donde los principales, que poseen tecnologías de supervisión idénticas, contratan agentes que difieren en la riqueza inicial. Principales y agentes se emparejan para formar sociedades, y las parejas están sujetas al problema de riesgo moral. En el equilibrio, los pagos se determinan endógenamente. Los agentes con mayor nivel de riqueza obtienen pagos mayores y todos los principales obtienen el mismo pago. Se analizan también los efectos de los cambios en el costo de supervisión y en la tasa de interés libre de riesgo sobre la supervisión óptima y el precio de los activos de las empresas.

Códigos JEL: D82, J33, J41.
Palabras claves: emparejamiento bilateral, estabilidad, contratos óptimos.

A GENERAL EQUILIBRIUM ANALYSIS OF THE CREDIT MARKET

Kaniṣka Dam*

Abstract

I analyse a model of incentive contracts where principals who each possesses the same monitoring technologies, contract with agents from a pool of individuals differing in their wealth endowments. Principals and agents are matched to form partnerships, and the matches are subject to a double-sided moral hazard problems. Agents need to borrow from the principals to finance their projects. In equilibrium, the payoffs to the principals and agents are determined endogenously. The wealthier agents consume higher payoffs, whereas all principals get the same payoff. I further analyse the effects of changes in the monitoring cost and the risk-free interest rate on the optimal monitoring and stock prices.

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1 Introduction

The credit contracts between the lenders/investors and borrowers/firms are, in general, subject to several market imperfections, among which the informational constraints play an important role. An investor-firm relationship is often subject to the moral hazard problem because of the inability to contract upon the borrower's choice of actions such as effort, investment, etc. Monitoring the borrowers by the lenders aims at ameliorating such moral hazard problem. In reality, many lenders are credit-constrained which results in the inability for the lenders to commit to a pre-specified level of costly monitoring. This gives rise to an additional moral hazard problem on the lenders' side. Such double sided moral hazard problem impedes the implementation of the first-best outcome in a lender-borrower relationship. The traditional agency theory (Grossman and Hart, 1983) analyses the optimal contract loan contracts from a partial equilibrium perspective, treating a lender-borrowe pair as an isolated entity. In this

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approach the outside option of the principal or the agent is taken as exogenously given. But in a market where many principals and agents interact, the outside option of any individual becomes endogenous. Such model thus calls for a general equilibrium approach. The main objective of the current paper is to propose a useful framework to analyse a general equilibrium model of a lender-borrower market.

I consider a market with a finite number of risk-neutral principals (investors or lenders) and agents (firms or borrowers) who are matched to form partnerships. Each agent has a project that costs 1 dollar to accomplish. Agents are differentiated with respect to their wealth endowment. No agent has sufficient wealth to cover the project cost, and hence needs to rely on external financing. A principal can lend money to at most one agent for which she receives state-contingent transfers (or interest payments). After obtaining loan from his financier, each agent chooses a non-verifiable action (say, effort) that determines the probability that the project is successful. Risk-neutrality along with a limited liability constraint (non-negative final wealth) give rise to a moral hazard problem in the agent's action choice. Each principal can choose to monitor her agent to ameliorate the moral hazard problem. Monitoring is costly, and hence no principal is able to pre-commit to a specific monitoring level. I assume that all principals are identical with respect to the cost of monitoring. This induces an additional moral hazard problem in the choice of monitoring. The general equilibrium approach to the market is treated in a two-sided matching game between the principals and the agents.[1] An allocation of the market is a matching rule (that specifies the assignment of an agent to a principal) and a vector of feasible contracts, one for each pair. My equilibrium concept is *stability*, which means that no individual or no principal-agent pair can improve their payoffs under alternative arrangements. Such modeling approach endogenises the principals' outside option, and hence the simultaneously determined equilibrium matching and contracts (consequently, the payoffs) are also endogenous.

First I show that all principals, being identical, consume the same payoffs. Differences in the agents' wealth endowment imply differences in liability in the sense that for an agent with deeper pocket the moral hazard in agent's action is less stringent. Therefore, the principals compete with each other for being matched with the wealthiest agent. Such competition is a Bertrand-like competition in which all agents except the least wealthy one appropriate any incremental surplus in a principal-agent relationship. Second, I show that all agents receive contracts that are the best contracts of them. Finally, a wealthier agent generates higher surplus in a match, and hence obtain strictly higher payoff than his less wealthy counterpart. My equilibrium analysis is amenable to further interesting comparative statics. I show that a decrease in the monitoring cost or a decrease in the risk-free interest rate improves the welfare of each agent, but leaves the situation of each principal unaltered. This is because of the fact that the any incremental surplus due to such changes accrues to the agent. Further, such change in the monitoring cost or risk-free rate increases the stock prices of all firms.

Theoretical literature on the effects of monitoring on optimal credit contracts is not scarce. Besanko and Kanatas (1993) show that substituting external financing by bank credit increases

[1] See Roth and Sotomayor (1990) for extensive analyses of two-sided matching markets.

the firm's stock price in equilibrium. My partial equilibrium model in Subsection 2.3 bears close resemblance with Besanko and Kanatas (1993). Repullo and Suárez (2000) consider a competitive equilibrium model of lender-borrower relationships where some firms may obtain loan from market and bank investors. Banks enjoy comparative advantage in monitoring over the market. They show that a rise in the risk-free rate reduces aggregate investment and widens the the interest rate spread. An important difference between the current model and that of Repullo and Suárez (2000) is that I consider an economy where each individual posses certain amount of market power. Incorporating principal-agent relationship into a two-sided matching model is of recent interest. Ackerberg and Botticini (2002) analyze the landlord-tenant contracts in renaissance Tuscany, and show that traditional view of more risk averse tenants getting fixed rent contracts may be reversed due to the nature of endogenous matching between the landlords and the tenants. A few other works have considered endogenous matching between principals and agents in a contracting environment. Besley and Ghatak (2005) analyze the sorting of motivated agents into mission-oriented firms. Chakraborty and Citanna (2005) show that, due to endogenous sorting effects, less wealth-constrained individuals choose to take up projects in which incentive problems are more important. Dam and Pérez-Castrillo (2006) also characterize a principal-agent economy in the presence of two-sided matching. Von Lilienfeld-Toal and Mookherjee (2007) consider matching between homogeneous principals and heterogeneous agents, and analyze the distributional impacts of a change in the personal bankruptcy law. The differences in the outside option has also been treated by Cantala (2004) where some individuals already have employment, and seek better jobs.

2 The Model

2.1 *Principals and Agents*

In the economy there are two groups of agents, a finite set of risk neutral principals $\mathbf{P} = \{1, \ldots, P\}$ with the generic element p and a finite set of risk neutral agents $\mathbf{A} = \{1, \ldots, A\}$ with the generic element a, where $P = A \geq 2$. The principals are lenders or investors and the agents are borrowers or firms. Principals are ex-ante identical, but agents differ in the amount of (verifiable) wealth endowment (or type). Each agent a has an initial wealth $w_a \in (0, 1)$ which is an element of a finite set $\{w_a\}_{a\in\mathbf{A}}$ with $w_a > w_{a'}$ for $a > a'$. For expositional simplicity, I assume that there is one agent of each type, but the model trivially extends to any general type distributions. A market or economy is denoted by $\xi = \{\mathbf{P}, \mathbf{A}, \{w_a\}_{a\in\mathbf{A}}\}$.

Each agent (or firm) has a project of fixed size 1. Agent's wealth is not sufficient to cover the total cost of the project, and hence each agent a requires to borrow $1 - w_a$ from a principal. Each project yields a high return $y_H > 0$ or a low return $y_L > 0$, where $y := y_H - y_L \in (0, 1)$. The returns are assumed to be independently distributed across projects. An agent may influence the probability of high return through his choice of action $e \in [0, 1]$. I assume that the probability of high return is a linear function of the agent's action, i.e., $p(e) = e$. In other words, each agent directly chooses this probability which is not verifiable by the principals, and hence is not contractible. All agents have the same cost of action $\Psi(e) = e^2/2$. If a principal agrees to lend the required amount to an agent, then she may choose to monitor the agent at a cost $\psi(e-e_0) = (e-e_0)^2/2m$, which is same for all principals. The parameter $m \in (0, 1)$ represents the cost of monitoring with higher values implying a lower marginal cost of monitoring, and e_0 is the agent's action or the probability of high project return if he is not monitored. It is assumed that a principal can induce an agent to choose a specific action e by monitoring him. Thus $e-e_0$ is the monitoring level in a project, which is assumed not to be contractible. Therefore, the non-verifiability of both agent's choice of action and principal's choice of monitoring induce a double-sided moral hazard problem in a principal-agent relationship. The opportunity cost of lending is the risk-free interest rate $r_f > 0$, which is also same for all principals.

2.2 *Matching and Contracts*

A principal-agent pair is formed according to a matching rule $\mu : \mathbf{P} \cup \mathbf{A} \longrightarrow \mathbf{P} \cup \mathbf{A}$ such that (i) $\mu(a) \in \mathbf{P} \cup \{a\}$ for each $a \in \mathbf{A}$, (ii) $\mu(p) \in \mathbf{A} \cup \{p\}$ for each $p \in \mathbf{P}$, and (iii) $\mu(a) = p$ if and only if $\mu(p) = a$ for all $(p, a) \in \mathbf{P} \times \mathbf{A}$. Part (i) and (ii) imply that an individual on one side of the market is either matched with an individual of the other side, or stays unmatched. Part (iii) implies that the matching is 'one-to-one', i.e., an agent can borrow only from one principal and a principal can lend only to one agent. When an arbitrary match (p, a) is formed, the principal and the agent write a binding contract $c(p, a) = (t_H(p, a), t_L(p, a))$ where t_θ specifies the state-contingent transfer made to the principal at state $\theta \in \{H, L\}$. Notice that such a contract may be interpreted as a mix of debt and equity, where t_H is the amount of risk-less debt and $t := t_H - t_L \in [0, y]$ is the total equity issued by the agent. For a principal-agent pair (p, a) matched under a given matching rule μ, i.e., $\mu(a) = p$, and for a contract $c(p, a)$, the expected payoffs to the agent and the principal are respectively given by

$$V_a(c(p, a)) := e(p, a)[y_H - t_H(p, a)] + (1 - e(p, a))[y_L - t_L(p, a)] - w_a - \frac{[e(p, a)]^2}{2},$$

$$U_p(c(p, a)) := e(p, a)t_H + (1 - e(p, a))t_L - \frac{[e(p, a) - e_0(p, a)]^2}{2m} - (1 + r_f)(1 - w_a).$$

Let us first describe the set of feasible contracts for a principal-agent pair (p, a). First, since the agent's action and the principal's monitoring activity are not contractible, they will choose the action and monitoring that must constitute a Nash equilibrium which gives rise to the following *Nash incentive compatibility constraints*.

$$e_0(p, a) = \arg\max_e \left\{ e[y_H - t_H(p, a)] + (1 - e)[y_L - t_L(p, a)] - w_a - \frac{e^2}{2} \right\}, \qquad \text{(ICA)}$$

$$e(p, a) = \arg\max_e \left\{ et_H + (1 - e)t_L - \frac{[e - e_0(p, a)]^2}{2m} - (1 + r_f)(1 - w_a) \right\}. \qquad \text{(ICP)}$$

Each principal p would accept a contract if it satisfies the following *individual rationality constraint* for the principal.

$$e(p,a)t_H + (1-e(p,a))t_L - \frac{[e(p,a)-e_0(p,a)]^2}{2m} - (1+r_f)(1-w_a) \geq v_p, \tag{IRP}$$

where $v_p \geq 0$ is the outside option of principal p that can be obtained from alternative matches. Also, each agent a would accept a contract if it guarantees non-negetive expected payoff to him, i.e.,

$$e(p,a)[y_H - t_H(p,a)] + (1-e(p,a))[y_L - t_L(p,a)] - w_a - \frac{[e(p,a)]^2}{2} \geq 0. \tag{IRA}$$

Finally, *limited liability* requires that the agent cannot have negative final wealth at any state of the nature.

$$y_\theta - t_\theta \geq 0, \quad \text{for } \theta \in \{H, L\}. \tag{LLC}$$

Let $\Omega(p,a)$ be the set of feasible action and tranfers for a pair (p,a), i.e., the contracts that satisfy (ICA), (ICP), (IRP), (IRA) and (LLC). Given a matching μ, let $\mathscr{C}$ be a $(P+A)-$vector of feasible contracts, one for each pair, compatible with μ. Thus $(\mu,\mathscr{C})$ denotes an allocation for the economy ξ.

2.3 *The A-Optimal Contracts*

The optimal contract $c^*(p,a)$ and probability of high return $e^*(p,a)$ for a given pair (p,a), called "A-optimal contract", are obtained by solving, subject to (ICA), (ICP), (IRP) and (LLC), the following maximization problem.

$$\Phi(w_a, v_p) := \max\{V_a(c(p,a))\}, \tag{$\mathscr{A}$}$$

where $\Phi(w_a, v_p)$ is the Pareto frontier for a pair (p,a), which represents the maximum payoff to agent a if principal p is to be guaranteed a minimum amount v_p. The following lemma characterises an A-optimal contract.[2]

LEMMA **1** *The A-optimal contracts have the following properties.*

(a) *For low values of v_p, only (IRP) binds at the optimum. The optimal monitoring level $e^*(p,a) - e_0^*(p,a) = 0$, the optimal equity $t^*(p,a) = 0$, and the optimal debt $t_L^*(p,a) \in (0, y_L)$,*

(b) *for high values of v_p, both (IRP) and (LLC) bind at the optimum. The optimal monitoring level $e^*(p,a) - e_0^*(p,a) > 0$, decreasing in w_a and increasing in v_p; the optimal equity*

[2] I omit the proof of these well-known results from the agency theory. Interested readers should refer to Bolton and Dewatripont (2005).

$t^*(p,a) \in (0,y)$, decreasing in w_a and increasing in v_p; and the optimal debt $t_L^*(p,a) = y_L$.

For low values of the principal's outside option, the limited liability constraint does not bind. Hence risk-neutrality induces the first-best outcome. This is equivalent to the case where agent's action would have been contractible. Thus, optimal monitoring level is zero. And the principal receives a fixed transfer at both states of the nature, i.e., the amount of equity is zero. For high values of v_p, both the participation and limited liability constraints bind, and the provision of incentives becomes costly. In such case, only the second-best contracts are implemented. Naturally, the level of monitoring and total equity decrease with agent's wealth. Often the wealth endowment is taken as a proxy for the agent's attitude towards risk, higher wealth implying lesser risk-aversion. Thus, a higher-wealth agent assumes higher risk by issuing lower equity.

3 The Market Equilibrium

In Subsection 2.3, I have analysed the optimal incentive contracts within a match, where only one principal and one agent are involved. In the above analysis a partnership is not treated as part of the principal-agent market. When there are many principals and many agents, the contracts analyzed in the previous section may not always be optimal since formation of other partnerships imposes externality on the contracts for a particular principal-agent pair. Thus, our main objective is to look at the equilibrium for the market with many principals and agents. We focus on two key issues associated with the market equilibrium. The first important aspect is the nature of the equilibrium payoffs. In the previous section optimal contracts have been solved taking the principal's outside as given. When many principals and agents interact, the outside options of a principal depend crucially on the other partnerships that are being formed in the market. Thus, unlike the standard principal-agent models, a principal's outside option is endogenous. We also see whether it is possible to rank these equilibrium payoffs, the ranking being dependent on agents' wealth endowments and principals' monitoring costs. Next, the optimal contract between a principal and an agent is influenced by the equilibrium matching. Thus, we would like to compare the contracts associated with two distinct matches in the market equilibrium.

The allocations of the market we describe here are endogenous. This endogeneity has two aspects. First, the contract signed by the principals and agents is endogenous. The second aspect is that the matching itself should be endogenous. I will approach this perspective in the same vein as the matching theory. One would require that a reasonable outcome should be immune to the possibility of being blocked by any principal-agent pair (as well as by any single individual). Consider an allocation $(\mu, \mathscr{C})$. If there is a principal-agent pair which can sign a feasible contract such that both the principal and agent are strictly better-off under the new arrangement compared to their situation in the allocation $(\mu, \mathscr{C})$, then such an allocation is not reasonable. This idea corresponds to the notion of stability.

DEFINITION 1 *An allocation* $(\mu, \mathscr{C})$ *is in the market equilibrium or is stable if there do not exist any principal-agent pair* (p, a) *and a feasible contract* $c'(p, a) \in \Omega(p, a)$ *such that, for both* $c(\mu(a), a)$ *and* $c(p, \mu(a))$ *in* $\mathscr{C}$*,* $U_a(c'(p, a)) > U_a(c(\mu(a), a))$ *and* $V_p(c'(p, a)) > V_p(c(p, \mu(p)))$*.*

The above definition asserts that if there is a feasible contract for a pair (p, a) which makes both strictly better-off as compared to the initial allocation $(\mu, \mathscr{C})$, then this pair would "block" the allocation, and hence the allocation is not in equilibrium. Now suppose that in a stable allocation $\mu(a) = p$. Then there cannot be any feasible contract for this particular pair with which they can block the outcome. This immediately implies that all the contracts in the market equilibrium must be (constrained) Pareto optimal. Thus, any contract in the market equilibrium must solve programme $(\mathscr{A})$. Let u_a and v_p be the payoffs to an agent a and a principal p, respectively. Thus in a market equilibrium one must have $u_a = \Phi\left(w_a, v_{\mu(a)}\right)$ and $v_p = \phi\left(w_a, u_{\mu(p)}\right)$, where ϕ is the quasi-inverse of Φ. A property of the stable allocations is that no principal can gain more than any of her counterpart does. That is the payoffs to all principals are equal. The following lemma proves this assertion.

LEMMA **2** *In any stable outcome* $(\mu, \mathscr{C})$*, all principals get the same payoff.*

The above lemma implies an important feature of the set of stable allocations of a two-sided matching game, namely, "the equal treatment of equals". It also allows me to write that $v_p = v$ for all $p \in \mathbf{P}$. This property would no longer be valid if we consider heterogeneity among the principals. All the homogeneous principals compete for the best agent to be matched with. This generates a Bertrand-like outcome in which all the incremental surplus accrues to the agents, pushing the payoff to each agent down to her outside option. This implies that each agent receives his A-optimal contract subject to a common value v of the outside option of the principals. The following proposition characterizes the set of stable allocations.

PROPOSITION 1 *An allocation* $(\mu, \mathscr{C})$ *is stable or in the market equilibrium if and only if the following three conditions hold.*

(a) *All principals and all agents are matched;* (b) $v \in [0, \phi(w_1, 0)]$ *for all* $p \in \mathbf{P}$*; and* (c) $u_a = \Phi(w_a, v)$ *for all* $a \in \mathbf{A}$ *with* $u_a > u_{a'}$ *for* $a > a'$*.*

Proposition 1(a) asserts that there is full employment in the economy. Notice that, given the same size of both sides of the market and restriction of the matching to be one-to-one, in any stable allocation all principals and agents are matched. Otherwise, an unmatched agent and an unmatched principal can easily block the allocation with a feasible contract that generates strictly positive payoffs to both. Part (b) describes the range of payoffs each principal can obtain in the market equilibrium. Since all principals consume the same expected payoff, they can obtain as low as zero but no more than $\phi(w_1, 0)$ which is the maximum payoff that can be consumed by the principal matched with the least wealthy agent if he were to receive zero payoff. It is worth noting that the least wealthy agent determines the payoff to each principal in a stable allocation. An important difference of the current approach with the standard competitive equilibrium approach is as follows. In the competitive equilibrium approach, the optimal contract for an agent is solved subject to the zero profit constraints of the principals. Since the outside option of each principal is endogenous, in a stable allocation the principals may obtain strictly positive payoffs, namely up to $\phi(w_1, 0)$. Thus, the payoff to each principal corresponding to each stable matching can take infinitely many values in a closed interval of the positive orthant of the real line. This is a typical feature of a two-sided matching game. There is an allocation where $v = 0$, which is the worst allocation for all the principals and the best for all the agents. On the other hand, $v = \phi(w_1, 0)$ is the allocation that is worst for all the agents and best for all the principals. Thus, the predictive power of the equilibrium allocations depends on which stable allocation is selected. As discussed earlier, Proposition 1(c) asserts that each agent receives his A-optimal contract with respect to the common payoff v to each principal. Thus, in a firm managed by a wealthier agent there is less monitoring and lower equity issued. Further, a wealthier agent consumes higher equilibrium payoff since otherwise this agent along with the principal matched with the less wealthy agent (getting higher payoff) can form a blocking pair, which contradicts the stability of the initial allocation.

One important fact regarding a stable allocation is worth mentioning. In a standard principal-agent model, the principal is able to make a take-it-or-leave-it offer to the agent since the agent's outside option is zero. For an arbitrary pair (p, a), call such contract the *P-optimal contract*. The Bertrand-like competition for the wealthier agents implies that the only set of contracts that emerges in the market equilibrium are the A-optimal contracts, in which the optimal probability of success in each match is higher than that in a P-optimal contract. Hence, stability induces the most efficient set of contracts.[3] This aspect of productive efficiency is an immediate consequence of a general equilibrium model of a principal-agent market.

4 Comparative Statics

In this section I study the general equilibrium effects of the changes in the monitoring cost and the risk-free rate on the market equilibrium. Often the lenders are liquidity-constrained, and hence that leads to sub-optimal level of monitoring due to a potential moral hazard problem on the investor-side of the market. A lower monitoring cost implied by an increase in the value of m relaxes such constraints, and should have favorable impact on welfare. On the other hand, a decrease in the risk-free rate lowers the opportunity cost of lending. It would have been interesting to see the impact of such changes on the optimal contracts had the loan size been variable. Yet with fixed loan size, interesting comparative statics results are obtained. Notice that if the first-best outcomes emerge in a stable allocation, then either of the above changes would not have any impact on the market equilibrium.

PROPOSITION 2 *For each match in a market equilibrium,* (a) *both a decrease in the monitoring cost and a decrease in the risk-free rate increase the payoff to each agent, but that of*

[3] See Dam and Pérez-Castrillo (2006) for a detailed discussion on efficiency.

each principal remains unaltered; (b) *a decrease in the monitoring cost increases the monitoring level and increases the firm's stock price; and* (c) *a decrease in the risk-free rate decreases the monitoring level and increases the firm's stock price.*

The Bertrand-like feature of competition for the better agents leads to the above important results. In each match, any incremental surplus due to a decrease in the monitoring cost or a decrease in the risk-free rate accrues to the agent, and not to the principal. A lower monitoring cost obviously increases the level of monitoring since it is marginally less costly. A firm's stock price is the present value of its expected cash flow, which is given by $q_a^* = [e(y-t) - w_a]/(1+r_f)$. A decrease in the monitoring cost has two effects through which it influences the stock price. First, it increases the probability of the high project return e. Second, it decreases the equity t, giving the agent higher claim on the final output. Both effects together increase the firm's stock price. On the other hand, a lower opportunity cost of lending leaves lower incentives for a principal to monitor her agent, and hence the monitoring level decreases. But it gives higher control rights to the agent through a decrease in t which in turn increases the probability of high return. Therefore, the stock price increases.

5 Conclusions

In this paper I have considered a two-sided model of principal-agent matching, and characterised the set of stable allocations. As opposed to the traditional partial equilibrium models of principal-agent relationships, matching between principals and agents generates a general equilibrium model taking into account the contract externality imposed on a particular relationship by the other partnerships being formed in the market. My model can be seen as a generalisation of the 'assignment game' of Shapley and Shubik (1971), where buyers and sellers are matched to trade indivisible goods in a market where matches are not subject to informational asymmetries. Using the restriction of limited liability should be taken as a very simple way to tackle the incentive problems. The findings can easily be applied to various other principal-agent economies that include risk-averse agents. As shown in Demange, Gale and Sotomayor (1986), the set of stable allocation can also be implemented via a mechanism similar to the ascending price auction.

I have assumed that the principals are identical. Although some of the conclusions of my analyses can immediately be extended to apply to economies with heterogenous principals, the characteristics of the contracts signed in the stable allocations can be quite different from those identified in the current work. On the one hand, the results that the contracts signed in a stable allocation are optimal and the matching itself is efficient (in the sense that it maximises the total surplus) hold also in a framework with heterogenous principals. On the other, there is no unique way to model the differences among the principals and the contracts will be different depending on the type of heterogeneity one would like to introduce. Further, introduction of coalitional externalities is a more interesting but difficult task. Often the action choice of one agent influences the payoffs to the principal and the agent in some other match. One such example is when the firms' projects are correlated. There is no unique way to define stability in this case. A proper definition of equilibrium and the characterisation of the stable allocations would be an interesting future research agenda.

Appendix

Proof of Lemma 1.[4] Since the expected payoffs to the agent and the principal are strictly concave in effort, both incentive compatibility constraints (ICA) and (ICP) can be replaced by the first order conditions of the maximisation problems as

$$e_0(p,a) = y - t(p,a), \tag{ICA$'$}$$
$$e(p,a) - e_0(p,a) = m\,t(p,a). \tag{ICP$'$}$$

Notice that the incentive compatibility and the limited liability at $\theta = L$ together imply the limited liability at $\theta = H$, and hence this constraint can be ignored throughout. Further, the participation constraint (IRP) binds at the optimum, otherwise the agent can reduce the transfers at both states a little bit, the principal would still accept the contract and the agent would be better-off. Substituting for $e(p,a)$ and $e_0(p,a)$ in the expressions for expected utilities of agent a and principal p, the maximisation problem reduces to the following.

$$\max_{t,t_L} \frac{1}{2}[y - t(p,a)]^2 - \frac{1}{2}m^2[t(p,a)]^2 + y_L - t_L(p,a) - w_a, \tag{1}$$

$$\text{subject to} \quad t(p,a)[y - t(p,a)] + \frac{1}{2}m[t(p,a)]^2 + t_L(p,a) - (1+r_f)(1-w_a) = v_p, \tag{2}$$

$$y_L - t_L(p,a) \geq 0. \tag{3}$$

Two cases might occur. First, the limited liability constraint does not bind at the optimum. The the first-best outcome emerges, which is obtained by maximising the total surplus net of v_p. This implies $t^{FB}(p,a) = 0$, i.e., the principal receives a fixed payment $t_H^{FB}(p,a) = t_L^{FB}(p,a)$ at both states of the nature. At the first best, since the agent's action can be contracted upon, one has zero monitoring, i.e., $e^{FB}(p,a) - e_0^{FB}(p,a) = m\,t^{FB}(p,a) = 0$. The optimal probability of success is given by $e^{FB}(p,a) = y < 1$, and the binding (IRP) determines the transfer at state H as $t_L^{FB}(p,a) = v_p + (1+r_f)(1-w_a) > 0$. The non-binding (LLC) implies that $v_p < y_L - (1+r_f)(1-w_a)$. Thus for low values of v_p, the first-best contracts are optimal.

The second case is where (LLC) binds at the optimum. Then the second-best (SB) outcome is achieved. The binding (LLC) determines the the transfer at state H as $t_L^{SB}(p,a) = y_L$. And the optimal equity is given by

$$t(p,a)[y - t(p,a)] + \frac{1}{2}m[t(p,a)]^2 + y_L - (1+r_f)(1-w_a) = v_p. \tag{4}$$

[4] I plan to omit the proof of this standard result from the main paper. This is intended only for the reviewers.

Given that $e(p,a) - e_0(p,a) = mt(p,a)$, differentiating the above expression and the optimal monitoring level with respect to w_a and v_p we get Lemma 1(b). $\|$

Proof of Lemma 2. Suppose that in a stable allocation $(\mu, \mathscr{C})$, one has $U_p(c(p,\mu(p))) > U_{p'}(c(p',\mu(p')))$. I show that there exists a contract $c'(p',\mu(p)) \in \Omega(p',\mu(p))$ such that with this contract the pair $(p',\mu(p))$ blocks the allocation $(\mu,\mathscr{C})$. Given any contract $c = (t_H, t_L)$, consider the contract $c' = c - \varepsilon = (t_H - \varepsilon, t_L - \varepsilon)$ with $\varepsilon > 0$ but very small. Notice that the incentive constraints are unaltered both under c and c'. Hence one has (i) $U_{p'}(c'(p',\mu(p))) = U_p(c(p,\mu(p))) - \varepsilon > U_{p'}(c(p',\mu(p')))$ and (ii) $V_{\mu(p)}(c'(p',\mu(p))) \geq V_{\mu(p)}(c(p,\mu(p))) + \varepsilon > V_{\mu(p)}(c(p,\mu(p)))$. This contradicts the stability of $(\mu,\mathscr{C})$. $\|$

Proof of Proposition 1. First I show that conditions (a)-(c) are necessary conditions for an allocation to be stable. (a) Suppose that $(\mu,\mathscr{C})$ is stable, and there is one agent a is unmatched. Then there must be a principal p unmatched, both consuming zero payoffs. Consider the A-optimal contract $c(w_a, 0)$, the contract between a and p in which $v_p = 0$. There exists a contract $c'(p,a) \in \Omega(p,a)$ such that $c'(p,a) = c(w_a,0) - \varepsilon$ at which $V_a(c'(p,a)) = \Phi(w_a,0) - \varepsilon > 0$ and $U_p(c'(p,a)) \geq \varepsilon > 0$. Thus the pair (p,a) blocks $(\mu,\mathscr{C})$, which is a contradiction. (b) According to Lemma 2, in a stable allocation, $v_p = v$ for all $p \in \mathbf{P}$. Suppose first that $v < 0$ for all p matched in a stable allocation $(\mu,\mathscr{C})$. Then each principal is better-off staying unmatched, and hence the allocation is blocked by any individual principal. Now suppose that $v > \phi(w_1, 0)$, where w_1 is the wealth level of the least wealthy agent. Notice that, by the definition of the Pareto frontier, $u_a = \Phi(w_a, \phi(w_a, u_a)$ for any agent a. Hence, the above implies that $\Phi(w_1, v) < \Phi(w_1, \phi(w_1, 0) = 0$ since $\Phi(w_1, v)$ is strictly decreasing in v. This is not possible in a stable allocation because of the constraint (IRA). (c) This condition asserts that all agents must get his A-optimal contract subject to the common payoff v to each principal. I have argued that any contract in a stable allocation must be Pareto optimal, i.e., it must solve $(\mathscr{A})$. Notice that $\Phi(w_a, v)$ is strictly increasing in w_a which implies that $u_a = \Phi(w_a, v) > \Phi(w_{a'}, v) = u_{a'}$ if $a > a'$.

I now prove that any allocation $(\mu,\mathscr{C})$ satisfying (a)-(c) is indeed stable. Consider any matched pair (p,a) under μ. Such election is possible because of (a). Clearly, (p,a) cannot block the allocation with any feasible contract in $\Omega(p,a)$. Indeed, there is no contract such that p gets more than v and a gets more than $\Phi(w_a, v)$ since the contract $c(p,a)$ is optimal by (c). $\|$

Proof of Proposition 2. Notice that the second-best equilibrium payoffs to principal p and agent a are respectively given by

$$u_a = \frac{1}{2}[y - t^{SB}(p,a)]^2 - \frac{1}{2}m^2[t^{SB}(p,a)]^2 - w_a, \tag{5}$$

$$v_p = t^{SB}(p,a)[y - t^{SB}(p,a)] + \frac{1}{2}m[t^{SB}(p,a)]^2 + y_L - (1+r_f)(1-w_a). \tag{6}$$

Further the equilibrium stock price of a firm a is the present value of the expected net cash flow, i.e.,

$$q_a^* = \frac{e^{SB}\left[y - t^{SB}\right] - w_a}{1 + r_f}. \tag{7}$$

Differentiation of (4) and the above three expressions with respect to m and r_f gives the desired results. ‖

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